ELASTIC-PLASTIC ANALYSIS OF A FINITE INTERNALLY CRACKED PLATE UNDER VARIOUS ASSUMPTIONS ON COHESIVE FORCE AT CRACK TIPS

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ABSTRACT

In this paper a modified Dugdale model for a finite internally cracked plate is discussed, in which the cohesive force is taken of variable distributions along plastic zones ahead of crack tips. Using the method of superposition and the results from our previous works (Chen, 1981, 1983), we find the proposed problem can be easily solved.

KEYWORDS

Crack problem; elastic-plastic analysis; modified Dugdale model; finite cracked plate; numerical technique.

INTRODUCTION

An early investigator (Dugdale, 1960) proposed an elastic-plastic model for an infinite internally cracked plate. In the model, he supposed that the cohesive force σ_c is constant along the plastic zone near the crack tip and the SIF value is equal to zero at the fictitious crack tip. Then, using these conditions, we can obtain the COD value and the plastic zone size.

In this paper the Dugdale model is extended to the following cases: (a) the cracked plate is finite, (b) the cohesive force is taken as $\sigma_c = \sigma_t$, $\sigma_c = \sigma_t|x|/a$ and $\sigma_c = \sigma_t(x/a)^2$ in the interval $c < |x| < a$, respectively, where σ_t is the yielding stress. Using the results of previous works (Chen, 1983; Harrop, 1978), all the derivations and calculations mentioned below can be easily completed.

ANALYSIS

Consider a square plate with a central crack of length 2c subjected to the uniform stress S on the upper and lower sides of the outer contour (Fig. 1). The cohesive force acting along the line $c<|x|<a$ is $\sigma_c = \sigma_t f(x)$, where σ_t is the yielding stress and f(x) is a given function. Clearly, this problem can be separated into three boundary value problems, namely, I , II and III as shown in Fig. 1. The problems I and III for the finite cracked plate can be easily solved by a proposed method (Chen, 1983). Consequently, now we concentrate our attention upon the problem II for an infinite plate. If these three crack problems are solved, the applied stress S can be found as follows

$$S = (K_{I,II} - K_{I,III})\,\sigma_t / K_{I,I} \qquad (1)$$

where $K_{I,I}$, $K_{I,II}$ and $K_{I,III}$ are the SIF value at the crack tip for the problems I , II and III respectively (Fig. 1).

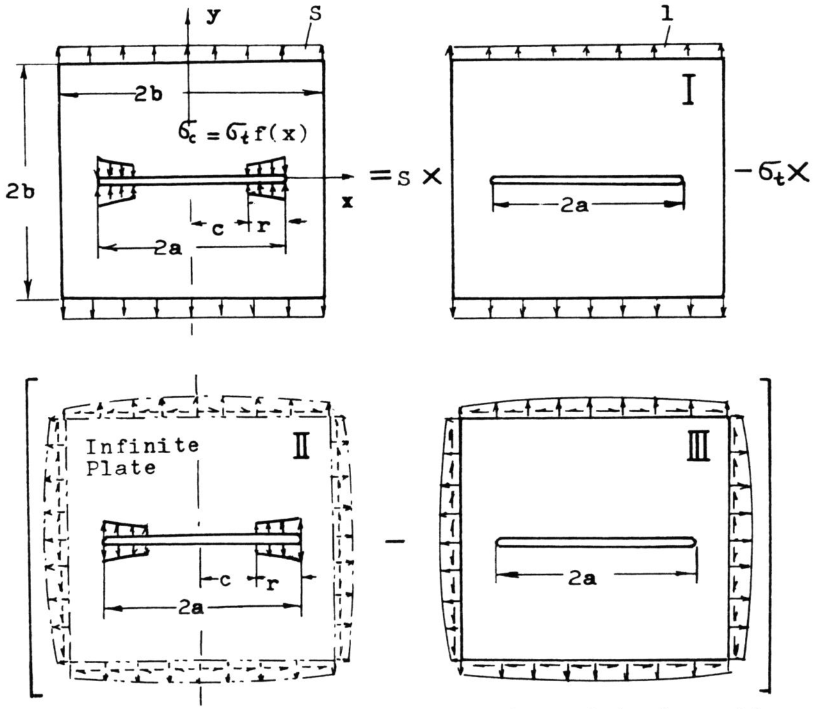

Fig. 1. A scheme showing how the original problem is separated into three boundary value problems I , II and III .

In problem II , using the conformal mapping

$$z = \omega(\zeta) = a(\zeta + \zeta^{-1})/2 \qquad (2)$$

we can express the stress and displacement components as

$$\sigma_{xx} + \sigma_{yy} = 4\,\mathrm{Re}\,\Phi(\zeta)$$

$$\sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} = 2\left[\overline{\omega(\zeta)}\,\Phi'(\zeta)/\omega'(\zeta) + \Psi(\zeta)\right] \qquad (3)$$

$$2G(u+iv) = \kappa\,\varphi(\zeta) - \omega(\zeta)\,\overline{\varphi'(\zeta)}/\overline{\omega'(\zeta)} - \overline{\psi(\zeta)} \qquad (4)$$

where $\varphi(\zeta)$ and $\psi(\zeta)$ are two analytical functions, $\Phi(\zeta) = \varphi'(\zeta)/\omega'(\zeta)$, $\Psi(\zeta) = \psi'(\zeta)/\omega'(\zeta)$, $\kappa = (3-\nu)/(1+\nu)$, ν is the Poisson's ratio and G is the shear modulus of elasticity.

In this paper, the following three cases: (A) $\sigma_c = \sigma_t$, $c<|x|<a$, (B) $\sigma_c = \sigma_t |x|/a$, $c<|x|<a$, (C) $\sigma_c = \sigma_t (x/a)^2$, $c<|x|<a$, are discussed. After using the previous results (Harrop, 1978) and completing some manipulations, we get

$$\varphi(\zeta) = -\frac{a\Theta}{\pi\zeta} + \frac{1}{2\pi i}\left\{(z-c)f_1(\zeta) + (z+c)f_2(\zeta)\right\} \qquad \text{for case (A)} \qquad (5a)$$

$$\varphi(\zeta) = \frac{\sqrt{a^2-c^2}}{4\pi}\left(\zeta - \frac{1}{\zeta}\right) + \frac{z^2-c^2}{4\pi a i}\left(f_1(\zeta) - f_2(\zeta)\right) \qquad \text{for case (B)} \qquad (5b)$$

$$\varphi(\zeta) = -\frac{a\Theta}{12\pi}\left(\frac{3}{\zeta} + \frac{1}{\zeta^3}\right) + \frac{a\sin 2\Theta}{24\pi}\left(\zeta - \frac{1}{\zeta}\right) + \frac{1}{6\pi a^2 i}\left\{(z^3-c^3)f_1(\zeta) + (z^3+c^3)f_2(\zeta)\right\}$$

$$\text{for case (C)} \qquad (5c)$$

$$\psi(\zeta) = \varphi(\zeta) - \frac{\zeta(\zeta^2+1)}{\zeta^2-1}\varphi'(\zeta) \qquad \text{for cases (A),(B),(C)} \qquad (6)$$

$$K_I = 2\sqrt{\pi/a}\;\varphi'(1) \qquad \text{for cases (A),(B),(C)} \qquad (7)$$

where

$$\Theta = \mathrm{Arc}\cos(c/a)$$

$$f_1(\zeta) = \mathrm{Ln}(\zeta - \exp(i\Theta)) - \mathrm{Ln}(\zeta - \exp(-i\Theta))$$

$$f_2(\zeta) = \mathrm{Ln}(\zeta + \exp(i\Theta)) - \mathrm{Ln}(\zeta + \exp(-i\Theta)) \qquad (8)$$

NUMERICAL RESULTS

Finally, after some rather complicated calculations, in the case of a square cracked plate we can express the obtained numerical results as

$$\frac{S}{\sigma_t} = f_1\left(\frac{a}{b}, \frac{r}{a}\right)\frac{2\theta}{\pi} \tag{9}$$

$$\frac{2G v(c, 0^+)}{\sigma_t} = f_2\left(\frac{a}{b}, \frac{r}{a}\right)\frac{(1+\kappa)a}{\pi}\cos\theta \ln(\sec\theta) \tag{10}$$

$$\frac{r}{a} = f_3\left(\frac{a}{b}, \frac{S}{\sigma_t}\right) \tag{11}$$

$$\frac{S}{\sigma_t} = f_4\left(\frac{c}{b}, \frac{a}{b}\right) \tag{12}$$

where a,b,c and r are the dimensions of the cracked plate as shown in Fig. 1, θ = Arc cos(c/a), 2v(c, 0^+) is the COD value. Owing to the limited space, only the numerical results for cases (A) and (B) are shown in Fig. 2 and 3, respectively.

Obviously, the curves shown in Fig. 2(c) and 2(d) are similar to those obtained by the previous investigators (Dugdale, 1960; Nisitani, 1978). In Fig. 2(d), a relation between the applied stress S and the plastic zone size (a-c) can be easily found. In addition, we also found that, in this case of Dugdale model instability in finite cracked plate can not occur. Furthermore, from Fig. 3(d) we see that there exists a maximum value in the S/σ_t versus a/b relation. That is to say, instability of plastic zone propagation may occur in some circumstances.

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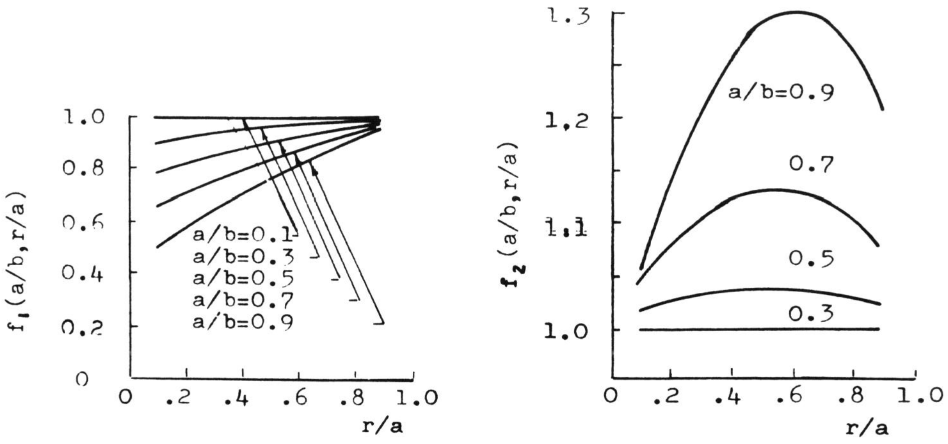
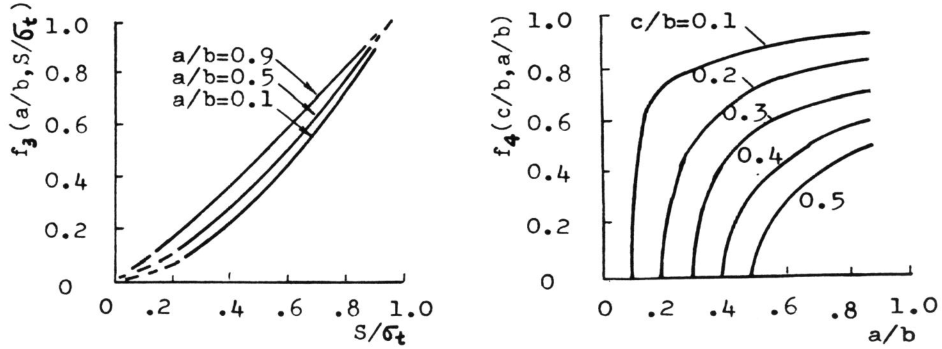
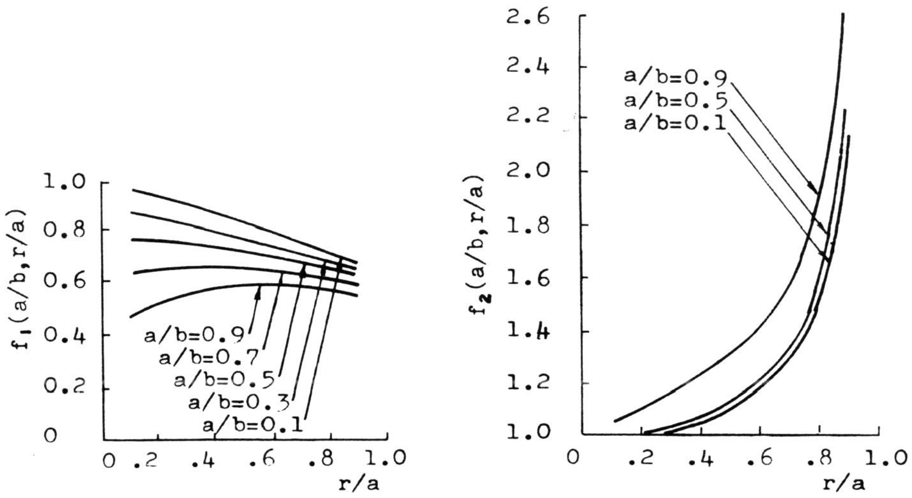


(a) f_1(a/b,r/a) values. (b) f_2(a/b,r/a) values.

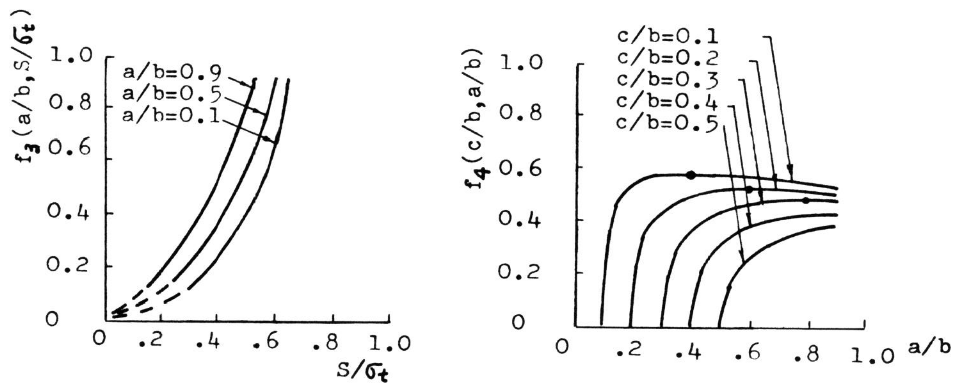


(c) f_3(a/b,S/σ_t) values. (d) f_4(c/b,a/b) values.

Fig. 2. Numerical results for the case (A) $\sigma_c = \sigma_t$.



(a) $f_1(a/b,r/a)$ values.   (b) $f_2(a/b,r/a)$ values.



(c) $f_3(a/b,S/\sigma_t)$ values.   (d) $f_4(c/b,a/b)$ values.

Fig. 3. Numerical results for the case (B) $\sigma_c=\sigma_t|x|/a$.