# EFFECT OF ELASTIC ANISOTROPY ON THE BRITTLE FRACTURE OF A POLYCRYSTALLINE SOLID

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ABSTRACT

A dislocation queuing model of fracture stress in an anisotropic brittle material is proposed. Using continuous distribution of dislocations and a crack initiation criterion at the tip of the dislocation pile up a modified Hall-Petch type relationship is obtained. Consequences of this relation for metals and their alloys are discussed.

KEYWORDS

Fracture stress; brittle material; elastic anisotropy; dislocation queuing; continuous distribution of dislocations.

## INTRODUCTION

Due to the natural inhomogeneities and local character of fracture initiation, a theory of strength of brittle fracture can be based on the concept of weakest link and/or on the critical concentration of defects. As is well known in case of many solids local plastic mechanisms play a role in crack initiation. The source of fracture can thus be associated with interaction of dislocation produced when load is applied rather than with other defects already present in the material.

Theory of dislocation queuing has been applied by many workers (Petch, 1953; Stroh, 1957; Cottrell, 1958) for explaining brittle fracture of polycrystals. Several years ago, Armstrong and Head (1965) proposed a dislocation queuing model for brittle fracture whereby the dislocation driven by an applied stress pile up on a grain boundary. Crack initiation and brittle fracture results when the leading dislocation is brought within a Burgers vector of the locked dislocation in the pile up. Their study revealed that the fracture stress of material depends not only on the grain size (in the familiar Hall-Petch fashion) but is also dependent on how strongly the dislocations are repelled or attracted by the image forces at the grain boundary. These image forces are however dependent on the elastic anisotropy of the material. Therefore, the familiar Hall-Petch relation must be modified to take into account the elastic anisotropy. Their treatment was essentially numerical. We present an analytical treatment and show that the anisotropy effect is relatively weak, but must be considered for superfine grain sizes. For such materials we propose a modified Hall-Petch type relationship.

## Proposed Model

Let us consider the brittle solid as an aggregate of randomly oriented grains. In some of these grains a dislocation pile up has formed under a constant stress as shown in Fig. 1.

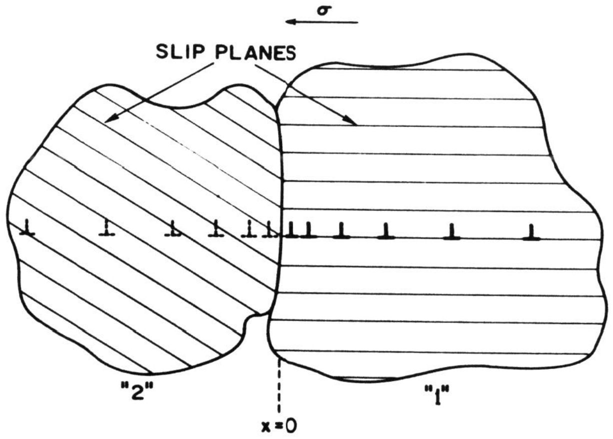


Fig. 1. Dislocation model used in the calculation. "1" and "2" refer to the two grains considered. Image dislocations are shown in grain "2".

Consider the plane x=0 as the grain boundary across which the orientation of the crystal changes. There will be an elastic interaction with the dislocations in "1" due to (a) the presence of the grain boundary and (b) elastic anisotropy. In such a situation the equations of equilibrium for n dislocations piled up at x=0 due to an applied stress $\sigma$ are:

$$\sum_{i=1}^{n} \frac{1}{x_j - x_i} + K \sum_{i=1}^{n} \frac{1}{x_j + x_i} = \frac{\sigma}{\pi A} \tag{1}$$

$$j = 1,2,\ldots n$$

Physically the strength of the image dislocations are K times the strength of the real dislocations K is a factor taking into account the anisotropy (K=1 if no anisotropy) where

$$K = \frac{G_1 - G_2}{G_1 + G_2}$$

$$\text{and} \quad A = \frac{Gb}{2\pi} \quad \text{(screw dislocations)}$$



$$\text{or} \quad A = \frac{Gb}{2\pi(1-\nu)} \quad \text{(edge dislocations)}$$

where b is the magnitude of the Burger's vector, G is the equivalent shear modulus $C_{44}H$, H is proportional to the energy factor, and $\nu$ is the Poissons' ratio. Equation (1) is exactly true only for screw dislocations (see Armstrong and Head, 1965). For our purpose (see below) we need to know the analytical expression for $x_1$, the distance of the leading dislocation from the grain boundary as a function of the applied stress. This is done by solving (1). An explicit solution of (1) can only be obtained in the continuum distribution approximation such that $f(x)\delta x$ dislocations are contained within any interval $\delta x_i$ leading to the integral equation:

$$\int_0^L \frac{f(x)dx}{x_0 - x} + K \int_0^L \frac{f(x)dx}{x_0 + x} = \frac{\sigma}{A} \qquad 0 < x < L \tag{2}$$

Barnett (1967) has given a solution for f(x) as

$$f(x) = \frac{\sigma}{\pi A} \left(\frac{2}{1-K}\right)^{1/2} \sinh\left[\left(\frac{1}{\pi} \cos^{-1} K\right) \cosh^{-1} \frac{L}{x}\right] \tag{3}$$

As in case of a regular pile up f(x) is infinite at x=0, but equal to zero at x=L. Most of the properties of the pile up under consideration can now be obtained from a knowledge of f(x). The length of the pile up can be obtained from the relation

$$n = \int_0^L f(x)dx = \frac{\sigma L}{\pi A} \frac{\cos^{-1} K}{(1-K^2)^{1/2}}$$

$$\text{or} \quad L = \frac{n\pi A}{\sigma} \frac{(1-K^2)^{1/2}}{\cos^{-1} K} \tag{4}$$

To compare this result with the numerical result obtained by Armstrong and Head (1965) we expand $\cos^{-1} K$ in a series in K and obtain

$$L = \frac{2nA}{\sigma} \left[1 + \frac{2}{\pi} K + 0(K^2) + \ldots\right] \tag{5}$$

which should be compared with the numerical result (Armstrong and Head, 1965)

$$L = \frac{2nA}{\sigma} \left[1 + 0.9K\right] \quad \text{for large } n \tag{6}$$

and the extrapolated expression of Chou (1965)

$$L = \frac{2nA}{\sigma} \left[1 + \left(\frac{\pi}{2} - 1\right)K\right] \tag{7}$$

We now proceed to calculate $x_1$, the position of the first mobile dislocation as a function of n and K. For this purpose it is convenient to write equation (3) in the equivalent form

$$f(x) = \frac{\sigma}{2\pi A} \left(\frac{2}{1-K}\right)^{1/2} \left(\frac{L}{x}\right)^{a} \{(1+y)^{a} - (1-y)^{a}\} \tag{8}$$

where $$a = \frac{1}{\pi}\cos^{-1}K$$

$$y = \left[1-\left(\frac{x}{L}\right)^2\right]^{1/2} \quad (9)$$

Near $x_1$, $x \ll L$ for sufficiently large n, therefore, $y \cong 1$

equation (8) becomes $$f(x) \cong \frac{\sigma}{2\pi A}\left(\frac{2}{1-K}\right)^{1/2}\left(\frac{2L}{x}\right)^a \quad (10)$$

The first dislocation position, $x_1$, is given by

$$\int_0^{x_1} f(x)dx = c \quad \text{where } c \cong 1$$

or

$$\frac{\sigma}{2\pi A}\left(\frac{2}{1-K}\right)^{1/2}(2L)^a \frac{x_1}{1-a} = c \quad (11)$$

provided $a<1$

$$x_1 = \left\{\frac{c(1-a)}{\frac{\sigma}{2\pi A}\left(\frac{2}{1-K}\right)^{1/2}}(2L)^{-a}\right\}^{\frac{1}{1-a}} \quad (12)$$

(As stated before, $c \cong 1$, $a<1$, $a = \frac{1}{\pi}\cos^{-1}K$, $K = \frac{G_1 - G_2}{G_1 + G_2}$

Writing L in terms of n

$$x_1 \sim n^{-\left(\frac{a}{1-a}\right)}$$

$$\sim n^{-(1-1\cdot27K+O(K^2))} \quad (13)$$

which should be compared with the numerical value (Armstrong and Head, 1965) for $x_1$

$$x_1 \cong n^{-(1-1.1K)} \quad \text{for large n.} \quad (14)$$

Finally we obtain the Hall-Petch relationship in the usual way by obtaining the applied stress for which $x_1 = b$ and obtain the Hall-Petch type relation as

$$\sigma = \sigma_o + K_o d^{-a} \quad (15)$$

where

$$K_o = 2\pi A\, c(1-a)b^{a-1}\left(\frac{2}{1-k}\right)^{-1/2}$$

and pile up length L identified with grain radius d/2.



Equation (15) is very similar to Hall-Petch relation. The exponent of d is, however, now seen to be $-a$ ($a=1/\pi \cos^{-1}K$) instead of $-1/2$. Depending on the value of K, a will vary from 0 to $-1$. Its exact value will depend on the anisotropy of the material. Preliminary calculations indicate that in most materials the value of a is within 15% of $-1/2$. In view of the simplified nature of the model and approximations used in the calculations, the quantitative predictions of the model are not expected to be in accurate accord with experiments and no such claim is implied. What we do claim is that our calculation implies a departure of the exponent of grain size in Hall Petch relation from $-1/2$ to a value between 0 and $-1$, depending on the elastic anisotropy.

This calculation is dependent upon the assumptions made and as such represents an initial attempt to incorporate the effects of elastic anisotropy on a model based on a continuous distribution of dislocations. Even though the model is strictly valid only for screw dislocations, it can be extended to edge dislocations with minimal error. The results obtained here can be viewed as a rough approximation to the correct solution based upon a model of two welded anisotropic half spaces.

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