APPLICATION OF LINE-SPRING MODEL TO A STIFFENED CYLINDRICAL SHELL CONTAINING AN AXIAL PART-THROUGH CRACK

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ABSTRACT

In this paper the line-spring model developed by Rice and Levy is used to obtain an approximate solution for a stiffened cylindrical shell containing an axial part-through surface crack. It is assumed that the stiffened shell is subjected to an internal pressure. To formulate the shell problem, Reissner's theory is used to account for the effects of the transverse shear deformations. The bending stiffness of the stiffener is assumed to be zero. The stre-s intensity factor is tabulated for open end and closed end shell for different shell and stiffener geometries.

KEYWORDS

Line-spring; cylindrical shell; part-through crack; stiffened cylindrical shell.

INTRODUCTION

Because of their potential applications to the strength and failure analysis of such structurally important elements as pressure vessels, pipes and great variety of aerospace and hydrospace components, in recent past the crack problems in shells have attracted considerable attention. These cracks in the shells generally start from surface scratches and in time they become through cracks.

Attaching a stiffener to a structure may inhibit crack growth, and thus increase the overall strength of the structure. The results found by Yahsi (1983) for loosely stiffened shell containing an axial part through crack clearly demonstrate that. Because of these reasons, from a practical point of view a solution for a stiffened cylindrical shell containing an axial part-through crack may be extremely important.

In the solution of this problem, the line-spring model developed by Rice and Levy (1972) is used to obtain approximate results. The results obtained from the model for plates and shells containing a part-through surface crack

(Delale and Erdogan 1981, 1982a) compare well with the solutions found from the finite element and the alternating methods (Raju and Newman 1979a, 1979b; Newman 1978, 1979). Another advantage of this model is its applicability to a great variety of shell structures with a relatively small computational effort.

In formulating the problem the solutions of a close end and open end shell without the crack for the internal pressure are obtained and the problem is reduced to a perturbation problem in which the self-equilibrating crack surface tractions are the only external loads.

To formulate the shell problem, Reissner's (Reissner and Wan 1969, Naghdi 1956) shell theory is used to account for the effects of the transverse shear deformations, and it is assumed that the cylindrical shell is thin-walled and shallow.

The bending stiffness of the stiffener is assumed to be zero. Similar to other crack problems, this mixed boundary value problem is reduced to a system of two simultaneous singular integral equations and they are solved numerically.

The stress intensity factor is tabulated for bending and membrane loading for different shell and stiffener geometries.

GENERAL FORMULATION

The part-through crack geometry for the stiffened cylindrical shell under consideration is shown in Fig. 1. This shell is under internal pressure.

Line-Spring Model

This model has been developed by Rice and Levy in 1972 to obtain an approximate solution for a plate containing a part-through surface crack. In a shell containing a part-through surface crack under membrane loading and bending, the stresses in the net ligament would have a constraining effect on the crack surface displacements. The basic idea in line-spring model consists of: a) representing the stresses in the net ligament by a membrane load N and a bending moment M and the crack surface displacements by an opening δ and a rotation θ, all referred to the midplane of the shell and continuously distributed along the length of the crack; b) by assuming that the relationship between (N,M) and (δ,θ) may be approximated by that of the plane strain results obtained from the solution of an edge-cracked strip or a ring; c) by using the boundary and the continuity conditions for the shell in the plane of the crack, reducing the problem to a pair of integral equations for the functions N and M or δ and θ.

The relation between (N,M) and (δ,θ) may be obtained from the expressions of the rate of change of potential energy in terms of crack closure energy and change of compliance for the related plane strain problem as given by Rice (1972), Delale and Erdogan (1981, 1982a),

$$\frac{1-\nu^2}{E} K^2 = \frac{1}{2}\left[\sigma h \frac{\partial\delta}{\partial L} + \frac{m h^2}{6}\frac{\partial\theta}{\partial L}\right] \tag{1}$$

where,

$$\sigma = N/h , \quad m = 6 M/h^2 \tag{2}$$

and K is the total mode I stress intensity factor at the crack tip, L is the depth of the edge crack and it is a known function of y.

Let the stress intensity factor for the strip be

$$K = \sqrt{h}\,(\sigma g_t + m g_b) \tag{3}$$

By noting that, $\delta = 2u(+0,y)$ and $\theta = 2\beta_x(+0,y)$ from (1) and (3) one can obtain the following result

$$\sigma(y) = E[\gamma_{tt}(y)\, u(+0,y) \pm \gamma_{tb}(y)\, \beta_x(+0,y)] ,$$

$$m(y) = 6E[\gamma_{bt}(y)\, u(+0,y) \pm \gamma_{bb}(y)\, \beta_x(+0,y)] \tag{4 a,b}$$

where + and - signs are to be used for the outer and the inner cracks, respectively, and

$$\gamma_{tt} = a\alpha_{bb}/h\Delta, \quad \gamma_{bb} = \alpha_{tt}/36h\Delta , \quad \gamma_{tb} = -\alpha_{tb}/6\Delta , \quad \gamma_{bt} = -a\alpha_{bt}/6h\Delta$$

$$\Delta = 6(\alpha_{tt}\,\alpha_{bb} - \alpha_{tb}^2)(1-\nu^2) , \tag{5 a-e}$$

$$\alpha_{ij} = \frac{1}{h}\int_0^L g_i\, g_j\, dL \qquad (i,j = t,b) . \tag{6}$$

The functions g_t and g_b, give the membrane and bending components of the stress intensity factor. They are obtained from the corresponding plane strain crack geometry. For the axial crack, the proper plane-strain problem would be that of a ring with a radial edge crack. But, as shown by Delale and Erdogan (1982b) for large h/R values the ring results are very close to the strip results. For small h/R values the convergence for the strip problem is very slow, therefore obtaining g_t and g_b becomes rather complicated. Because of these reasons in this paper the edge-cracked strip results will be used for the axial crack problem.

For the strip the functions g_t and g_b are obtained from the results given by Kaya and Erdogan (1980) which are valid for $0 < L/h < 0.8$ and may be expresse as

$$g_t(\xi) = \sqrt{\pi\xi}\,(1.1216 + 6.5200\,\xi^2 - 12.3877\,\xi^4 + 89.0554\,\xi^6$$
$$- 188.6080\,\xi^8 + 207.3870\,\xi^{10} - 32.0524\,\xi^{12}), \tag{7a}$$

$$g_b(\xi) = \sqrt{\pi\xi}\,(1.1202 - 1.8872\,\xi + 18.0143\,\xi^2 - 87.3851\,\xi^3$$
$$+ 241.9124\,\xi^4 - 319.9402\,\xi^5 + 168.0105\,\xi^6), \tag{7b}$$

where,

$$\xi = L(x_2)/h = L(ay)/h \tag{7c}$$

From (6) and (7) the compliance coefficients α_{ij}, (i,j = t,b) are found to b

$$\alpha_{tt} = \xi^2 \sum_{n=0}^{12} C_{tt}^{(n)} \xi^{2n} , \quad \alpha_{bb} = \xi^2 \sum_{n=0}^{12} C_{bb}^{(n)} \xi^n , \quad \alpha_{tb} = \alpha_{bt} = \xi^2 \sum_{n=0}^{18} C_{tb}^{(n)} \xi^n \tag{8a-c}$$

where $C_{ij}^{(n)}$ values are given by Delale and Erdogan (1981).

Stiffened Cylindrical Shell With an Axial Part-Through Crack

Stiffened cylindrical shell containing an axial part-through crack is under the effect of internal pressure only. Since the crack surfaces are free of load, the problem can be considered as the superposition of the following two problems; a) uncracked shell under the given load; b) cracked shell under the effect of stresses acting on the crack surfaces which are negative of the stresses obtained from the solution of problem (a).

In case (a) all the stresses are finite and there is no crack, so the stress intensity factors will be zero. Since, we are interested in finding the stress intensity factors, the main problem of interest is, therefore, the stress perturbation problem defined in (b).

In the formulation of the crack problem for the shell, the derivatives of the crack surface displacement and rotation are used as the unknown functions, which are defined by

$$\partial u(+0,y)/\partial y = G_1(y), \quad \partial \beta_x(+0,y)/\partial y = G_2(y) \tag{9 a,b}$$

The notation and the dimensionless quantities are given in Fig. 1 and in Appendix A of the work done by Yahsi and Erdogan (1983). It is shown by Yahsi and Erdogan (1983) that the problem of stiffened cylindrical shell containing an axial through crack may be reduced to the following system of integral equations:

$$\int_{-1}^{1} \frac{H_1(\tau)}{\tau-\eta} d\tau + \int_{-1}^{1} \sum_{j=1}^{2} k_{1j}(\tau,\eta) H_j(\tau) d\tau = 2\pi\, F_1(\eta), \quad -1 < \eta < 1 , \tag{10a}$$

$$(1-\nu^2) \int_{-1}^{1} \frac{H_2(\tau)}{\tau-\eta} d\tau + \int_{-1}^{1} \sum_{j=1}^{2} k_{2j}(\tau,\eta) H_j(\tau) d\tau = \frac{2\pi h \lambda^4}{a} F_2(\eta), \quad -1 < \eta < 1 , \tag{10b}$$

subject to the following single valuedness conditions

$$\int_{-1}^{1} H_i(\tau) d\tau = 0 , \qquad i = 1,2 , \tag{10c}$$

where

$$\eta = y+s , \quad \tau = t+s , \quad H_i(\tau) = G_i(\tau - s), \ i = 1,2$$

$$s = (b+d)/2a , \quad -d/a < y,t < -b/a , \quad -1 < \eta,\tau < 1 . \tag{11a-c}$$

and kernels $k_{ij}(\tau,\eta)$, $(i,j = 1,2)$ are known functions.

Since we are working with a perturbation problem the self-equilibrating crack surface tractions are the only external loads. In this case the loads can be expressed as follows

$$F_1(\eta) = N_{xx}(+0,\eta) , \quad -1 < \eta < 1 , \tag{12a}$$

$$F_2(\eta) = M_{xx}(+0,\eta) , \quad -1 < \eta < 1 . \tag{12b}$$

Let $N_\infty(x_2)$ and $M_\infty(x_2)$ be the stress and the moment resultants at the crack surface due to the internal pressure and $N(x_2)$ and $M(x_2)$ be the net ligament stress and moment resultants.

The input functions in (10) may then be expressed as,

$$F_1(\eta) = \sigma_\infty/E + \sigma/E , \qquad F_2(\eta) = m_\infty/6E \pm m/6E \tag{13a,b}$$

where (+) and (-) signs are to be used for the outer and the inner crack respectively, and

$$\sigma(\eta) = N(x_2)/h = N(ay)/h = N(\eta)/h , \tag{14a}$$

$$m(\eta) = 6M(x_2)/h^2 = 6M(ay)/h^2 = 6M(\eta)/h^2 . \tag{14b}$$

From (9), (13) and the singular integral equations (10) one can obtain the following singular integral equations

$$-\gamma_{tt}(\eta) \int_{-1}^{\eta} H_1(\tau) d\tau \pm \gamma_{tb}(\eta) \int_{-1}^{\eta} H_2(\tau) d\tau + \frac{1}{2\pi} \int_{-1}^{1} \frac{H_1(\tau)}{\tau-\eta} d\tau$$

$$+ \frac{1}{2\pi} \int_{-1}^{1} \sum_{j=1}^{2} k_{1j}(\eta,\tau) H_j(\tau) d\tau = -\frac{\sigma_\infty}{E} , \quad -1 < \eta < 1 , \tag{15a}$$

$$\pm \gamma_{bt}(\eta) \int_{-1}^{\eta} H_1(\tau) d\tau - \gamma_{bb}(\eta) \int_{-1}^{\eta} H_2(\tau) d\tau + \frac{a(1-\nu^2)}{2\pi h \lambda^4} \int_{-1}^{1} \frac{H_2(\tau)}{\tau-\eta} d\tau$$

$$+ \frac{a}{2\pi h} \int_{-1}^{1} \sum_{j=1}^{2} k_{2j}(\eta,\tau) H_j(\tau) d\tau = -\frac{m_\infty}{6E} , \quad -1 < \eta < 1 \tag{15b}$$

where (-) and (+) signs are to be used for the outer and the inner crack, respectively.

SOLUTION AND RESULTS

The solution of the singular integral equations (15) is of the following form

$$H_i(\tau) = \frac{h_i(\tau)}{\sqrt{1-\tau^2}} , \quad i = 1,2 , \quad -1 < \tau < 1 \tag{16}$$

where $h_i(\tau)$, $(i=1,2)$, $-1 < \tau < 1$ are unknown and bounded functions. In these circumstances, the singular integral equations (15a), (15b) subject to single valuedness conditions (11) are solved by using Gauss-Chebyshev integration formulas given by Erdogan and Gupta (1972); Krenk (1978); Theocaris (1973). After solving for $h_1(\tau)$ and $h_2(\tau)$, the net ligament stress and moment resultants σ and m may be determined from the equations (4-6) and (9). The stress intensity factor $K(x_2) = K(y)$ is then determined from (3) and (7). In the examples considered in this paper Poisson's ratio ν is assumed to be 0.3 .

The numerical results are obtained only for a semi-elliptic surface crack given by

$$L(x_2) = L(ay) = L(a\eta - s) = L(\eta) = L_0 \sqrt{1-\eta^2} \tag{17}$$

with different L_0/h ratios.

For various crack geometries and loading conditions the calculated stress intensity factors are shown in Table 1, and Fig. 2,3. In Fig. 4 stress intensity factors at different penetration points are given. In these tables and figures k_m and k_b show the normalized membrane and bending components of the stress intensity factor. They are normalized with the corresponding edge-crack value for the plane-strain problem under tension or bending. This normalizing stress intensity factor k_0 is given by

$$k_0 = K_0/\sqrt{\pi} = N_\infty \sqrt{h}\, g_t\,(\xi_0)/\sqrt{\pi}\, h \quad , \quad \xi_0 = L_0/h \tag{18}$$

for membrane loading, and

$$k_0 = K_0/\sqrt{\pi} = 6M_\infty \sqrt{h}\, g_b\,(\xi_0)/\sqrt{\pi}\, h^2 \quad , \quad \xi_0 = L_0/h \tag{19}$$

for bending.

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Table 1. Normalized stress intensity factor k/k_0 at the deepest penetration point $L=L_0$, of a semi-elliptic axial crack in a cylindrical shell under internal pressure; $h/R = 1/10$, $c/a = 1.5$, $A_s/h=1$, $\nu = 0.3$.

	L_0/h	a/h		k_{mm}: 2	4	10	k_{bb}: 2	4	10
k_{mm} / k_{bb}	0.2	Outer Crack	A*	0.834	0.989	1.020	0.247×10^{-1}	0.295×10^{-1}	-0.100×10^{-2}
			B**	0.902	0.988	1.020	0.864×10^{-2}	0.103×10^{-1}	-0.351×10^{-3}
		Inner Crack	A	0.819	0.971	1.010	-0.242×10^{-1}	-0.289×10^{-1}	0.976×10^{-3}
			B	0.885	0.970	1.010	-0.847×10^{-2}	-0.101×10^{-1}	0.342×10^{-3}
	0.6	Outer Crack	A	0.344	0.540	0.716	0.628×10^{-2}	0.139×10^{-1}	-0.607×10^{-3}
			B	0.372	0.542	0.716	0.220×10^{-2}	0.487×10^{-2}	-0.212×10^{-3}
		Inner Crack	A	0.308	0.465	0.643	-0.521×10^{-2}	-0.112×10^{-1}	0.371×10^{-3}
			B	0.333	0.467	0.643	-0.182×10^{-2}	-0.393×10^{-2}	0.130×10^{-3}
	0.8	Outer Crack	A	0.112	0.207	0.351	0.480×10^{-3}	0.370×10^{-2}	-0.168×10^{-3}
			B	0.121	0.209	0.350	0.168×10^{-3}	0.129×10^{-2}	-0.587×10^{-4}
		Inner Crack	A	0.100	0.169	0.282	-0.126×10^{-3}	-0.230×10^{-2}	0.178×10^{-4}
			B	0.108	0.169	0.281	-0.440×10^{-4}	-0.804×10^{-3}	0.622×10^{-5}

*A means open end cylindrical shell, *B means closed end cylindrical shell.

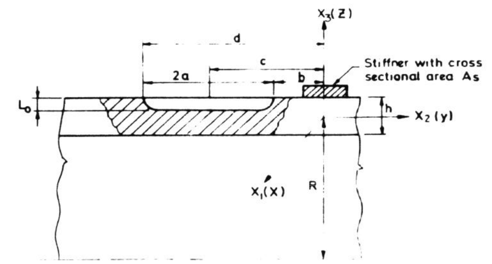

Fig. 1. The geometry of an axial part-through surface crack in a stiffened cylindrical shell.

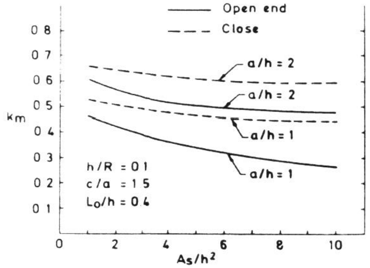

Fig. 2. Membrane component of the normalized stress intensity factor k_m at the deepest penetration $L=L_0$, of an outer semi-elliptic axial crack in a cylindrical shell under internal pressure, $\nu=0.3$.

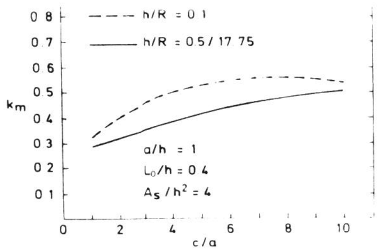

Fig. 3. Membrane component of the normalized stress intensity factor k_m at the deepest penetration $L=L_0$, of an outer semi-elliptic axial crack in an open end cylindrical shell under internal pressure.

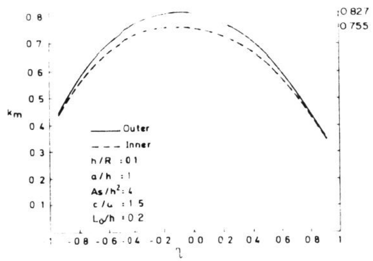

Fig. 4. Distribution of normalized stress intensity factor k_m along the crack front in an open end, stiffened cylindrical shell containing an inner or outer axial semi-elliptic surface crack, $\nu = 0.3$.