AN EFFICIENT FINITE-ELEMENT-ALGORITHM FOR THREE-DIMENSIONAL FRACTURE PROBLEMS

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ABSTRACT

On the basis of a hybrid stress model special crack tip elements have been elaborated according to the approach outlined by Pian and Moriya. Their efficiency and accuracy in describing the crack tip singularity are compared with those of other crack tip elements. Various examples of a three-dimensional fracture mechanical analysis are given.

KEYWORDS

Finite element method; fracture mechanics; three-dimensional crack problems; crack tip elements; hybrid stress model; stress intensity factors.

INTRODUCTION

In many practical cases the fracture mechanical assessment of a structural component with crack requires a three-dimensional mechanical analysis. At present the finite element method (FEM) is the mostly used numerical tool for solving such complicated boundary value problems in elastostatics. Since conventional finite element types are not capable of reproducing the singular stress state at the crack tip, special crack tip elements or refined techniques have been developed to determine the stress intensity factors. Especially for three-dimensional crack problems the search for more efficient finite-element-algorithms is necessary to reduce the high computational expenditure. In the present paper crack tip elements are proposed on the basis of a hybrid stress model. Their efficiency and accuracy in describing the crack tip singularity are compared with those of other crack tip elements by various examples.

CRACK TIP ELEMENTS

From analytical investigations (Kassir and Sih, 1975) it is known that for a crack fully embedded in a 3D solid, the asymptotic stress and displacement fields at each point of the crack front are as follows:

$$\sigma_{ij}(r,\theta,t) = r^{-1/2}\left[K_I(t)f_{ij}^{I}(\theta)+K_{II}(t)f_{ij}^{II}(\theta)+K_{III}f_{ij}^{III}(\theta)\right] \quad (1)$$

$$u_i(r,\theta,t) = r^{1/2}\left[K_I(t)g_i^{I}(\theta)+K_{II}(t)g_i^{II}(\theta)+K_{III}g_i^{III}(\theta)\right] \quad (2),$$

where (r,θ,t) are cylindrical coordinates and t lies parallel to the crack front.

If one wants to take advantage of equations (1) and (2) for constructing special crack tip elements the following dilemma arises: (a) either the compatibility with the neighbouring standard elements is violated, or (b) the analytical knowledge can only partly be used if the compatibility is maintained. This particularly holds for the majority of crack tip elements developed by means of a displacement model. Among these, so-called "quarter-point" elements (Barsoum, 1976) are mostly applied, since they can be obtained by a simple modification of isoparametric standard elements. The mid-edge nodes are shifted to the quarter-point position, whereby an $r^{-1/2}$-singularity of the strain state is achieved. The angular distribution of equations (1) and (2), however, cannot be taken into account.

Hybrid element formulations enable the above mentioned difficulties to be overcome, since the field variables in the interior of the element and those at its boundary are treated independently of each other. The continuity of displacements and tractions at the transition to the neighbouring elements is guaranteed by the variational principle in an integral manner. For the construction of crack tip elements the known crack tip solutions can be completely employed for the element volume, whereas at the boundary such displacement functions are assumed which are identical with those of the adjacent standard elements. The "displacement-compatible" crack tip elements developed in this manner can be incorporated into any FEM-program system. A review about hybrid crack tip elements was given by Atluri and Tong, 1977.

On the basis of a hybrid stress model special crack tip elements have been elaborated according to the approach outlined by Pian and Moriya, 1978. A detailed description was given by Kuna, 1982 and 1983.

The shape of the crack tip elements is a 20-node hexahedron. Four hybrid elements are arranged around each segment of the crack front line (see Fig. 1). The remaining part of the body is modelled by standard elements.

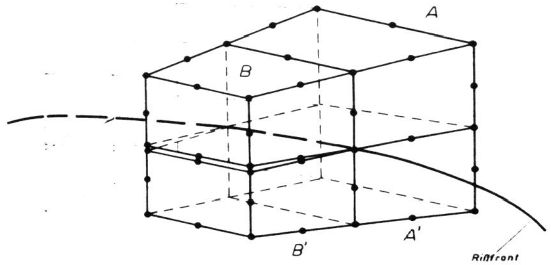

Fig. 1. Arrangement of hybrid crack tip elements around the crack front

The assumed stress state in the elements consists of regular polynomial terms together with equation (1) containing the stress intensity factors K_I, K_{II} and K_{III} as unknown variables. It satisfies the equilibrium equations and the traction boundary conditions on the crack faces. Besides quadratic isoparametric shape functions also special terms are used for the boundary displacements taking into account the radial and angular dependence of the crack tip solution (2). In the computational implementation these hybrid elements were combined with the FEM-program elaborated by Altenbach and Wiltinger, 1981. From the resulting solution for the nodal point displacements the values of the stress intensity factors are directly calculated for each segment of the crack front.

COMPARISON OF CRACK TIP ELEMENTS

For the purpose of comparison the problem of a cylindrical bar with an internal penny-shaped crack was chosen, loaded by axial tension. Because of axial symmetry only a wedge-shaped finite element network had to be constructed, the faces of which were subjected to appropriate displacement constraints (see Fig.2). For the two hexahedron elements directly at the crack tip the following element types were used:

a) isoparametric standard elements;
b) quarter-point hexahedron elements;
c) hybrid elements of A and B type (see Fig. 1)

From the resulting opening displacements of the nodes on the crack face a local stress intensity factor K_I^* was evaluated by means of equation (2). In Fig. 3 the dependence of K_I^* on the distance from the crack tip is shown and compared with the solution of $K_I = 0.6396\,\sigma\sqrt{\pi c}$ given by Kassir, 1975.

The results point out that standard elements fail in describing the crack tip singularity (drop of K_I^* near the crack tip), whereas quarter-point elements are much better suited. The best behaviour (nearly straight line to the exact value) stems

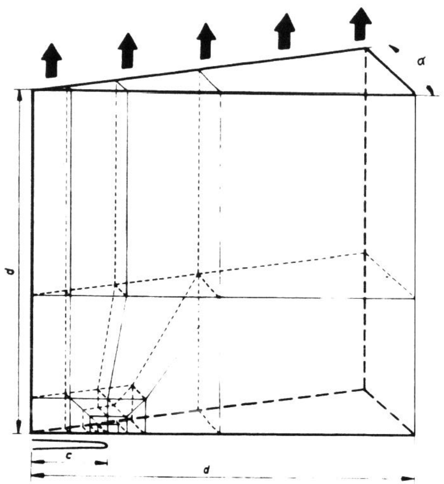

Fig. 2. FEM-mesh for a cylindrical bar with internal penny-shaped crack (20 elements, 177 nodes, $\alpha = 2.3^{o}$, $c/d = 0.2$)

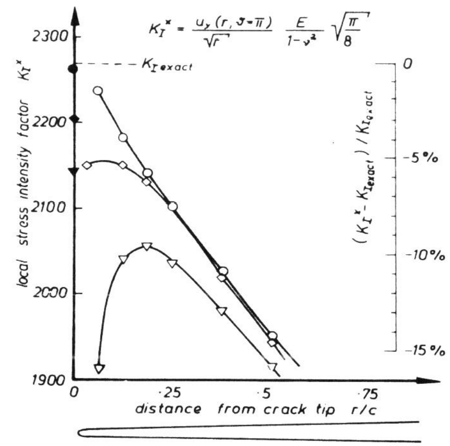

Fig. 3. Comparison of results for penny-shaped crack using isoparametric elements (▽), quarterpoint elements (◇) or hybrid elements (○)

from the hybrid elements. The commonly used extrapolation of K_I^* towards $r = 0$ leads to K_I-values (full symbols in Fig. 3) with errors of -5.5 %, -3 % and -0.5 % for the element types (a), (b) and (c), respectively.

Another testing example was the compact tension fracture test specimen (see e.g. Tada, 1973). Crack length a was half the specimen width W, thickness B=W/2. Due to double symmetry only one quarter of the geometry was considered using different networks and crack tip elements. The coarsest mesh consisting of 4x2x5 elements is drawn in Fig. 4. The finest mesh had 6x4x5=120 elements and 733 nodes. Around the crack front ten hybrid elements or quarter-point elements were located. The calculated distributions of the stress intensity factor across the thickness of the specimen are summarized in Fig. 5. For comparison the 2D solution (Tada, 1973; ---) and one of the most accurate 3D solutions (Yamamoto and Sumi, 1978; ——) are included in the figure. The K_I-values obtained by means of the hybrid elements (△ fine mesh, O coarse mesh) are in good agreement with the results of Yamamoto and Sumi, 1978. The application of quarter-point elements (▲ fine mesh, ● coarse mesh) resulted in a loss of accuracy by 5 % and 30 %, respectively. In this case K_I was determined by displacement extrapolation. If applied to the hybrid solution of the fine mesh, this method gave the results indicated by ▽ symbols.

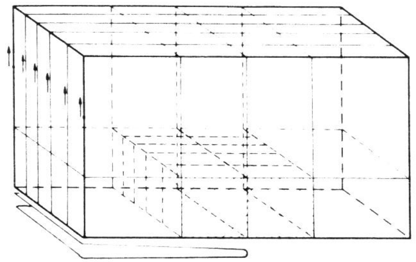

Fig. 4. Finite element mesh for one quarter of the compact tension specimen (40 elements, 297 nodes)

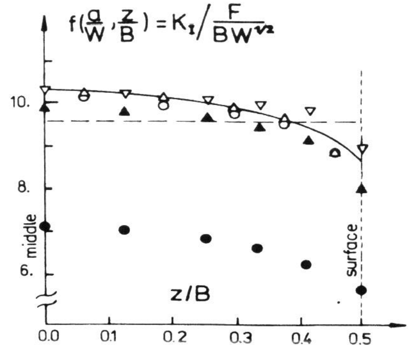

Fig. 5. Normalized stress intensity factor K_I across the thickness of the CT-specimen

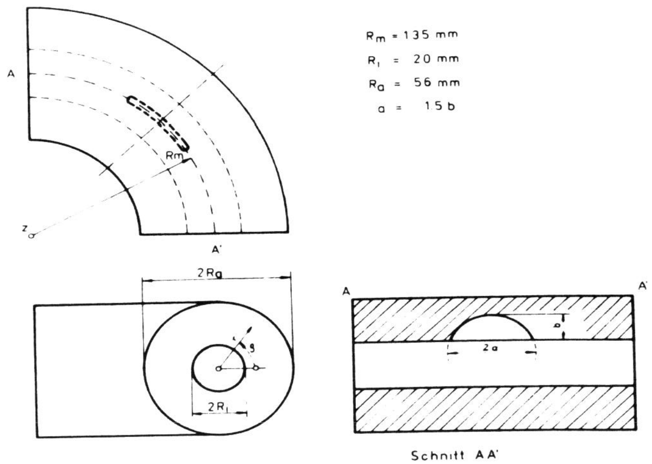

Fig. 6. 90-degree bow with inner semi-elliptical surface crack;
lower right:
cross section along the bending line;
bending radius = 135 mm

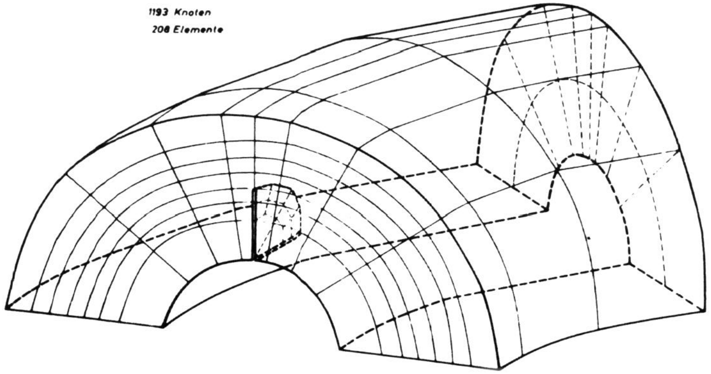

Fig. 7. FEM-mesh for the bow with inner semi-elliptical surface crack (one quarter, 208 elements, 1193 nodes)

APPLICATIONS

Finally, a more complicated technical crack geometry was investigated. At first, a thick-walled cylindrical tube under internal pressure of p=1 MPa was analysed, containing a semi-elliptical inner surface crack of an axial ratio of a/b = 1.5. The outer and inner radii amounted to R_i = 20 mm and R_a = 56 mm, respectively. Crack depth was half the wall thickness. Second, a 90-degree bow of such a tube was considered, see Fig. 6. In Fig. 7 the network for one quarter of the geometry is depicted. The crack front was surrounded by 16 hybrid elements. The calculated distribution of K_I along the crack front is shown in Fig. 8 for all cases analysed. In the case of the bow only negligible values of K_{II} and K_{III} occurred. The pressure acting on the crack faces was taken into account. Without it (see solution ---) the stress intensity decreased by about one half. Apart from the region close to the inner surface, the correspondence of our result for the straight tube with the estimating formula derived by Williams, 1980, is good. The K_I-distribution for the bow does not significantly differ from the straight tube solution, i.e. the curvature of the bow has no great influence.

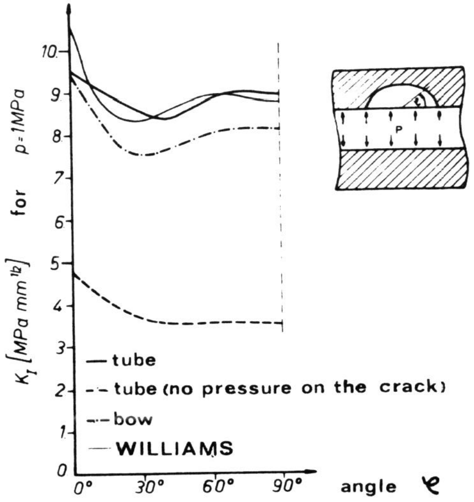

Fig. 8. K_I-distributions along the crack front for a semi-elliptical surface crack in a pressurized tube or bow

CONCLUSIONS

In contrast to other 3D crack tip elements, the proposed hybrid elements have the advantage that their shape functions contain the complete singular crack tip solution and fulfil the interelement compatibility requirements. Furthermore, the stress intensity factors K_I, K_{II} and K_{III} are obtained directly and separately. Compared with quarter-point elements, the hybrid elements reproduce much better the crack tip singularity and give a higher accuracy of the stress intensity factor solution. For instance with the rather coarse mesh of Fig. 4 a sufficiently exact K_I-solution could be achieved for the CT-specimen. The most promising technique for 3D crack analysis would be a combination of hybrid elements with the virtual crack extension method, as it is usually done for quarter-point elements. The extension to crack problems under mixed mode loading conditions, thermal strains and body forces does not imply difficulties. Problems arise - as for all 3D crack analysis today - if the crack front intersects a surface of the body, because the fundamental crack tip solution of equations (1, 2) becomes invalid at this point. The use of the embedded crack tip solution outside a small boundary layer is generally supposed to be justified.

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