A DISLOCATION MODEL OF THE DELAYING EFFECTS OF OVERLOAD

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ABSTRACT:

A dislocation model of the delaying effects caused by overload is described. The model is based on BCS model and superposition method of crack problems. An exact expression for the crack opening displacement is derived within the framework of the proposed model. Small scale yielding approximations and some numerical solutions are also given. The model predicts that during crack propagation through the overload plastic zone its growth rate decreases to a minimum at first and then increases to the appropriate value which would result if the overload had not been applied. Using this model the fatigue crack growth delayed retardation behaviour resulting from the application of the overload may be explained rationally. Results given by the dislocation model are the same as that given by McCartney based on a Dugdale model. Advantage of the dislocation model is that the defining expression for the crack tip opening displacement is very clear in physical meaning and consistent with the model itself.

KEYWORDS

BCS model; superposition method; fatigue; overload delayed retardation.

INTRODUCTION

Most theoretical investigations on the fatigue crack growth retardation resulting from the application of single or multiple overloads have been reported in the literature. Among which McCartney (1978) analytically studied the delayed retardation behaviours resulting from overload, using the Dugdale model. His work supplied a theoretical explanation for the delaying effects of overloads. In the literature, several investigators have studied the fatigue crack growth by means of the dislocation model, but they mainly investigated the constant amplitude fatigue. There have been also investigations on the behaviours of overload retardation and crack closure, such as Kanninen and co-workers (1976, 1980) with superdislocation pair model. The present paper generalized the BCS model to problems of the delaying effects of overloads and the fundamental method to be used is the superposition method. It is shown that results given by the present paper and by McCartney (1978) are the same. An advantage of the dislocation model is that the defining expression of crack tip opening displacement (COD) is very clear in physical meaning and has more generality. In addition, the dislocation model can consider combined effects of residual stresses, crack closure factors, etc.

PLASTIC DEFORMATION RESULTING FROM THE OVERLOAD

The overload often produces intense plastic deformation. Both the stationary and growing crack are analysed as follows:

Stationary Crack

A so-called stationary crack means that the crack length remains constant during cyclic-loadings. Let the length of a stationary crack be $2c_o$. Consider the applied stress sequence $\sigma \to \sigma_1 \to \sigma_2 \to \sigma$, as shown in Fig. 1, i.e., after an overload the subsequent cycle is between the stress levels σ_2 and σ. For convenience, we introduce stress ratios r and R, defined as follows

$$r = \sigma_1/\sigma \geq 1, \qquad R = \sigma_2/\sigma \leq 1 \tag{1}$$

First, consider the state A in Fig. 1, i.e., the applied stress is monotonic increases from zero to σ_1, the associated dislocation density and extent of the plastic zones are $f_1(x)$ and $\pm a_1$ respectively. The problem can be solved by using the BCS model. It is known that the BCS model makes use of a double pile-up of dislocations to simulate elastoplastic cracks (see Fig. 2). It is assumed that a crack occupying the $|x| \leq c_o$ in an infinite plate which is subjected to stress σ_1 at infinity. Instead of supposing the dislocations created by the source at x=y=0 to be blocked at $|x| = c_o$ as in Fig. 2. We now allow them to move into the material in the regions $|x| > c_o$. The maximum range is $x = a_1$. We suppose that a frictional resistance σ_o opposed the motion of dislocation is the material's flow stress. In order to solve the problem, two equations are required. One is the equilibrium equation for the double pile-up of dislocations in terms of the density $f_1(x)$ and the other one is that requirement of $f_1(x)$ to be bounded at $x = a_1$. The two equations are

$$AP\int_{-a_1}^{a_1} \frac{f_1(x')\,dx'}{x - x'} = -\sigma(x) = \begin{cases} -\sigma_1 & (-c_o \leq x \leq c_o) \\ -(\sigma_1 - \sigma_o) & (c_o \leq |x| \leq a_1) \end{cases} \tag{2}$$

$$\int_{-a_1}^{a_1} \frac{(x)\,dx}{(a^2 - x^2)^{1/2}} = 0 \tag{3}$$

with

$$A = \mu b / 2\pi(1 - \nu)$$

where μ is the shear modulus; ν is the Poissons ratio; b is the Burgers vector; $\sigma(x)$ is the applied effective stress; P indicated that the integral is a Chauchys principal-value integral. Substituting $\sigma(x)$ in equation (2) into (3), the extent of the plastic zones can be determined

$$a_1/c_o = \sec(r\theta), \qquad \theta = \pi\sigma/2\sigma_o \tag{4}$$

Equation (2) has definite solutions, provided condition (3) holds. The general solution has the form

$$f_1(x) = -\frac{1}{A\pi^2}(a_1^2 - x^2)^{1/2} \; P\int_{-a_1}^{a_1} \frac{\sigma(x')}{(a_1^2 - x'^2)^{1/2}} \frac{dx'}{x - x'} \tag{5}$$

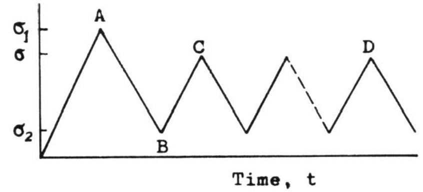

Fig. 1. The applied stress sequence

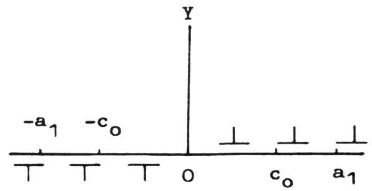

Fig. 2. BCS model

The relative displacement at any point x on the slip plane is equal to the sum of the Burgers vectors of all the dislocation passed through x, i.e.

$$\delta(x,a) = -b\int_a^x f(x)\,dx = V(x) - V(a) \tag{6}$$

where "a" refers to the extent of the plastic zones in general and V(x) is the displacement function. For convenience of analysis, the solutions of V(x) are given at first. Using equations (2), (4), (5), and (6), we get

$$V_1(x)/V(c) = Z(c_o, a_1, x), \qquad V(c) = 2(1-\nu)\sigma_o/\pi\mu \tag{7}$$

where

$$Z(c_o, a_1, x) = c_o z^*(c_o, a_1, x) - x z(c_o, a_1, x) \tag{8}$$

$$z(c_o, a_1, x) = \mathrm{Ln}\left|\frac{x(a_1^2 - c_o^2)^{1/2} + c_o(a_1^2 - x^2)^{1/2}}{x(a_1^2 - c_o^2)^{1/2} - c_o(a_1^2 - x^2)^{1/2}}\right| \tag{9}$$

$$z^*(c_o, a_1, x) = \mathrm{Ln}\left|\frac{(a_1^2 - c_o^2)^{1/2} + (a_1^2 - x^2)^{1/2}}{(a_1^2 - c_o^2)^{1/2} - (a_1^2 - x^2)^{1/2}}\right| \tag{10}$$

Secondly, consider state B in Fig. 1, i.e., the applied stress is reduced from σ_1 to σ_2 and friction stress σ_o is reversed in direction. The relevant quantities are denoted by subcript "2". The equilibrium equation is

$$AP\int_{-a}^{a_1} \frac{f_2(x')\,dx'}{x - x'} = \begin{cases} -\sigma_2 & (-c_o \leq x \leq c_o) \\ -(\sigma_2 + \sigma_o) & (c_o \leq |x| \leq a_2) \end{cases} \tag{11}$$

The value $\sigma(x)$ of the right-hand side in equation (11) is not known for $a_2 \leq |x| \leq a_1$; however, we do know that $f_1(x) = f_2(x)$ in the region. Therefore, we can use the superposition principle. Subtracting (11) from (2) we obtain

$$AP \int_{-a_2}^{a_2} \frac{f_1(x') - f_2(x')}{x - x'} dx' = \begin{cases} -(\sigma_1-\sigma_2) & (-c_o \leqslant x < c_o) \\ -(\sigma_1-\sigma_2)-2\sigma_o & (c_o \leqslant |x| < a_2) \end{cases} \quad (12)$$

This equation is identical with the original BCS equation (2), provided we make the changes $f_1(x) \rightarrow f_1(x)-f_2(x)$, $\sigma_1 \rightarrow \sigma_1-\sigma_2$, $\sigma_o \rightarrow 2\sigma_o$ and $a_1 \rightarrow a_2$, thus all of the above results can be applied immediately. In particular

$$a_2 / c_o = \sec ((r-R)\theta /2) \quad (13)$$

$$V_2(x)/V(c) = Z(c_o,a_1,x) - 2Z(c_o,a_2,x) \quad (14)$$

here $\pm a_2$ is the extent of reversed plastic zones for overload.
Finally, consider state C in Fig. 1, i.e., the applied stress increases again from σ_2 to σ. The relevant quatities are denoted by subcript "3". The equilibrium equation for this case is easily written. Subtracting (11) from it and once making use of results of the BCS model, we get

$$a_3 /c_o = \sec ((r-R)\theta /2)$$
$$V_3(x)/V(c) = Z(c_o,a_1,x) - 2Z(c_o,a_2,x) + 2Z(c_o,a_3,x) \quad (15)$$

here $\pm a_3$ is the extent of the cyclic plastic zones.

Growing Crack

During the stress-cycling between values σ and σ_2 after the overload, fatigue crack growth will occur. Now consider some state D after state C in Fig. 1. Assume the crack length to be $2c \geqslant 2c_o$. Three situations must be considered therein

Current plastic zones ($c_o < x < a$) extend into the reversed plastic zones of the overload ($c_o \leqslant x \leqslant a_2$).

The equilibrium equation in terms of the density f(x) is

$$AP \int_{-a_1}^{a_1} \frac{f(x')\, dx'}{x - x'} = \begin{cases} -\sigma & (-c \leqslant x \leqslant c) \\ -(\sigma - \sigma_o) & (c \leqslant |x| \leqslant a) \end{cases} \quad (16)$$

Subtracting (11) from (16) we get

$$AP \int_{-a}^{a} \frac{f(x')-f_2(x')}{x - x'} dx' = \begin{cases} -(\sigma - \sigma_2) & (-c_o \leqslant x \leqslant c_o) \\ -((\sigma - \sigma_2) - \sigma_o) & (-c_o \leqslant x \leqslant c) \\ -((\sigma - \sigma_2) - 2\sigma_o) & (c \leqslant |x| \leqslant a) \end{cases} \quad (17)$$

From equations (3), (5), (17) we get

$$a = \frac{(c^2 - 2cc_o \cos((1-R)\theta) + c_o^2)^{1/2}}{\sin((1-R)\theta)}$$

$$V(x)/V(c) = Z(c_o,a_1,x) - 2Z(c_o,a_2,x) + Z(c_o,a,x) + Z(c,a,x) \quad (18)$$

Current plastic zones extend into the overload plastic zones ($a_2 \leqslant |x| \leqslant a_1$)

In this case the effect of the overload plastic zones is considered only. Based on similar consideration as above, the equilibrium equation is just (16). Similar, subtracting (16) from (2) and making use of (3) we obtain

$$a = \frac{(c^2 - 2cc_o \cos((r-1)\theta) + c_o^2)^{1/2}}{\sin((r-1)\theta)}$$

$$V(x)/V(c) = Z(c_o,a_1,x) - Z(c_o,a,x) + Z(c,a,x) \quad (19)$$

The current plastic zones exceed the overload plastic zones ($a \geqslant a_1$).

The effects of the overload may be ignored in this case. Thus, results of the BCS model can be used directly as follows:

$$a/c = \sec\theta$$
$$V(x)/V(c) = Z(c,a,x) \quad (20)$$

CRACK TIP OPENING DISPLACEMENT (COD)

Exact Solutions for COD

We take x=c in equation (6), then the COD can be obtained. Such definition of COD is more general than conventional. Under monotonic increasing loading there is $\delta(c,a) = V(c)$, which is just the conventional definition for COD, due to $V(a) = 0$. But under cyclic-loading, the use of definition (6) seems especially important. It should be pointed out that by making use of other models rather than dislocation model, such as Dugdale model, to study the effects of crack closure or overload, similar definition as (6) has to be introduced directly or presupposions. Clearly, this is not concordant. Making use of the associated expressions of V(x), the exact solutions for COD can be obtained within the framework of the proposed model:
for $a_3 \leqslant a \leqslant a_2$

$$\delta(c,a)/V(c) = Z(c_o,a_1,c) - Z(c_o,a_1,a) - 2Z(c_o,a_2,c) + 2Z(c_o,a_2,a) + Z(c_o,a,c) + Z(c,a,c) \quad (21)$$

for $a_2 \leqslant a \leqslant a_1$

$$\delta(c,a)/V(c) = Z(c_o,a_1,c) - Z(c_o,a_1,a) + Z(c,a,c) - Z(c_o,a,c) \quad (22)$$

for $a \geqslant a_1$

$$\delta(c,a)/V(c) = Z(c,a,c) \quad (23)$$

Small Scale Yielding Approximation

In the case of small scale yielding, i.e., $r \ll 1$ and $\omega_1 = a_1 - c_o \ll c_o$, the size of the plastic zone and COD for a center crack in a infinite plate subjected to uniformly tensile stress σ in a direction normal to the plane of the crack are as follows:

$$\omega_o = \frac{1}{2} c_o \theta^2$$

$$\delta_o = 4(1-\nu)\omega_o \sigma_o / \pi\mu \qquad (24)$$

We introduce the parameter t defined by

$$t = (c-c_o)/\omega_o \qquad (25)$$

Thus, it is not difficult to show that

$$\begin{aligned} & a_3 \leqslant a \leqslant a_2 \longrightarrow 0 \leqslant t \leqslant (1-R)(r-1) \\ & a_2 \leqslant a < a_1 \longrightarrow (1-R)(r-1) \leqslant t \leqslant r^2-1 \end{aligned} \qquad (26)$$

It can be shown that in the case of small scale yielding conditions the approximations of COD may be written in the form:

for $0 < t < (1-R)(r-1)$

$$\begin{aligned} \delta(c,a)/\delta_o = {} & r^2(H(t/r^2) - H(s^{*2}/4r^2)) - ((r-R)^2/2)\,(H(4t/(r-R)^2) \\ & -H(s^{*2}/(r-R)^2)) + (s/2)^2 + (s^*/2)^2\, H(4t/s^{*2}) \end{aligned} \qquad (27)$$

$$s = t/(1-R) - (1-R), \qquad s^* = t/(1-R) + (1-R)$$

for $(1-R)(r-1) \leqslant t < r^2-1$

$$\begin{aligned} \delta(c,a)/\delta_o = {} & r^2\,(H(t/r^2) - H(s^{*2}/4r^2)) + (s/2)^2 \\ & -(s^*/2)^2\, H(4t/s^{*2}) \end{aligned} \qquad (28)$$

$$s = t/(r-1) - (r-1), \qquad s^* = t/(r-1) + (r-1)$$

for $r^2-1 \leqslant t < \infty$

$$\delta(c,a)/\delta_o = 1 \qquad (29)$$

The function H appearing in the relations (27) and (28) is defined by

$$H(x) = (1-x)^{1/2} - (x/2)\,\mathrm{Ln}\,\frac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}} \qquad (30)$$

Hence, by using the dislocation model we have obtained the same results given by McCartney (1978). However, the definiting expression based on the dislocation model for COD is more clear in physical meaning and consistent with the model itself.

Crack Tip Closure Behaviours and the Delayed Retardation Effects

Let the first derivative of equation (27) with respect to t equal to zero, we find, when

$$t = (1-R)^2 \qquad (31)$$

equation (27) will give a minimum value of zero. It means that during cyclic-loadings after overload the crack tip begins to close at $t = (1-R)^2$ thus the growth will stop. Based on the effective range of t in (27): $0 \leqslant t \leqslant (1-R)(r-1)$, it is clear that value t in (31) must be in the same range too. According to this, the precondition in which equation (27) has

a minimum value of zero is

$$r + R \geqslant 2 \qquad (32)$$

Similarly, it can be shown that when

$$t = (r-1)^2 \qquad (33)$$

and precondition (32) holds, equation (28) has a minimum value of zero. So far we can conclude that during cyclic-loading after overloads the precondition resulting in crack closure or stop is the equation (32). When $(r+R)=2$, thus $(r-1) = (1-R)$, then positions of minimum value of zero for (27) and (28) coincide at a point, $t = (1-R)(r-1)$, i.e., at the boundary of reversed plastic zones for overload. The variations of COD with t are illustrate in Fig. 3 for conditions $(r+R) \geqslant 2$. For comparition, curves obtained by the conventional expression $\delta(c) = V(c)$ are also shown in Fig. 3. When

$$r + R < 2 \qquad (34)$$

the crack tip closure never occurs but there is minimum value at $t=(1-R)(r-1)$. Behaviours for this case are illustrated in Fig. 4. If COD is the governing mechanical parameter on fatigue crack growth and supposing that crack growth takes place at the point of maximum value of the applied stress, then the overload delayed retardation effects may be well explained by equations (27) and (28). It can be seen from Fig. 3 and 4 that during cyclic-loading following the overload, with increasing of the crack length $\delta(c,a)$ at first gradually decreases to a minimum value and then subsequently increases again. When $t \geqslant (r^2-1)$, the effects of overloads disapear. That is, these effects may be divided into two distinct stages. For example, with $(r+R) \leqslant 2$, the stage of overload retardation is in the interval of $(1-R)(r-1) \leqslant t \leqslant r^2-1$, and the stage of overload delayed retardation is in the interval $0 \leqslant t \leqslant (1-R)(r-1)$. These results quantitatively agree with numbers of experimental facts. Finally, we consider effects of the self-unloading. It can be shown that as the applied stress decreases to $\sigma' \geqslant \sigma_2$, condition in which crack tip closure occurs is

$$\delta(c,a')/\delta_o = (1-R')^2/2 \qquad (35)$$

where a' is the extent of the plastic zones corresponding to σ' and $R'=\sigma'/\sigma$. Similarly, when the applied stress increases from σ_2 to $\sigma'' \leqslant \sigma$, crack tip reopening just occurs, then we have

$$\frac{\delta(c,a'')}{\delta_o} = \frac{(1-R)^2}{2}\left(1-H\left(\left(\frac{R''-R}{1-R}\right)^2\right) - \frac{(R'-R)^2}{2}\right) \qquad (36)$$

where a'' is the extent of the plastic zones corresponding to σ'' and $R''=\sigma''/\sigma$. Following the concept of crack closure, the effective stress intensity factor range may be written as

$$\Delta K_{eff}/\Delta K = (\sigma_{max} - \sigma_{op})/(\sigma_{max} - \sigma_{min})$$

For the present case $\sigma_{max} = \sigma$, $\sigma_{min} = R\sigma$, $\sigma_{op} = \sigma'' = R''\sigma$. Thus we get

$$\Delta K_{eff}/\Delta K = (1-R'')/(1-R) \qquad (37)$$

where R" can be solved from equation (36). Therefore, based on the present dislocation model ΔK_{eff} may be evaluated in priciple. After that, using some appropriate formula for the fatigue crack propagation rates, the numbers of cycles for the overload delayed retardation can be evaluated.

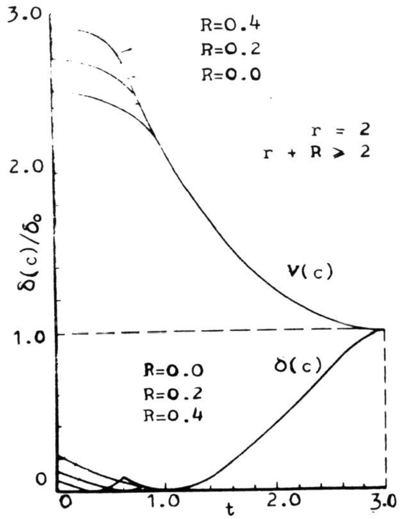

Fig. 3. The dependence of $\delta(c)$ and V(c) on parameter t

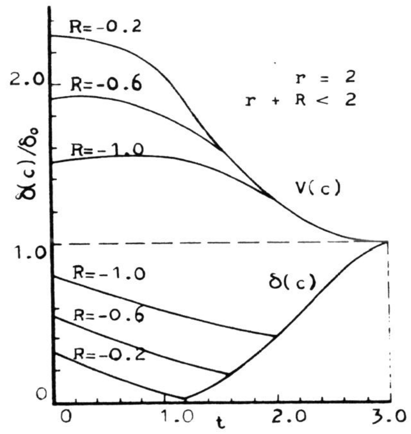

Fig. 4. The dependence of $\delta(c)$ and V(c) on parameter t

CONCLUSIONS

Definition (6) for COD given by the dislocation model is more general. Under cyclic-loadings it seems especially important. By using the BCS model and superposition method to analyse the overload delayed retardation, the effects of overload can be resonably explained. Making use of the dislocation model, the combined effects of the residual stresses, the crack closure factors, etc, can be taken into account.

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