

THRESHOLDS FOR FATIGUE CRACK PROPAGATION: QUESTIONS AND ANOMALIES

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ABSTRACT

Many studies into the propagation of fatigue cracks have been focussed in recent years on the so-called near-threshold regime, where growth rates per cycle approach atomic dimensions and stress intensity ranges approach a threshold, below which crack extension cannot be experimentally detected. Such studies in general have shown that the many mechanical, microstructural and environmental factors which can influence the fatigue threshold are effective via extrinsic mechanisms, such as crack closure and crack deflection processes, which locally perturb the crack driving force. At the same time, however, they have questioned the validity of a threshold concept for such situations as variable amplitude loading or in the presence of short cracks. In the present article, the current physical interpretations of near-threshold fatigue are reviewed, with reference to variable amplitude loading effects and the apparent anomalous behavior of small cracks. Further, through comparison with classical S/N and Mode III data, the relationships between the fatigue limit and long and short crack thresholds are discussed in terms of crack closure and the characterization of crack tip fields for both small and growing cracks.

KEY WORDS

Fatigue crack propagation, threshold stress intensity, crack closure, crack deflection, variable amplitude loading, small cracks, non-stationary cracks.

INTRODUCTION

In 1970, Paris first demonstrated, in the context of fracture mechanics terminology, the existence of a fatigue threshold representing a limiting stress intensity range, ΔK_0 , for crack extension, although the concept had been suggested earlier from both theoretical (McClintock, 1963) and experimental (Frost, 1966) studies. Since that time the topic of fatigue thresholds and associated near-threshold crack growth behavior, below

typically 10^{-6} mm/cycle, has received significant attention in the literature, particularly recently as evidenced by the proceedings of two international conferences devoted specifically to the subject (Bäcklund, Blom and Beevers, eds., 1982; Davidson and Suresh, eds., 1984).

Values of the fatigue thresholds, ΔK_0 , represent the nominal stress intensity ranges ($\Delta K = K_{\max} - K_{\min}$) below which fatigue cracks remain dormant or grow at experimentally-undetectable rates, although it is now realized that this definition must be restricted to so-called "long" cracks. Experimentally, thresholds are generally measured in Mode I tests where the crack growth rate (da/dN) is progressively decreased, usually at a fixed load ratio ($R = K_{\min}/K_{\max}$) under decreasing ΔK (load shedding) conditions. However, at high load ratios, alternative methods of cycling at constant K_{\max} with increasing K_{\min} (i.e., varying R) have also been used (Liaw and others, 1984). Using the resulting da/dN vs. ΔK data, the threshold is then operationally defined using a regression fit to data points at the lowest growth rates, e.g., below $\sim 10^{-6}$ mm/cycle, or in terms of a maximum growth rate based on the resolution of the crack monitoring technique and the number of cycles spent at the lowest ΔK where no growth was detected. With these procedures, thresholds have been defined at maximum growth rates of between 10^{-7} and 10^{-8} mm/cycle at conventional frequencies (i.e., < 500 Hz) and between 10^{-10} and 10^{-11} mm/cycle at ultrasonic frequencies (i.e., ~ 21 kHz). Since these limiting growth rates imply increments of crack advance between 0.5 and 0.00001 of a lattice spacing per cycle, clearly the extension of fatigue cracks at near-threshold levels cannot be locally continuous in nature, and presumably occurs by a non-uniform, segmented advance of the crack front.

From an academic viewpoint, the study of fatigue thresholds and near-threshold behavior in engineering materials has highlighted many important effects of load history, frequency, load ratio, crack size, strength level, slip mode, microstructure and environment in influencing crack growth, effects which can differ sharply from those documented for higher growth rates and from those inferred from classical S/N or low cycle fatigue results. Furthermore, it has provided insight into the many mechanisms of fatigue crack closure and the understanding of variable amplitude behavior (e.g., Suresh, 1983; Suresh and Ritchie, 1984a). Practically speaking, however, the concept of a fatigue threshold has met with limited success in structural and engineering design, although it has proved useful in the analysis of certain problems involving high frequencies, such as cracking in turbine shafts, blades and alternator rotors, and acoustic fatigue of welds in nuclear gas circuitry (Wanhill, 1982; Lindley and Stewart, 1982). This limited practical use of the threshold stems in part from the fact that it represents a far too conservative design criterion in many applications, and in part from fundamental questions regarding its uniqueness as a "material parameter," particularly in the light of experimental measurement, variable amplitude loading and crack size problems. Since the mechanistic and phenomenological aspects of fatigue thresholds have been the subject of several recent reviews (Ritchie, 1979; Beevers, 1982; Suresh and Ritchie, 1984a), it is the problems, and in some cases the anomalies, concerning near-threshold behavior which form the basis of the current paper. Specifically, the validity of the threshold concept is examined for certain critical situations, such as involving tension or compression overloads and in the presence of short cracks. Further, through comparison with classical S/N and Mode III data, the relationship between the fatigue limit and long and short crack thresholds are discussed in terms of crack

closure, crack deflection and the characterization of crack tip fields for both small and growing cracks.

GENERAL ASPECTS OF THE FATIGUE THRESHOLD

Mechanisms

Fatigue threshold values and corresponding low growth rate data are now available on a wide range of engineering materials, including steels, aluminum, titanium, nickel and copper alloys, although information is still relatively scarce for near-threshold behavior in aggressive environments. Based on these data, which show a dependence of the threshold on such factors as load ratio, yield strength, grain size, slip mode, crack length environments, two major extrinsic mechanisms can be identified which have a profound influence on the value of ΔK_0 , namely crack deflection and crack closure. Both mechanisms act to reduce the nominal driving force, such as the far-field ΔK computed globally from applied stress, crack size and geometry measurements, to some lower effective value locally experienced at the crack tip. Furthermore, due to the small crack tip opening displacements and predominately crystallographic or single shear nature of crack advance mechanisms at low growth rates, both processes are prominent at near-threshold levels.

Crack deflection. With rare exception, most Mode I crack growth data have been correlated in terms of the stress intensity K_I , or non-linear elastic parameters such as J , assuming a linear crack, usually orientated perpendicular to the plane of maximum tensile stresses. However, frequently crack paths deflect or meander from this plane (Fig. 1) due to load excursions, metallurgical and/or environmental interactions with the consequence that the local driving force may be reduced (Suresh, 1983b). Analysis by Bilby and others (1977), for an elastic crack subjected to both tensile and shear loads give the local Mode I and Mode II stress intensity factors, k_1 and k_2 , at the tip of the deflected crack in terms of the nominal stress intensities for the linear crack, K_I and K_{II} , the angle of branching, θ , and angular functions $a_{ij}(\theta)$, as:

$$\begin{aligned} k_1 &= a_{11}(\theta)K_I + a_{12}(\theta)K_{II}, \\ k_2 &= a_{21}(\theta)K_I + a_{22}(\theta)K_{II}, \end{aligned} \quad (1)$$

such that the effective near-tip driving force can be considered as:

$$k_{\text{eff}} = (k_1^2 + k_2^2)^{1/2}. \quad (2)$$

For a simple 45° deflected crack, where the length of the branch is small compared to crack length (Fig. 1a), solutions to Eqs. (1) and (2) suggest roughly a 20 pct reduction in the local Mode I stress intensity factor, with $k_1 \approx 0.79K_I$, $k_2 \approx 0.33K_I$ and $k_{\text{eff}} \approx 0.86K_I$ (Suresh, 1983b). Thus crack deflection can both modify the effective driving force and induce a significant Mode II shear component at the crack tip.

In fatigue, crack deflection is promoted microstructurally in duplex structures where a hard phase may induce extensive crack path meandering (Suresh, 1983b; Dutta and others, 1984; Yoder and others, 1982) and at lower ΔK levels, especially in coarse planar slip materials, where crack advance becomes crystallographic in nature, involving primarily a single

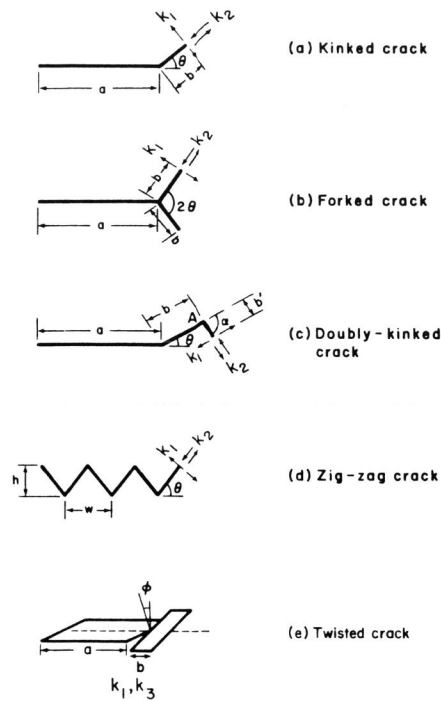


Fig. 1. Schematic illustration showing possible types of fatigue crack deflection and corresponding nomenclature to describe stress intensity factors (after Suresh, 1983b).

active shear band mechanism (akin to Forsyth's Stage I). Furthermore, significant crack deflection is often observed following tensile overloads (Suresh, 1983a) and with certain alloy/environment systems which induce crack branching, crystallographic growth or intergranular cracking.

In addition to modifying the local driving force, crack deflection also results in a longer crack path such that the measured growth rate of the deflected crack appears lower than that of the linear crack. Based on the two-dimensional elastic model of a non-linear crack consisting of linear segments of length, S , and segments deflected through an angle θ of length D , Suresh (1983b) has calculated that the stress intensity ΔK_{NL} and apparent (measured) growth rate $(da/dN)_{NL}$ for the two dimensional non-linear crack are given in terms of ΔK_L and $(da/dN)_L$ for the corresponding linear crack as:

$$\Delta K_{NL} \approx \left[\frac{D \cos^2(\theta/2) + S}{D + S} \right]^{-1} \Delta K_L$$

$$\text{and } (da/dN)_{NL} \approx \left[\frac{D \cos \theta + S}{D + S} \right] \left(\frac{da}{dN} \right)_L \tag{3}$$

Moreover, under cyclic loading in non-inert environments, the process of deflection induces rough irregular fracture surfaces and inelastic Mode II crack tip displacements, factors which promote the development of crack closure which further enhances the discrepancy between local and global crack driving forces.

Fatigue crack closure. As originally described by Elber (1970), fatigue crack closure can develop from the elastic constraint of material surrounding the plastic zone envelope in the wake of the crack tip acting on material elements previously plastically stretched at the tip. Since the cracks cannot propagate whilst closed, the resulting interference between mating fracture surfaces can lead to a reduction of the nominal driving force, i.e., $\Delta K = K_{max} - K_{min}$, to some lower near-tip effective value, $\Delta K_{eff} = K_{max} - K_{cl}$, where K_{cl} is the closure stress intensity at which fracture surface contact first occurs on unloading. With increasing load ratio, K_{cl} tends to K_{min} , with the result that closure effects become less significant as $\Delta K_{eff} \rightarrow \Delta K$.

It is now realized that such plasticity-induced closure is most effective under plane stress conditions (e.g., Lindley and Richards, 1974; Newman, 1983; Blom, 1984) and thus is relevant primarily at higher ΔK levels. At near-threshold growth rates where invariably plane strain conditions exist, crack closure is generated by a variety of alternative mechanisms, principally involving crack surface corrosion product formation (Ritchie and others, 1980; Stewart, 1980), irregular fracture surface morphologies coupled with inelastic crack tip shear displacements (Walker and Beevers, 1979; Minakawa and McEvily, 1981; Suresh and Ritchie, 1982), fluid-induced pressure inside the crack (Endo and others, 1972; Tzou and others, 1984) and metallurgical phase transformations (Hornbogen, 1978; Suresh and Ritchie, 1984a), as shown schematically in Fig. 2.

Closure induced by crack surface corrosion deposits becomes important where the size-scale of such deposits approaches the scale of crack tip opening displacements, and thus is very significant at near-threshold levels. The mechanism is most potent at low load ratios in moist, oxidizing environments, particularly in lower strength materials where fretting processes between the crack walls during the fatigue cycle can facilitate oxidation and promote excess debris formation. The resulting closure, which has been simply modelled in terms of the thickness of the oxide film, s , the location of the peak thickness from the crack tip, 2ℓ , Poisson's ratio, ν , and Young's modulus, E , as (Suresh and others, 1982):

$$K_{cl} \approx \frac{sE}{4\sqrt{\pi\ell}(1 - \nu^2)} \tag{4}$$

tends to offset the embrittling influence of an aggressive environment which might otherwise accelerate crack growth. Thus in material/environment systems where the susceptibility to classical corrosion fatigue is not large, fatigue thresholds may appear higher in the seemingly more aggressive environments. An example of this is in lower strength ferritic steels where oxide thickness measurements using Auger spectroscopy and direct experimental closure measurements have clearly shown that

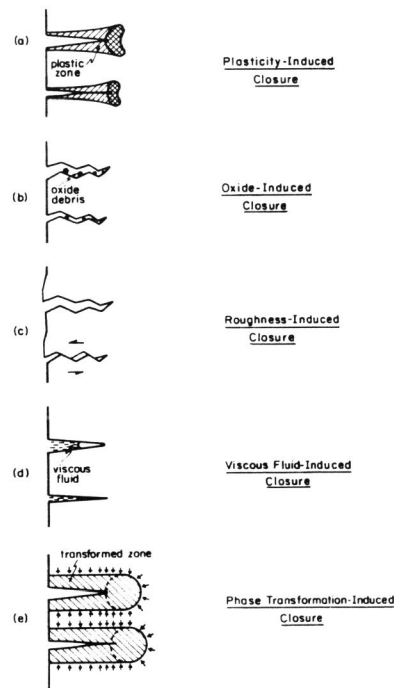


Fig. 2. Schematic illustration showing mechanisms of fatigue crack closure.

observations of ΔK_0 values at low R being smaller in dry helium gas and larger in water or steam, compared to room air, can be primarily attributed to enhanced oxide-induced closure in the moist environments (e.g., Suresh and others, 1981).

The morphology of the crack path can also promote crack closure when the size-scale of the fracture surface asperities becomes comparable with the crack tip opening displacements (Walker and Beevers, 1979; Minakawa and McEvily, 1981). Where crack paths are crystallographic or subjected to significant deflection or meandering (Fig. 3), the resulting irregular fracture surfaces, coupled with the induced Mode II shear component of the crack tip displacements, can lead to roughness-induced closure from mismatch and premature contact between asperities during unloading. The mechanism has been simply modelled in terms of the ratio of asperity height to width, γ , and the ratio of Mode II to Mode I displacements, u , as (Suresh and Ritchie, 1982):

$$K_{cl} \approx \sqrt{\frac{2\gamma u}{1 + 2\gamma u}} K_{max} \quad (5)$$

It is promoted at low load ratios in coarse grained and planar slip materials, particularly at lower ΔK levels where crack tip displacements

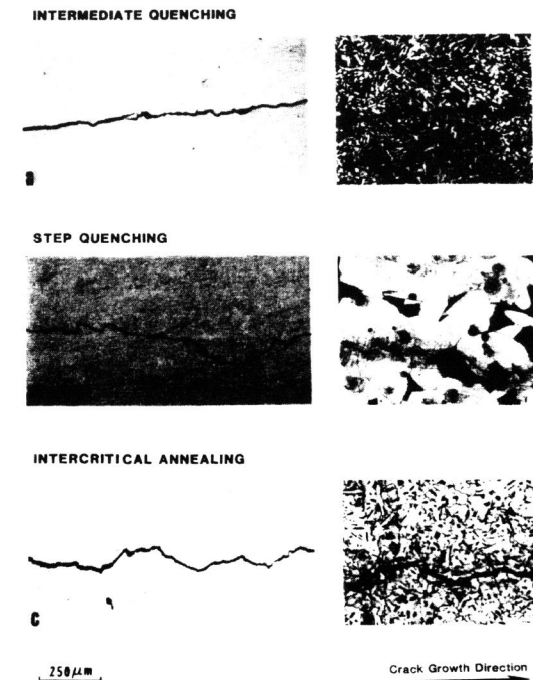


Fig. 3. Crack path morphology of near-threshold fatigue cracks in a dual phase steel, with duplex martensitic/ferritic microstructure, showing a relatively linear path in the intermediate quenched structure compared to meandering, deflected crack paths in the step quenched and intercritical annealed structures (after Dutta, Suresh, and Ritchie, 1984).

are small and crack growth tends to be crystallographic or faceted in nature, following load excursions or in duplex or other microstructures (e.g., Fig. 3) which promote crack deflection leading to tortuous crack paths (Suresh, 1983b; Dutta and others, 1984), and in certain hydrogen-containing environments which induce intergranular fracture (Esaklul and others, 1983).

Crack closure can also be generated in liquid environments from the hydrodynamic wedging effect of the fluid inside the crack during cyclic loading (Endo and others, 1972). Such fluids induce an internal pressure which acts to oppose the opening and principally the closing of the crack walls, such that if K_{max} is the maximum stress intensity from the fluid pressure, the effective stress intensity range at the tip is reduced from ΔK to $\Delta K_{eff} = \Delta K - K_{max}$ (Tzou and others, 1983). This mechanism results in fatigue crack growth rates in inert fluids being sensitive to frequency and viscosity, although the precise trend in behavior is difficult to predict since the higher viscosity fluids, which can induce higher internal

pressures, can be kinetically restricted in their ability to fully ingress into the crack. Recent theoretical analyses of this mechanism (Tzou and others, 1984) consider both the extent of fluid penetration, d , into a crack of length, a , modelled by capillary flow in terms of the average crack opening width, $\langle h \rangle$, surface tension, γ , density, ρ , wetting angle, β , and kinematic viscosity, η , as:

$$d^2(t) \approx \frac{\gamma \cos \beta}{3 \eta \rho} \int_0^t \langle h \rangle dt, \quad (6)$$

and the resulting internal fluid pressure $p(x)$ as a function of distance x along the crack (Fig. 4):

$$p(x) = 6 \eta \rho \frac{\langle \dot{h} \rangle}{\langle h \rangle^3} x(x-d), \quad \text{for } d/a < 1$$

$$= 6 \eta \rho \frac{\dot{h}_{\max}}{3} a^2 \ln(1-x/a), \quad \text{for } d/a = 1, \quad (7)$$

where \dot{h}_{\max} is the crack mouth opening width. Computations of the resultant closure indicate that for fluid viscosities in the range 5 to 100,000 cS, peak values of K_{\max} approach the mean stress intensity ($\frac{1}{2}(K_{\max} + K_{\min})$), such that the magnitude of this closure mechanism typically saturates at K_{c1}/K_{\max} of order 0.5. The mechanism is thus far less potent than oxide or roughness-induced closure.

Finally, crack closure can result from deformation-induced metallurgical phase transformations which induce a positive volume change, e.g., the martensite transformation in austenitic stainless steels. Since the constraint of surrounding elastic material will place regions of transformed material in compression, once entering such regions the crack will experience a resultant stress intensity tending to close it. Although not analyzed for deformation conditions involving deviatoric strains or cyclic inelasticity, recent studies for purely dilatant transformations in ceramics under monotonic loading suggest (McMeeking and Evans, 1982):

$$K_{c1} \approx \frac{0.22}{(1-\nu)} V_f \epsilon_T E \sqrt{r_t}, \quad (8)$$

where ϵ_T is the transformation strain in the transformed zone of width r_t , and V_f is the associated volume change. This analysis further implies that the full effect of transformation-induced closure is only felt once the crack has penetrated the transformed zone over a distance comparable to ~ 5 times the width of the zone, indicating a "resistance curve" effect of less closure at shorter crack lengths.

Interpretations

As discussed in some detail elsewhere (e.g., Suresh and Ritchie, 1984a), considerations of crack deflection and primarily crack closure have provided a convenient framework to explain many behavioral characteristics of the fatigue threshold and near-threshold growth rates, both under constant amplitude and variable amplitude conditions. However, quantitative

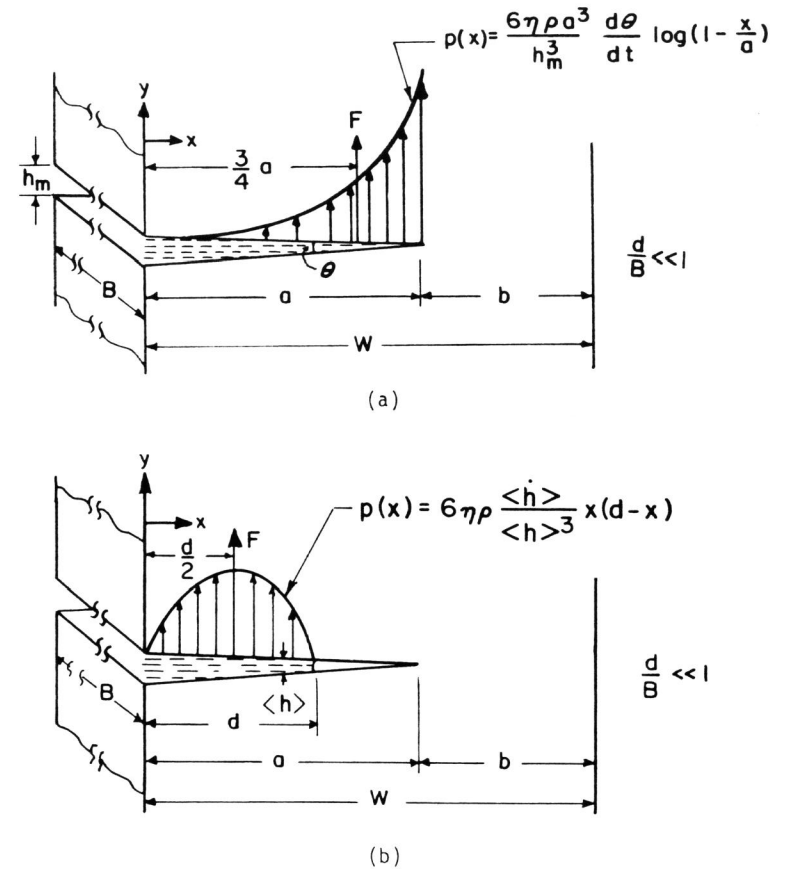


Fig. 4. Schematic illustration of the internal pressure distribution $p(x)$ and resultant force F due to a) full penetration and b) partial penetration of a fluid inside an edge crack (after Tzou, Hsueh, Evans, and Ritchie, 1984).

assessments of the implications of such mechanisms have not in every case been developed. Accordingly, many explanations of near-threshold behavior attributed to closure phenomena, both here and in the literature, may be regarded as being somewhat superficial. Nevertheless, the contribution of crack closure mechanisms to the growth rate characteristics of fatigue cracks close to ΔK_0 is clearly dominant, as briefly summarized below in the context of the major mechanical, microstructural and environmental factors affecting near-threshold behavior.

Effect of load ratio (R). As recognized in early studies by Elber (1970), the load ratio dependence of growth rates, at all but very high stress intensities approaching final failure, is strongly associated with closure.

This can be seen in Fig. 5 where numerical computations (Blom, 1984) on plasticity-induced closure indicate a progressive reduction in closure (i.e., $K_{C1}/K_{max} \rightarrow K_{min}/K_{max}$) with the increasing crack opening displacements associated with high load ratios. Similarly, experimental crack surface oxide measurements for a low alloy steel fatigued in moist air, dry hydrogen and distilled water environments (Fig. 6) clearly show a direct correlation between the extent of oxidation, and hence oxide-induced closure, and the fatigue threshold ΔK_0 , with change in R (Suresh and Ritchie, 1983). Here, in the moist air and hydrogen atmospheres where oxide thicknesses comparable with crack opening displacements only form at the lower load ratios (by fretting oxidation mechanisms), the R-dependence upon ΔK_0 values is quite marked. In contrast, the more oxidizing water environments permit significant oxide formation at all load ratios (by thermal mechanisms), and the R-dependence on ΔK_0 is correspondingly far shallower (Fig. 6). With such data, it can be shown that the physical existence of a threshold is consistent, at least in steels, with the cyclic crack tip opening displacements approaching the excess oxide, or asperity, thickness. Thus, with the less oxide debris and greater opening displacements at high R, ΔK_0 values must decrease (Suresh and others, 1981).

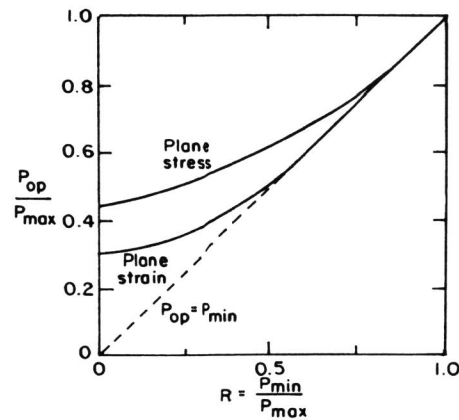


Fig. 5. Numerical predictions of the crack opening loads P_{op} , normalized by the maximum load P_{max} , as a function of load ratio, P_{min}/P_{max} , for plasticity-induced closure under plane stress and plane strain conditions (after Blom, 1984).

Effect of variable amplitude loading. The transient growth rate response of Mode I fatigue cracks has also been attributed to major contributions from closure and deflection processes. For example, recent analyses (Suresh, 1983a) interpret the retardation (and sometimes arrest) following tensile overloads to an initial branching of the crack at the overload, which reduces the local driving force through crack deflection (e.g., Eqs. 1 and 2), and which further induces fracture surface roughness leading to

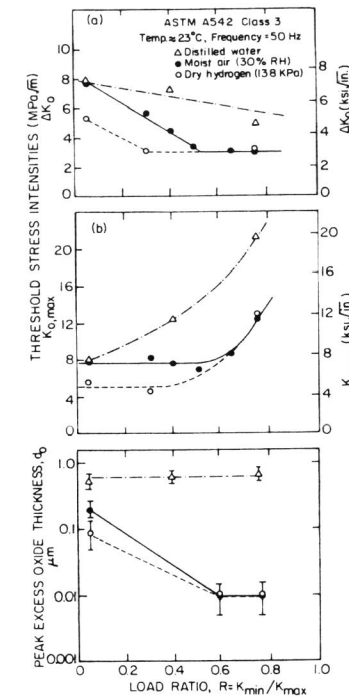


Fig. 6. Variation of alternating and maximum stress intensities at threshold (ΔK_0 and $K_{0,max}$, respectively) and the maximum excess oxide thickness with load ratio for a $2\frac{1}{2}\text{Cr}-1\text{Mo}$ steel tested in moist air, distilled water and dry hydrogen gas (after Suresh and Ritchie, 1983).

enhanced closure (Fig. 7). Once retardation has been established, due to these local reductions in driving force, crack growth can be considered to be essentially occurring at near-threshold levels. There are two major consequences of this. First, the post-overload retardation period can be prolonged by development of the various near-threshold mechanisms of closure. For example, there is ample evidence that fracture surfaces show abrasion and excess corrosion deposits in the post overload crack growth regime, indicating an enhanced role of oxide-induced closure (Hertzberg, 1976; Suresh, 1983a). Second, in ranking the crack growth resistance of alloys, it is generally observed that microstructures with good near-threshold resistance show similar good resistance to variable amplitude loading spectra. For example, planar slip materials such as underaged aluminum alloys, which are inferior under overaged microstructures above $\sim 10^{-5}$ mm/cycle, are far superior under near-threshold and variable amplitude loading conditions because of their tendency to follow crystallographic or meandering crack paths, and hence to enhance closure and deflection, at lower growth rates.

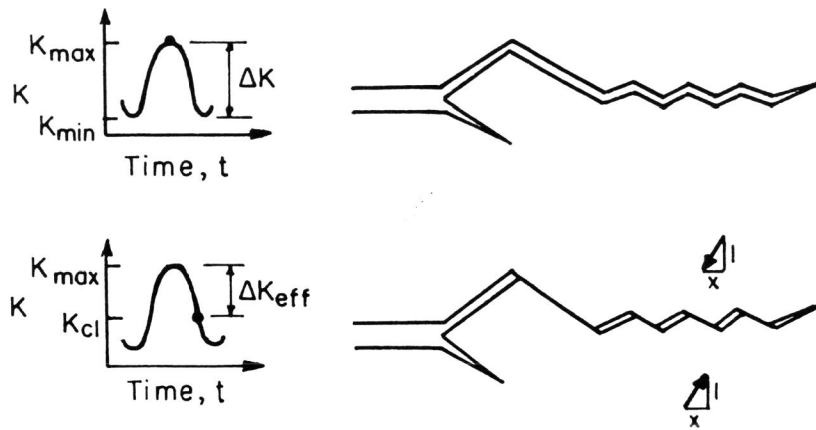


Fig. 7. Schematic illustration of the mechanisms for retardation following a tensile overload cycle, showing both crack deflection and closure due to the roughness of the post-overload fracture surface (after Suresh, 1983a).

Effect of strength level. The marked influence of yield strength on fatigue thresholds, particularly in ferrous alloys, can also be attributed to a strong function of closure. Recent studies by Ritchie, Suresh and Liaw (1982) found that the decrease in ΔK_0 with increasing strength level, in a wide spectrum of steels, was consistent with a sharp decrease in measured crack surface oxide debris and hence with less oxide-induced closure. The formation of less oxide in higher strength materials is presumably associated with less fretting between the crack surfaces due to less plasticity-induced closure and the fact that the fretting surfaces are much harder. Although unproven as yet, the lower ΔK_0 values in higher strength steels may also be associated with less roughness-induced closure, as the martensitic or bainitic microstructures associated with such high strength tend to be of a far finer scale than the ferritic and pearlitic structures associated with lower strength conditions. Such explanations based on closure arguments are consistent with observations that at high load ratios, where closure influences are minimized, the variation in ΔK_0 with strength level is much reduced (Ritchie, 1979).

Effects of microstructure. Of the many microstructural factors which may influence near-threshold behavior, three stand out as having major effects on growth rates, namely those of grain size, slip mode and duplex structures. For example, there are now many results showing increased threshold levels in coarser grained structures (e.g., Gray and others, 1983), in planar slip materials (e.g., Carter and others, 1984) and in dual phase alloys (e.g., Dutta and others, 1984), and that such microstructural improvements in crack growth resistance predominate at lower load ratios. The role of closure and deflection processes is again essential since coarse grains, planar slip and duplex structures all promote rougher, more faceted crack paths, i.e., crystallographic or deflected at grain or phase boundaries, such that slower growth rates result from a lower local crack driving force due to i) a longer path length, ii) crack deflection mechanics, and principally iii) enhanced roughness-induced closure. These effects can be extremely large, as shown in Fig. 8, where low load ratio

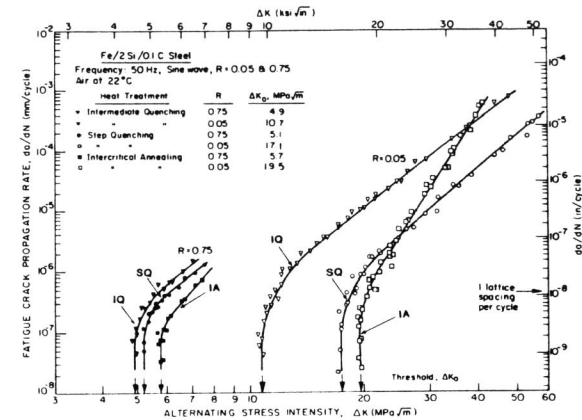


Fig. 8. Fatigue crack propagation rates (da/dN) as a function of ΔK in a dual phase steel, showing the marked increase in ΔK_0 (at $R = 0.05$) in the step-quenched (SQ) and intercritical annealed (IA) structures, compared to the intermediate quenched (IQ), due to crack path meandering (see Fig. 3) (after Dutta, Suresh, and Ritchie, 1984).

ΔK_0 values for an Fe-2Si-0.1C dual phase steel have been increased to almost 20 MPa√m by utilizing duplex ferritic-martensitic microstructures which induce a tortuous crack path (Fig. 3).

Effects of environment. The question of environmental factors novel to the near-threshold regime have been analyzed recently by Gangloff and Ritchie (1984). Here, although conventional ideas of corrosion fatigue dictate an accelerating influence of an active environment on crack extension, through such mechanisms as hydrogen embrittlement, metal dissolution and so forth, environmental factors may simultaneously promote crack closure and/or deflection mechanisms, which can offset this acceleration particularly at lower growth rates. For example, in lower strength steels, threshold ΔK_0 values, at low R , have been observed to be considerably lower in dry environments, such as dry helium or hydrogen gases, compared to room air, due to the formation of excess corrosion deposits, and hence more closure, in moist atmospheres (Ritchie and others, 1980; Stewart, 1980). Conversely, in high strength steels where fretting oxide formation is restricted, threshold ΔK_0 values may be higher in dry hydrogen gas, compared to room air, in part because the hydrogen induces intergranular fracture which enhances roughness-induced closure (Toplosky and Ritchie, 1981; Liaw and others, 1983; Esaklul and others, 1983). Moreover, environmental effects may also induce hardening or softening (e.g., Hirth, 1980) thereby affecting plasticity-induced closure, cause crack branching thereby promoting crack deflection, or can result in fluid-induced closure (Tzou and others, 1983) from the presence of an aqueous or viscous fluid within the crack. Thus, in the general sense, interpretation of the role of a specific environment in influencing fatigue threshold behavior may be extremely complex since the contribution to crack advance from environmentally-assisted mechanisms must be balanced with the possible

retarding influence of closure and deflection mechanisms which result from this environmental interaction.

UNIQUENESS OF FATIGUE THRESHOLDS

The usefulness of fatigue threshold data has often been questioned by uncertainties in the uniqueness of ΔK_0 values. For example, thresholds have been shown to vary in a given system with such factors as measurement technique, specimen geometry, thickness and especially crack size. Furthermore, in the presence of variable amplitude loading, ΔK_0 values are known to depend markedly on the extent of tension or compression overload cycling. These questions are now examined in the light of our understanding of the near-threshold crack growth mechanisms of closure and deflection discussed above.

Effect of Measurement Procedure

Current ASTM suggested procedures for measuring ΔK_0 values involve load shedding at a rate designed not to yield premature arrest (e.g., Bucci, 1981). Too rapid a reduction in ΔK_0 values on the approach to ΔK_0 (e.g., by greater than 10%) is known to result in overly high threshold values by mechanisms analogous to the retardation following high-low block loading, i.e., presumably by enhancing plasticity-induced closure. However, in certain alloys tested under corrosive conditions, too slow a reduction in ΔK can similarly result in higher threshold values (Bucci, 1981), this time by allowing excess corrosion debris to promote oxide-induced closure. In fact, recent studies by Cadman, Nicholson and Brook (1984) confirm that the variation in ΔK_0 value with load shedding rate does indeed go through a minimum. However, over the range of load shedding rates recommended by ASTM (i.e., $1/K(dK/da)$ between -0.08 and -0.024 mm^{-1}) the variation in threshold values has generally been found to be negligible (Donald and Schmidt, 1980).

Where constant K_{max} /increasing K_{min} cycling is alternatively employed, lower bound threshold values are generally claimed (Schulte and others, 1980; Radon, 1982). Such loading spectra, however, yield very high load ratios close to ΔK_0 , thereby minimizing closure effects and hence lowering the threshold.

Effect of Specimen Geometry

Somewhat surprisingly few studies have explicitly examined the role of specimen geometry and size on near-threshold behavior. Radon (1982), however, noted a large influence of testpiece thickness on ΔK_0 in a low alloy steel (yield strength 386 MPa). Although thresholds were unchanged at high load ratios (i.e., $R = 0.9$), values varied between 10 and 3 MPa $\sqrt{\text{m}}$ for thicknesses of 12 and 50 mm, respectively, at $R \sim 0.1$. The lack of an effect at high R is certainly consistent with an explanation based on crack closure, yet evidence to support any change in K_{C1} with thickness was not obtained. Furthermore, Radon's observations have yet to be confirmed by other workers.

Similarly, Horng and Fine (1984) claim an effect of specimen geometry on threshold behavior in an HY80 dual phase steel ($R_C 38$). Here, for tests in argon at $R = 0.05$, ΔK_0 values were found to vary between 6.5 and 5.4 MPa $\sqrt{\text{m}}$ for tests on single-edge-notched and center-cracked sheet specimens, respectively, when analyzed in terms of nominal ΔK values, yet to be identical when analyzed in terms of ΔK_{eff} through corrections for experimentally measured closure. Such experiments clearly imply that the effect of specimen configuration on ΔK_0 values originates from variations in crack closure, yet the mechanism is unexplained, the reported differences in ΔK_0 are small, and the results is as yet unconfirmed by other investigations.

Effect of Crack Surface Removal

Recent studies in P/M 7090 (McEvily and Minakawa, 1984) and I/M 7150 (Zaiken and Ritchie, 1984) aluminum alloys have shown that progressive machining, by electro-discharge or mechanically, of material along the crack face to within 1 to 0.5 mm behind the tip can cause a previously arrested threshold crack to recommence propagation at the nominal ΔK_0 value (Fig. 9a). Such results confirm that the existence of a threshold at low load ratios is intimately linked to the development of crack closure. Furthermore, by monitoring K_{C1} values during the sequential removal of material, the physical location of such closure along the crack can be deduced. For example, for a 7150-T6 alloy at the threshold ($R = 0.1$), the closure was found to be dispersed along the entire crack length but to be primarily developed in the immediate vicinity of the crack tip, i.e., 50% of the original K_{C1} value is developed within 1 mm of the tip (Fig. 9b). Similar removal of closure, and hence subsequent recommencement of crack growth at the threshold, can be achieved through the application of large compression overloads (Zaiken and Ritchie, 1984), as discussed below.

Effect of Crack Size

Perhaps the major variable leading to variations in threshold values is the question of crack size, where crack lengths become small compared to the scale of microstructure or scale of local plasticity, i.e., typically $< 1 \text{ mm}$ (e.g., Suresh and Ritchie, 1984b). There is now a large body of evidence indicating that "short" crack growth occurs at stress intensities below the "long crack" threshold at generally progressively decreasing growth rates (Fig. 10), until arresting or merging into the expected long crack behavior (e.g., El Haddad and others, 1979; Lankford, 1982; James and Morris, 1983). However, such behavior is largely specific to low load ratio conditions (McCarver and Ritchie, 1982).

Although there is a multitude of factors leading to differences in behavior between long and short cracks, including the use of inappropriate crack tip characterizing parameters (e.g., Dowling, 1977), notch effects (e.g., Hammouda and Miller, 1979), differing local strain fields (e.g., Lankford, 1983), microstructure effects (e.g., Schijve, 1982) and environmental effects (e.g., Gangloff, 1981)...for recent reviews see Suresh and Ritchie (1984b) for overview and Gangloff and Ritchie (1984) for the specific role of environment...the major factor for near-threshold behavior pertains to differing local crack tip fields resulting from such mechanisms as deflection and particularly closure. For example, the local retardations experienced by microstructurally-short cracks as they encounter different phases or grain boundaries can be assessed in terms of crack deflection

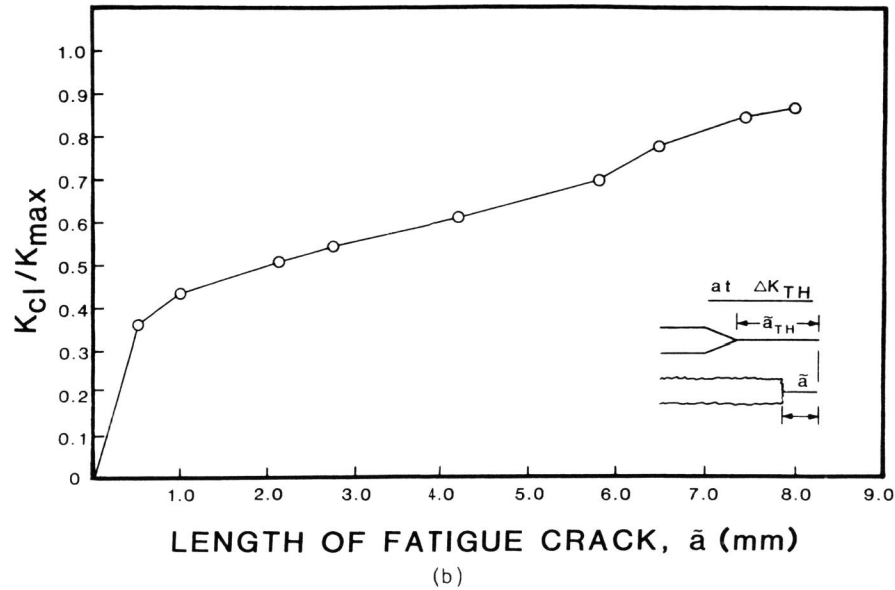
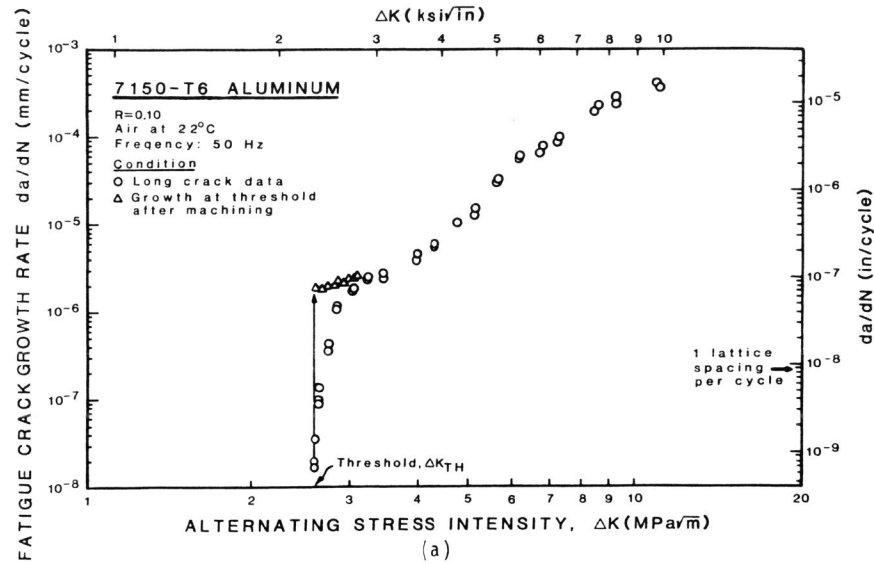


Fig. 9. a) Fatigue crack propagation rates (da/dN) and b) experimental closure measurements (K_{C1}) for 7150-T6 aluminum alloy at threshold ($\Delta K_{TH} = \Delta K_0$) following machining away of material in the wake of the crack tip. Note how b) 50% of the closure is within 1 mm of the crack tip and that once such closure has been removed, a) the previously arrested crack re-commences to grow (after Zaiken and Ritchie, 1984).

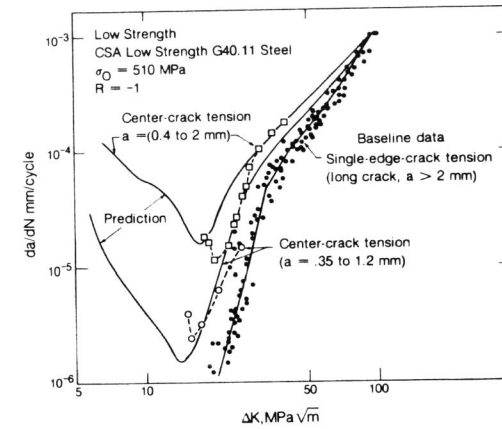
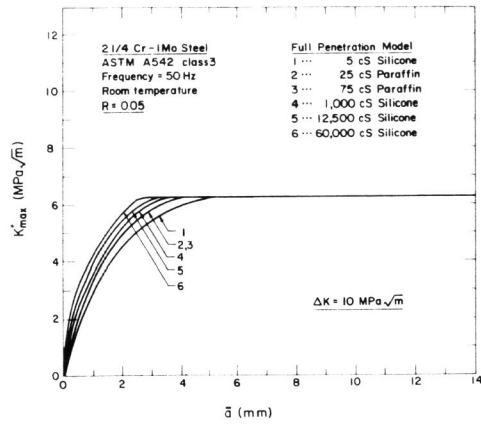


Fig. 10. Comparison of experimental results and numerical predictions for the propagation rates of small cracks in a low strength structural steel (after Newman, 1983).

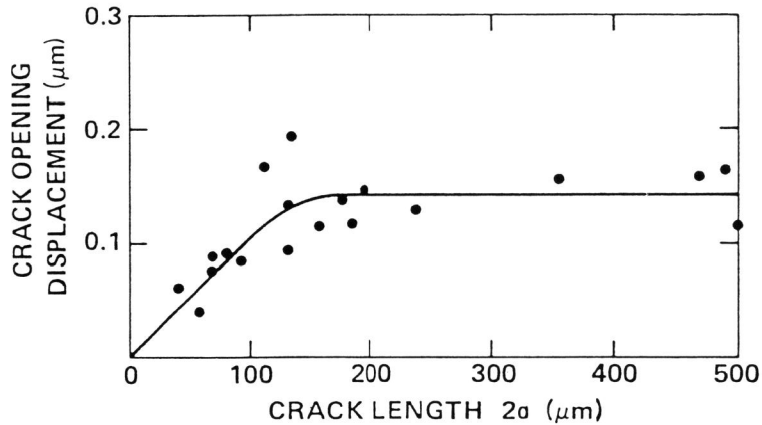
(Suresh, 1983b), although as yet there is no crack length dependence on the magnitude of the effect (e.g., in Eqs. 1-3) and no consideration of the role of crack tip plasticity. More importantly, due to their restricted wake, short flaws are far less influenced by crack closure, which normally acts on this wake behind the crack tip. Although there are currently few quantitative models for closure (e.g., Eqs. 4, 5 and 8) which incorporate any crack length dependence (fluid-induced closure is an exception, Eqs. 6 and 7), experimental evidence for roughness-induced closure in titanium alloys (James and Morris, 1983) and theoretical predictions for fluid-induced closure in steels (Tzou and others, 1984), all show a marked reduction in the magnitude of the closure (e.g., K_{C1}) at small crack sizes (Fig. 11). This implies that at the same nominal driving force, e.g., same ΔK or ΔJ , the short crack will experience a larger local (near-tip) driving force, e.g., higher ΔK_{eff} , than the long crack due to a reduced influence of closure.

Thus, due to this higher effective driving force, the short crack is to be expected to be able to propagate below the "long crack" threshold ΔK_0 . However, as it grows in size, the corresponding development of closure will generally outweigh the normal increase in nominal ΔK (due to increased length), leading to progressively decreasing growth rates until arrest (i.e., non-propagating cracks) or merging with long crack behavior. Theoretical studies on near-threshold short cracks by Newman (1982), where plastically-induced closure was modelled numerically, can reproduce such behavior (Fig. 10). Moreover, experimental studies by Tanaka and Nakai (1983) on short cracks emanating from notches in a structural steel clearly show that the higher sub-threshold growth rates of the short cracks, when compared with long crack behavior in terms of nominal ΔK values, disappear (i.e., long and short crack results coincide) when the data are reanalyzed in terms of ΔK_{eff} from experimental closure measurements (Fig. 12).

Thus, the uniqueness of fatigue threshold values can be expected to be challenged where crack sizes become small enough to limit the development



(a)



(b)

Fig. 11. Effect of small crack size, a , in reducing the extent of crack closure. a) theoretical predictions of the stress intensity due to fluid-induced closure K_{max} for 2 1/4Cr-1Mo steel in silicone and paraffin oils (after Tzou, Hsueh, Evans, and Ritchie, 1984) and b) experimental measurements of the crack opening displacements at zero load due to roughness-induced closure in a titanium alloy (after James and Morris, 1983).

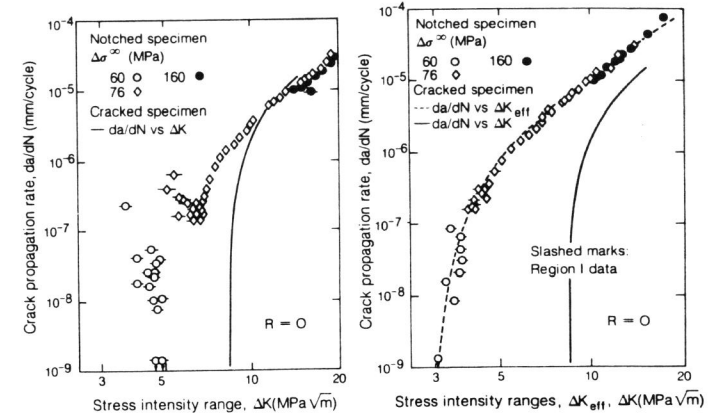


Fig. 12. Variation in crack propagation rates with a) nominal, ΔK and b) effective stress intensity range, ΔK_{eff} , respectively, for small cracks (data points) and long cracks (solid curves) in a structural mild steel. Note how anomalous (sub-threshold) behavior of short cracks is brought into correspondence with the long crack data when closure is accounted for (after Tanaka and Nakai, 1983).

of crack closure, i.e., over crack sizes ranging from 10 μm in high strength alloys to above 1 mm in low strength materials. This fact clearly places the concept of a fatigue threshold in question from the perspective of applications in engineering design. However, since crack closure is a dominant mechanism, the "short crack problem" is largely confined to low load ratio behavior, and to the first approximation thresholds for short flaws can be expected to be of the order of long crack thresholds measured at high load ratios (i.e., above $R \sim 0.75$).

"ANOMALIES"

The fatigue threshold has sometimes been described as the crack propagation analogue of the fatigue limit (or endurance strength), since both parameters represent limiting conditions for fatigue failure. Whereas the former defines a stress intensity range below which pre-existing long cracks will not grow in the fracture mechanics based defect-tolerant fatigue design approach, the latter represents a stress range below which failure of a smooth bar will not occur (or exceed say 10^8 cycles) in the classical stress/life (S/N) approach. Kitagawa and Takahashi (1976) first attempted to unify these two concepts by plotting threshold stress, $\Delta\sigma_{TH}$, versus crack size, a , and showing how the limiting condition of a constant stress intensity (equal to the threshold ΔK_0) at large crack sizes becomes one of a constant stress (approaching the fatigue limit $\Delta\sigma_e$) at small crack sizes (Fig. 13).

However, there are certain anomalies with respect to such factors as microstructure and overload cycling. For example, in general i) coarser

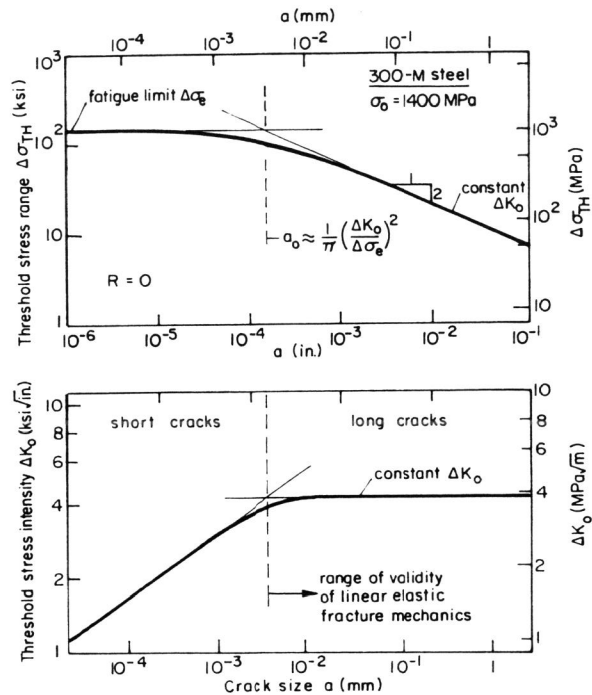


Fig. 13. Variation with crack size, a , of threshold stress range, $\Delta\sigma_{TH}$, and threshold stress intensity range, ΔK_0 , for no crack growth in ultrahigh strength Si-modified 4340 (300-M) steel at $R = 0$.

microstructures promote high thresholds whereas finer microstructures promote high fatigue limits, ii) lower yield strength promotes high thresholds (at least in steels) whereas higher yield strength promotes high fatigue limits, iii) thresholds can be reduced by periodic compression overloads, whereas fatigue limits can be reduced by periodic tension overloads. These apparent paradoxes pose a real problem to the fatigue designer who may need to select different materials for the same application depending upon whether a defect-tolerant or a stress/life design approach is being used. However, explanations for such anomalies can again be found within the context of crack closure.

Effect of Microstructure

In simple terms, both increasing grain size and decreasing tensile strength tend to decrease fatigue limits (in the absence of environmental or notch sensitivity effects) by enhancing the plastic flow required for the evolution of a crack. Whereas such factors may have a similar influence on the continued propagation of an existing flaw, their parallel effect in promoting crack closure clearly outweighs this such that thresholds in




general are increased, at least for long cracks at low load ratios. Observations i) that the variation in ΔK_0 with either strength or grain size is far less apparent at high load ratios (e.g., Ritchie, 1979), ii) that the extent of oxide-induced closure increases with decreasing strength (Ritchie and others, 1982) and iii) that the extent of roughness-induced closure increases with increasing grain size (Gray and others, 1983) are clearly consistent with this notion. Thus the anomalous effects of microstructure on the fatigue threshold, as opposed to the fatigue limit, result principally from an effect on closure. Furthermore, since such closure is principally a long crack, low load ratio phenomenon, it is to be expected that thresholds for short cracks will not follow the same trend but rather to vary in similar fashion to fatigue limits.

Effect of Overload Cycling

It is well known that a single tensile overload can markedly reduce, or even eradicate, a fatigue or endurance limit in smooth specimen S/N tests (Brose and others, 1974). Recently, however, Au, Topper and El Haddad (1983) have shown that compression overloads markedly reduce threshold ΔK_0 values, based on tests on center-cracked specimens of 2024-T351 aluminum alloy. In the latter experiments, ΔK_0 values of approximately 3 MPa \sqrt{m} at $R = 0$ were reduced to less than 1 MPa \sqrt{m} by intermittent compression overloads, the effect being dependent upon the magnitude of the overload and its frequency of occurrence. Such compression overloads can leave residual tension ahead of a crack, although the effect is thought to be primarily associated with a reduction in closure during the compressive portion of the cycle due to the flattening out of fracture surface asperities. Blom (1984) has experimentally measured such a reduction in roughness-induced closure for cycling at $R = -1$ compared to $R = 0$. Moreover, Zaiken and Ritchie (1984) have monitored such a reduction following a single compressive overload (magnitude ~ 5 times ΔK_0) applied to an arrested crack at ΔK_0 . Similar to Fig. 9a, the arrested crack then recommenced to propagate following the compressive overload.

This effect is a further example of the differing response of crack propagation (da/dN vs. ΔK) and smooth specimen stress-life (S/N) tests due to a varying influence of closure. For example, it is now well established that single tensile overloads can cause reduction, or even arrest, in long crack growth rates, which accordingly increases lifetimes. However, such overloads in a smooth specimen S/N test are known by Miner's Law to decrease life (e.g., Hertzberg, 1976). Although overloads can be considered to promote "damage," thereby reducing S/N life, their occurrence in the presence of "long" Mode I cracks additionally promotes crack deflection and crack closure (Fig. 7). The latter mechanisms tend to dominate long crack behavior such that tensile overloads generally lead to an increase in life (e.g., Hertzberg, 1976; Lankford and Davidson, 1981; Suresh, 1983a). However, in the case of the growth of short cracks, where the role of closure is minimized, such transient retardations due to variable amplitude loading are hardly apparent (Potter, 1983). Torsion experiments on fatigue crack growth in anti-plane shear also show this effect particularly clearly since, apart from a sliding interference between crack surfaces (Tschegg and others, 1983), crack closure cannot develop for Mode III crack advance. Nayeb-Hashemi, McClintock and Ritchie (1983) compared the response of Mode I and Mode III cracks to various simple loading spectra in rotor steels and correspondingly found markedly contrasting behavior (Table I).

TABLE 1 Transient Growth Rate Response of Mode I and Mode III Fatigue Cracks Following Simple Loading Spectra

Load Sequence	Transient Growth Rate Response	
	Mode I	Mode III
	da/dN decreases	da/dN increases
	above effect reduced	above effect enhanced
	da/dN slower*	da/dN faster*

*Compared to steady-state da/dN at lower load level.

Whereas the initial growth rates of Mode I cracks were reduced following the application of a single positive overload, the initial growth rates of Mode III cracks were increased (Fig. 14). For fully reversed overloads (e.g., tension followed by compression), the effect was reduced in Mode I yet enhanced in Mode III. Similarly the response to high-low block loading sequences was exactly opposite in Mode III compared to Mode I. Clearly, the transient response of long Mode I cracks to variable amplitude loading, i.e., retardations following positive overloads and so forth, is dominated by closure, whereas for S/N tests, short cracks or Mode III cracks, where closure influences are minimized, distinctly different behavior, as predicted by Miner's Law or damage accumulation arguments, is to be expected.

Such examples serve to illustrate the difficulty in generalizing fatigue behavior as reported for a single type of test. The micro-mechanisms governing the propagation of long cracks clearly can be far different from those involved in crack initiation or the growth of short cracks, particularly where crack closure effects are important.

CONCLUDING REMARKS

In this review, an attempt has been made to highlight how the fatigue threshold and associated near-threshold crack growth behavior are uniquely influenced by such mechanisms as crack deflection and particularly crack closure. It has been shown that mechanical factors, such as load ratio and variable amplitude loading spectra, microstructural factors, such as strength, grain size and slip mode, and certain environmental factors all influence crack extension at ultralow growth rates through their effect on such mechanisms. Further, although they play a role at higher growth rates, both deflection and closure appear to be far more potent close to ΔK_0 , particularly for long cracks, at low load ratios and following variable load excursions. Moreover, a consideration of these mechanisms provides a basis for explaining the contrasting microstructural effects observed for crack initiation as opposed to crack growth, and thus provides a unifying link between classical S/N and long crack propagation data.

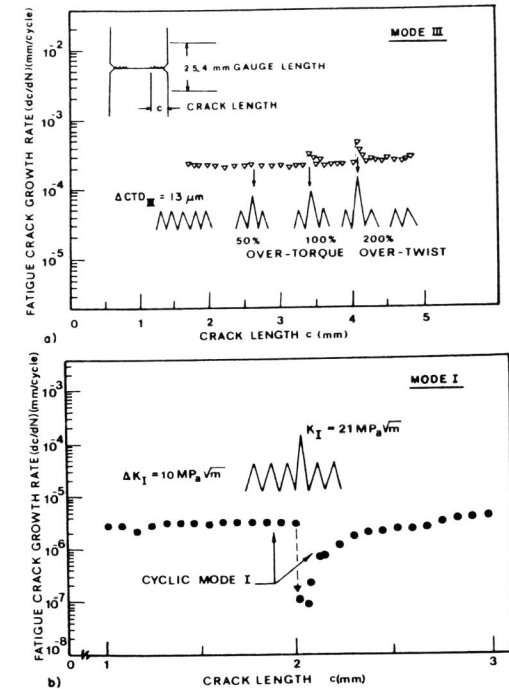


Fig. 14. Experimental fatigue crack growth rate data in A469 rotor steel showing a) the acceleration of Mode III cracks and b) the deceleration of Mode I cracks, following the application of a single positive overload (after Nayeab-Hashemi, McClintock, and Ritchie, 1983).

Both crack deflection and crack closure act to vary the local, as opposed to the global or nominal, driving force. In this regard, they serve to promote the invalidity of the fracture mechanics similitude concept whereby cracks of different lengths, subjected to the same nominal driving force, are assumed to advance at equal rates; a concept which currently forms the basis of scaling of laboratory data to predict component life. Although such scaling has been established for long fatigue and statically loaded cracks extending at higher growth rates in essentially benign environments, stress intensity based similitude must also be modified to account for several other phenomena. These include i) excessive crack tip active plasticity, where the size of the plastic zone compared to crack size necessitates the use of non-linear fracture mechanics (e.g., Dowling, 1977), ii) plasticity in the wake of a moving crack, where the crack tip strain singularity is weakened from a $1/r$ -type for a stationary crack to a $\ln(1/r)$ type for a non-stationary crack (Drugan and others, 1982), and iii) environmental effects, where the chemical driving force is crack size, shape and applied stress sensitive due to specific effects on mass

transport and electrochemical reaction (e.g., Gangloff, 1981; Gangloff and Ritchie, 1984).

For these reasons, care must be taken when applying laboratory data based on a nominal stress intensity characterization to predict in-service component life. This is particularly true for fatigue threshold data, since many of the prominent mechanical, microstructural and environmental factors which govern its value cannot be uniquely accounted for by the use of current crack tip field parameters.

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