THERMAL CRACK TIP BLUNTING IN HIGH SPEED CRACKS

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ABSTRACT

An elastic model of crack tip blunting is developed in order to describe the effect of decreasing yield stress. This is used in conjunction with a two-parameter fracture criterion to describe rapid increases in K and G at both very long and very short times. At the latter, adiabatic crack tip heating is invoked to give the decrease in yield stress. The theory gives good agreement with observations in both impact tests and on high speed cracks. A criterion for crack bifurcation arises from the analysis.

KEYWORDS

High speed cracks; thermal; blunting; impact; slow crack blunting.

INTRODUCTION

This work was prompted by measurements of G_c-a curves for high speed cracks in polymers (Crouch and Williams). High speed here is taken as greater than 1 m/s and was typically in the region 100-700 m/s. The curves for several polymers at several test temperatures had much the same form with G_c remaining constant at about the initiation value, or somewhat below, until at some speed in the 300-500 m/s region it increased very rapidly giving increases of several times in G_c for very small crack speed increases. The method used to determine these curves will be described elsewhere (Crouch and Williams), but it should be mentioned that there is always some measure of doubt about G_c (or K_c) calculations in this crack speed range because of inertial effects. However, the crack speeds are rarely more than half half the shear wave velocity, so errors are unlikely to be very large. In addition, the G_c-$\dot{a}$ curves are very steep, so the form will not be changed significantly by G_c calculation errors.

This view is supported by observations by other workers using very different methods.(Kobayashi and co-workers 1980a, 1980b) have reviewed Homolite 100 and Araldite B data obtained using a photoelastic method. These results, together with his own data from a caustic method, have also been reviewed by (Kalthoff, 1983). All these results show the same very steep rise following an initially almost constant value. Because this phenomenon appears to happen in a wide range of polymers, including both thermoplastics and thermosets, and is so distinct a change in behaviour, one is led to speculate if it is a true reflection of a micromechanism. Rather, perhaps, it is a reflection of a transition in the mechanics of the crack propagation process.

The situation is closely parallel to that observed (Williams and Hodgkinson, 1981) in the impact testing of polymers. Here, G_c was determined as a function of loading time and a similar abrupt transition was noted from almost constant values for $t > 0.1$ ms to an almost vertical rise for $t < 0.1$ ms. This was explained in terms of thermal blunting mechanism at the crack tip. The energy dissipated in the plastic zone was retained locally at short times and gave a consequent local rise in temperature. This in turn, resulted in a drop in both yield stress and modulus which led to a blunting of the crack tip from the local plastic deformation and a consequent rise in G_c. The predictions of this analysis generally explained the observations quite well and there is an obvious parallel with the high speed crack data. A note of caution is in order here, however, since the likely time-scales for the running crack are very much less than those recorded in impact. If plastic zone sizes are taken as being of the order of 25 μm, then for a crack speed of 250 m/s the time is approximately 10^{-7} s $(t \approx r_p/\dot{a})$ compared with a minimum of 10^{-4} s in impact.

The crack blunting theory drew on some much earlier work (Constable, Culver, and Williams, 1970) and, recently, this has been successfully applied to data obtained on a series of epoxy resins to explain stick-slip crack propagation (Kinloch and Williams, 1980). The same basic argument was used in that a decreasing yield stress, from whatever cause, resulted in an increasing crack opening displacement (COD) and consequent crack blunting. Direct measurements were made using mechanically blunted notches and those for which the blunting was calculated from the COD, and excellent agreement was obtained. While this was a very useful analysis, it was not completely consistent with that used in the thermal case in that the thermal analysis did not include a full coupling of the deformation and energy questions. Further anomalies also arose with regard to the very successful constant COD criterion used for slowly running cracks (Marshall, Coutts and Williams, 1974) which is clearly at variance with a blunting analysis based on an increasing COD. The opportunity provided by the attempt to extend the analysis to the high speed crack case has therefore been taken to try and remove some of these deficiencies in earlier versions which have become apparent with the advantage of hindsight.

CRACK BLUNTING ANALYSIS

This analysis is based on the assumption that the behaviour of a blunted crack can be described by the elastic stress field for a crack with a finite tip radius, R, in the same manner as the K field is used for sharp cracks. From the solution for an elliptical hole (Williams, 1980b) subjected to a stress p normal to its semi-major axis, a, we may derive an expression for the stress p_c a distance r_c from the crack tip as:

$$\frac{p_c}{p} = \frac{2\sqrt{a}\,(R + r_c)}{(R + 2r_c)^{3/2}} \qquad (R, r_c << a) \qquad (1)$$

(see Fig. 1). For the sharp crack, $R = 0$, we have:

$$\frac{p_c}{p} = \sqrt{a/2r_c}$$

so that the K_c criterion may be written as:

$$p\sqrt{\pi a} = p_c\sqrt{2\pi r_c} = K_c \qquad (2)$$

where fracture is defined by the single parameter K_c, but combines a critical stress at a critical distance. If we assume that failure occurs at the <u>same</u> conditions for the blunt crack, then we may write an expression for the apparent K, K_b, given by:

$$K_b = p\sqrt{\pi a} = K_c\,\frac{(1 + R/2r_c)^{3/2}}{(1 + R/r_c)} \qquad (3)$$

It should be noted that <u>two</u> fracture parameters are needed when R is finite, K_c and r_c (or p_c). Since R and r_c are taken as very small, then in a manner similar to that employed in LEFM we may use the elastic stress field associated with K_b to determine the behaviour of the small scale yielding and, if we use the line zone model, then we may write:

$$\text{COD}, \quad \delta = \frac{K_b^2}{E\, p_y} \tag{4}$$

and:

$$\text{zone length}, \quad r_p = \frac{\pi}{8} \frac{K_b^2}{p_y^2}$$

A further assumption is now introduced in stating that δ determines R for an initially sharp crack, so that:

$$R = \alpha\, \delta \tag{5}$$

where α depends on the blunting process. Figure 2 shows different forms of crack tip deformations which may be deduced from simple power-law models (Williams, 1984) and suggests that for smooth blunting ($\alpha = \frac{1}{2}$) we have work-hardening and that as the work-hardening decreases, α increases. Initiation data suggest that $\alpha \simeq 1$ for some thermoset resins (Kinloch and Williams, 1980).

Combining equations (2), (4) and (5):

$$\frac{R}{r_c} = (2\pi\, \alpha) \left(\frac{K_b}{K_c}\right)^2 \frac{p_c^2}{E\, p_y} \tag{6}$$

which, in association with equation (3), gives a relationship between the observed toughness, K_b, the true sharp notch value, K_c, and the deformation parameter, $E\, p_y$.

The form of the blunting equations may be expressed conveniently in terms of the two parameters:

$$x = \frac{2}{p_c} \sqrt{(E\, p_y)/(\pi\, \alpha)} \quad \text{and} \quad y = \frac{K_b}{K_c} \tag{7}$$

so that equation (3) becomes:

$$y = \frac{(1 + 4y^2/x^2)^{3/2}}{(1 + 8y^2/x^2)} \tag{8}$$

The relationship between x and y is shown in Fig. 3, where it can be seen that $y \to \infty$ as $x \to 1$ and decreases rapidly as x increases, becoming unity at $x \simeq 1.6$. Thereafter, y decreases to a minimum of about 0.92 at $x \simeq 2.5$

and then rises to unity for large x. This decrease below the sharp crack toughness has been observed in thermoset resins (Kinloch and Williams, 1980) and will be discussed later, but a useful approximation to the function is:

$$y \simeq 1 \;, \quad x > 2$$
$$y \simeq \frac{x}{2\sqrt{x - 1}} \;, \quad 1 \leqslant x < 2 \tag{9}$$

It is also convenient to introduce some further parameters. The yield strain, e_y, for several polymers is often independent of time and temperature, so that E may be removed via:

$$E = \frac{p_y}{e_y}$$

and if we refer p_y to some reference value p_o, we may write x in the form:

$$x = \frac{2}{\sqrt{\pi\, e_y}} \frac{p_o}{p_c} \frac{p_y}{p_o}$$

and we have:

$$N\, x = \frac{p_y}{p_o} \quad \text{with} \quad N^2 = \frac{\pi}{4} e_y \left(\frac{p_c}{p_o}\right)^2 \tag{10}$$

p_c/p_o is the constraint factor in the crack tip region and would have a minimum of unity and a maximum of up to ten. Since $e_y \simeq 0.03$ for most polymers, this gives a range for N of .15 to 1.5 with a value of 0.5 for $p_c/p_o = 3$.

Observations on the slow propagation of sharp cracks have shown very clearly that they often occur at approximately a constant COD (Marshall, Coutts and Williams, 1974), although direct measurements by Döll (1983) have shown that this is an over-simplification and that at long times δ increases rapidly. In other cases (e.g. [Chan and Williams, 1983]) there are substantial variations in COD, particularly for low yield stress materials. A model consistent with this behaviour is obtained by assuming that for sharp crack, we have a constant COD, δ_o, and that this provides the constant length criterion and not r_c. If p_c is now taken as constant, then:

$$K_c^2 = \delta_o\, p_y\, E = 2\pi\, r_c\, p_c^2 \tag{11}$$

and r_c decreases as $E\,p_y$ decreases with p_c constant and, since $R = \alpha\,\delta_o$ for sharp cracks, it is this decrease in r_c which gives rise to the blunting effect via equation (3). In physical terms, the two fracture criteria are probably the attainment of a critical stress, p_c, at some internal point in front of the crack tip (Williams, 1980b). This stress concentration is achieved by constraint and can only occur at some distance greater than a minimum value because of crack front plane stress effects (see Fig. 1). The limiting condition may be written as:

$$r_{c,min} = \delta_o \left(\frac{\alpha}{8}\right) \qquad (12)$$

The observed COD is thus:

$$\delta = \frac{K_b^{\,2}}{E\,p_y} = \left(\frac{K_b}{K_c}\right)^2 \delta_o \quad ; \quad \text{i.e.} \quad \frac{\delta}{\delta_o} = y^2 \qquad (13)$$

and from Figure 3, it can be seen that $\delta \simeq \delta_o$ until $E\,p_y$ decreases to the critical value when a rather rapid increase in δ would be expected.

It is now convenient to express the various K parameters in terms of x and y:

$$K_c^{\,2} = \delta_o\,p_y\,E = p_o^{\,2}\left(\frac{\delta_o}{e_y}\right)\left(\frac{p_y}{p_o}\right)^2$$

i.e.
$$K_c = K_o\,N\,x \quad \text{where} \quad K_o = \sqrt{\delta_o/e_y}\;p_o \qquad (14)$$

and:
$$K' = \frac{K_b}{K_o} = N\,x\,y \qquad (15)$$

In terms of G, we have:

$$G' = \frac{G_b}{G_o} = N\,x\,y^2 \quad \text{where} \quad G_o = \delta_o\,p_o \qquad (16)$$

TIME DEPENDENT EFFECTS

The effect of time is to decrease both E and p_y as time increases by viscoelastic relaxation. This can be expressed in a power-law form:

$$N\,x = \frac{p_y}{p_o} = \left(\frac{t}{t_o}\right)^{-n} \qquad (17)$$

where n and t_o are material constants. For a stationary crack, we may thus deduce δ as a function of t via equations (8), (13) and (17). Fig. 4(a) shows such data for PMMA at 23°C (Döll, 1983) where $n \simeq 0.06$, and the line is fitted using $x = 1$ at 10^7 s and $\delta_o = 2.0$ µm.

For a moving crack, the time-scale is determined by the crack speed, the zone length and the displacement distribution (Williams, 1984). This may be written as:

$$\dot{a} = \beta\,\frac{r_p}{t} \quad \text{where} \quad \beta \simeq \frac{1}{5} \quad \text{for } n \ll 1 \qquad (18)$$

i.e.
$$\dot{a} = \beta\,\frac{\pi}{8}\,\frac{K_b^{\,2}}{p_y^{\,2}}\,\frac{1}{t} = \beta\,\frac{\pi}{8}\left(\frac{\delta_o}{e_y}\right)\frac{y^2}{t} \qquad (19)$$

and the expression for x becomes:

$$N\,x = \left(\frac{Z_o}{y^2}\right)^n \qquad (20)$$

where:
$$Z_o = \frac{\dot{a}}{\dot{a}_o} \quad \text{and} \quad \dot{a}_o = \beta\,\frac{\pi}{8}\,\frac{\delta_o}{e_y\,t_o}$$

Figure 4(b) shows the COD for PMMA at 20°C (Döll, 19) for a running crack and the blunting data have been fitted to give a minimum at 10^{-8} m/s and with $\delta_o = 3.1$ µm. The general trend is in accord with the theory, but the δ_o is clearly larger than for the stationary crack. From these data, it is clear that δ changes only slowly for $\dot{a} > 10^{-8}$ m/s and the usual constant COD criterion is a useful criterion. From equations (15) and (20), we may deduce a K-$\dot{a}$ relationship in this region using $y = 1$:

$$K' = Z_o^{\,n}$$

giving a linear log K-log $\dot{a}$ graph, as shown schematically in Fig. 5. The same form of curve will result for initiation time data from equation (17) with $Z_o = t_o/t$. When blunting occurs, y increases rapidly and for stationary cracks the limit as $x \to 1$ is given by:

$$\check{Z}_o = N^{1/n} \tag{21}$$

while for the moving crack: $Z_o = N^{1/n} y^2$

in the limit, giving: $K' = N y = N^{1-\frac{1}{2}n} Z_o^{\frac{1}{2}}$

Both of these asymptotic solutions are shown in Fig. 5 where the distinct difference between the rapidly rising stationary crack case and that for the moving crack is apparent. The interaction of time-scale and the changing zone size results in a minimum in the crack speed which can be achieved. An approximation to this minimum may be deduced from asymptotic form, equation (9) and equation (20), and is:

$$Z_{o,min} = \frac{(1+2n)^{\left(\frac{1+2n}{n}\right)}}{(1+n)^{\left(\frac{1+n}{n}\right)}} \frac{N^{1/n}}{4n} \tag{22}$$

The onset of blunting is approximately when $x = 2$ and $y = 1$, i.e. $Z_o = (2N)^{1/n}$, and this is also shown in Fig. 5.

THERMAL EFFECTS

The temperature rise in the zone at the crack tip may be modelled by using the solution for an infinite strip of width b in which energy is dissipated at a constant rate per unit volume (Carslaw and Jaeger, 1978). The solution is:

$$\Delta T = \tilde{\Delta T}\,[1 - 4i^2\, erfc\, \xi] \tag{23}$$

where $i^2\, erfc$ is the second integral of $(1 - erf)$, where erf is the error function, and:

$$\tilde{\Delta T} = \frac{G_c}{\rho\, c\, b} \quad \text{and} \quad \xi = \frac{b}{4} \frac{1}{\sqrt{\kappa\, t}} \tag{24}$$

(ρ is density, c specifies heat, and κ thermal diffusivity, $k/(\rho\, c)$, where k is the thermal conductivity). G_c/b represents the energy per unit volume dissipated, and $\tilde{\Delta T}$ is the maximum (adiabatic) rise achievable since $\Delta T \to \tilde{\Delta T}$ as $\xi \to \infty$ $(t \to 0)$.

b is a crucial factor in this analysis since it controls $\tilde{\Delta T}$ and the rate at which it is achieved. For slow moving cracks with crazes, it would be expected to be some fraction of the COD, say a 1/3 factor because of the approximately parabolic displacement distribution within the craze and 1/2 for the volume fraction of voids, giving $b \simeq \delta_o/6$. Since $G_c \simeq p_o\, \delta_o$, without blunting, then $\tilde{\Delta T} \simeq (6\, p_o)/(\rho\, c)$. For most polymers, $\rho\, c \simeq 2 \times 10^6$ $J/m^3\,°K$ and, since $p_o \simeq 10^8$ J/m^3, we have $\tilde{\Delta T} \simeq 300°C$. The time scale to achieve ΔT may be approximated by $\xi = 1$ and, since $\kappa \simeq 10^{-7}$ m^2/s, we have $t \simeq 0.2$ μs for $\delta_o = 3$ μm and $\dot{a} \simeq 40$ m/s, which is of the order of the observed high crack speeds. It seems likely, therefore, that in the high high speed region, the crack tip zones are adiabatic but the transition to this behaviour can be modelled by considering small values of ξ at $y = 1$[1].

The series for equation (23) gives:

$$\Delta T \simeq \frac{4}{\sqrt{\pi}} \tilde{\Delta T}\, \xi$$

which is shown in Fig. 6 compared with the full solution. This form does not depend on b and we have:

$$\Delta T = \frac{G_c}{\sqrt{\pi}\, \rho\, c\, k} \frac{1}{\sqrt{t}} \tag{25}$$

while for the asymptotic case for large blunting:

$$\Delta T \to \frac{G_b}{\rho\, c} \frac{6}{\delta} \tag{26}$$

A tractable form of temperature dependence may be written as:

$$N\, x = \left(\frac{t}{t_o}\right)^{-n} - \frac{\Delta T}{T_o} \tag{27}$$

where T_o is the softening temperature at which $p_y = 0$. Combining this with equations (25) and (26) gives:

$$N\, x = \left(\frac{t}{t_o}\right)^{-n} - N\, x \left(\frac{t}{t_1}\right)^{-\frac{1}{2}} \tag{28}$$

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[1] The formulation of the complete solution is given in Appendix A.

where: $$t_1 = \left(\frac{p_o \delta_o}{T_o}\right)^2 \frac{1}{\pi \rho c k}$$

and: $$N x = \left(\frac{t}{t_o}\right)^{-n} - N x \psi \qquad (29)$$

with $\psi = (6p_o/(\rho cT_o))$ for the asymptotic case. These expressions may now be written in terms of a dimensionless thermal speed factor:

$$Z_1 = \left(\frac{t_1}{t}\right) = \left(\frac{\dot{a}}{\dot{a}_1}\right)$$

where: $$\dot{a}_1 = \beta \frac{\pi}{8} \left(\frac{\delta_o}{e_y}\right) \left(\frac{T_o}{p_o \delta_o}\right)^2 \pi \rho c k$$

and the parameter $Q = (\dot{a}_1/\dot{a}_o)^n = (t_o/t_1)^n$. For stationary cracks:

$$N x = \frac{Q Z_1^n}{1 + Z_1^{\frac{1}{2}}} \qquad (30)$$

and for moving cracks: $$N x = \frac{Q (Z_1/y^2)^n}{1 + (Z_1/y^2)^{\frac{1}{2}}} \qquad (31)$$

The onset of blunting for a moving crack is approximately when $x = 2$ and $y = 1$ and equation (31) has two roots, one at low speeds and one at high. For PMMA, we have $p_o \simeq 65$ MPa at 10^5 s (Döll, 1983), so that $\dot{a}_o \simeq 2 \times 10^{-11}$ m/s and $p_c/p_o \simeq 3.5$ from $N = 0.76$. Since $\pi \rho c k \simeq 10^6$ $J^2/m^4{}^\circ K^2 s$, we have $\dot{a}_1 \simeq 3$ m/s and $Q = 4.7$, giving values for the roots of 2×10^{-8} m/s and 18 m/s. The former is the viscoelastic blunting threshold discussed previously, while the latter is the onset of thermal effects. Between these two limits, there is no blunting but there is a maximum in K given by:

$$K' = \frac{Q Z_1^n}{1 + Z_1^{\frac{1}{2}}} \quad \text{and} \quad \frac{dK'}{dZ_1} = 0$$

i.e. $$\hat{Z}_1 = \left(\frac{2n}{1 - 2n}\right)^2 \quad \text{and} \quad \hat{K}' = Q (2n)^{2n} (1 - 2n)^{(1 - 2n)}$$

This peak in K has been discussed in some detail in previous work (Marshall, Coutts and Williams, 1974).

For PMMA, $\hat{Z}_1 \simeq 0.02$ so that $\dot{a} \simeq 6 \times 10^{-2}$ m/s and $K \simeq 1.4$ MPa$\sqrt{m}$, which agrees well with observations [(Marshall, Coutts and Williams, 1974) and (Doll, 1983)]. The peak frequently leads to instabilities since K' decreases as $\dot{a}$ increases because of the heating effect.

The solutions for the two cases at high speeds (and short times) are shown in Fig. 7 using the PMMA values. The trend to infinity at some time for the stationary crack case is apparent, as at long times, and the limiting value of speed for the moving crack also occurs. This corresponds to 96 m/s here with a corresponding temperature rise of 420°C assuming $T_o = 100$°C. (Fuller, Fox and Field, 1975) reported about 500°C for $200 < \dot{a} < 650$ m/s using infra-red measurements and our estimate from b was about 300°C. Adjusting b to $\delta_o/10$ gives 500°C and, similarly, by increasing T_o the calculated $\dot{a}$ increases to about 200 m/s.

The limiting K' may be derived and is:

$$\hat{K}' = \frac{N}{2} \left(\frac{1 + 2n}{1 + n}\right)^2 \left(\frac{1 + n}{n}\right)^{\frac{1}{2}}$$

and the minimum in K' on the curve is when $x = 4/3$ and $y = 2/\sqrt{3}$, i.e.

$$K'_{min} = \frac{8}{3\sqrt{3}} N$$

so we may write: $$\frac{\hat{K}}{K_{min}} = \frac{3\sqrt{3}}{16} \left(\frac{1 + 2n}{1 + n}\right)^2 \left(\frac{1 + n}{n}\right)^{\frac{1}{2}} \qquad (32)$$

This type of limit on the speed curve is frequently associated with bifurcation phenomena (Thomson, 1982) and, it is proposed, should be used as a criterion here.

As an illustration of the analysis, two final sets of data are given. In Fig. 8, data for Homolite 100 (Metcalf and Kobayashi, 1980) is fitted approximately by the theory with $\dot{a}_1 = 28$ m/s, $n = 0$ and $y = 1$, and clearly gives a reasonable form. n is very low for this material (see Metcalf and Kobayashi, 1980) and the branching prediction of $n = 0.01$ is shown using equation (32) to determine the ratio of the plateau to the observed branching K value. In Fig. 9, some impact G_c data for PMMA are given as a function of loading time (Williams and Hodgkinson, 1981). The lines are drawn for $t_1 = 22$ μs and $G_o = 0.4$ kJ/m^2. This value of t_1 agrees well with equation (28) with these rather high values of p_o δ_o. The equivalent values for running cracks are much lower and may reflect differences in the impact test. It is this difference in the plateau toughness value which is mainly responsible for the time-scale discrepancies in impact and fast running cracks noted originally.

CONCLUSIONS

The use of a constant COD fracture criterion coupled with a blunting mechanism leads to a model of fracture behaviour in polymers which describes a wide range of phenomena. Variations in COD at low rates are as predicted and long time blunting is described. The inclusion of thermal effects gives high speed blunting and the data are in reasonable quantitative agreement with several observations on both high speed cracks and in impact. Much more detailed study is required, but this initial evaluation is promising

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APPENDIX A

The full thermal blunting analysis may be written in terms of the following equations:

$$y = \frac{(1 + 4y^2/x^2)^{3/2}}{(1 + 8y^2/x^2)} \qquad (A.1)$$

$$K' = \frac{K_b}{K_o} = N\,x\,y \quad \text{or} \quad G' = \frac{G_b}{G_o} = N\,x\,y^2 \qquad (A.2)$$

$$N\,x = \left(\frac{t}{t_o}\right)^{-n} - \frac{\Delta T}{T_o} \quad \text{or} \quad N\,x = \left(\frac{\dot{a}}{\dot{a}_o\,y^2}\right)^{n} - \frac{\Delta T}{T_o} \qquad (A.3)$$

$$\Delta T = N\,x\,\hat{T}\,[1 - 4i^2\,\mathrm{erfc}\,y^2\sqrt{\hat{t}/t}] \qquad (A.4)$$

where $\hat{T} = p_a/(\lambda/4)^2\,\delta_o^{\,2}/K$ and $\lambda = b/\delta$, a constant. For crack speeds, $\sqrt{\hat{t}/t} = \sqrt{\hat{\dot{a}}/(\dot{a}\,y^2)}$, where $\hat{\dot{a}} = (\beta\,2\pi)/\lambda^2\,\kappa/(\delta_o\,e_y)$. A curve of K' or G' versus $\dot{a}$ may be calculated by iteration from the four numbered equations (Crouch and Williams, to be published).

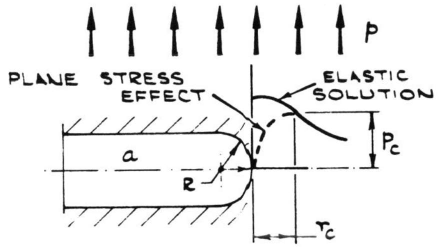
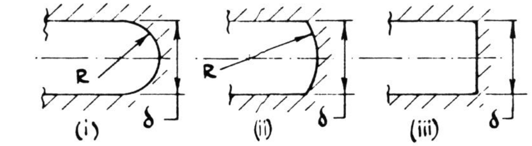


FIG. 1. CRACK TIP GEOMETRY & LOADING



SMOOTH BLUNTING $\alpha = 1/2$ HIGH HIGH WORK HARDENING

INTERMEDIATE CASE $\alpha \sim 1$

NON WORK HARDENING $\alpha \to \infty$

FIG. 2. CRACK TIP BLUNTING PROFILES

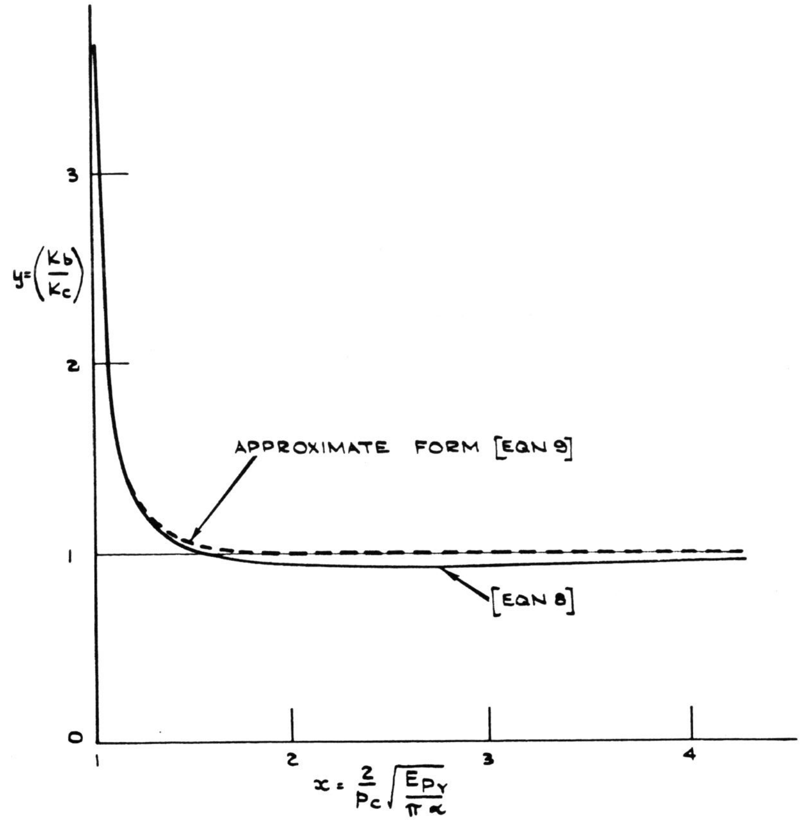
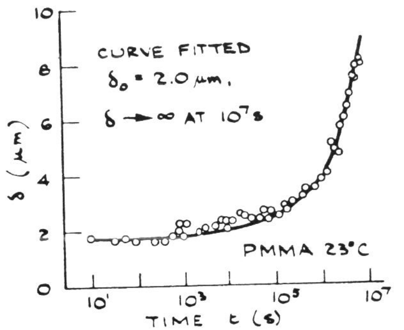
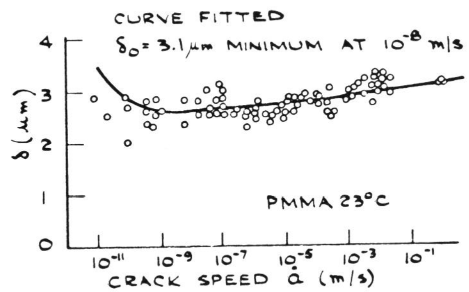


FIG. 3. THE VARIATION OF APPARENT TOUGHNESS WITH YIELD STRESS AND MODULUS.



(a) CRACK OPENING DISPLACEMENT VS. TIME FOR A STATIONARY CRACK.



(b) CRACK OPENING DISPLACEMENT VS. CRACK SPEED

FIG.4. DATA FOR PMMA AT 23°C TAKEN FROM [11]

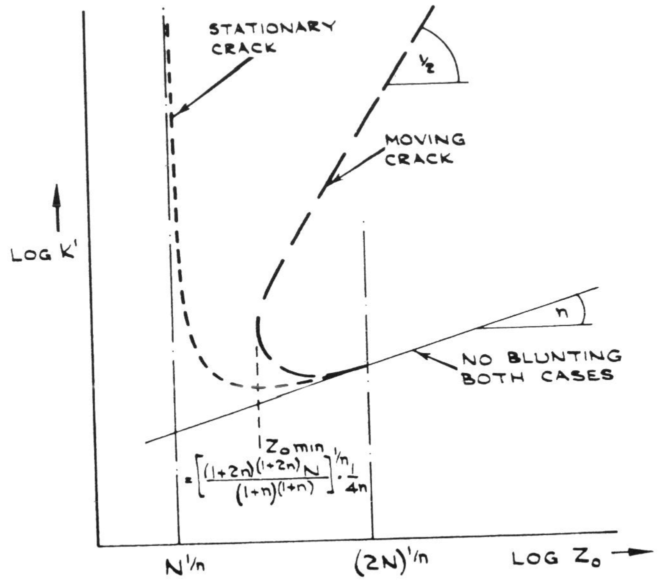


FIG.5. SCHEMATIC DIAGRAM OF LOW SPEED (LONG TIME) BLUNTING.

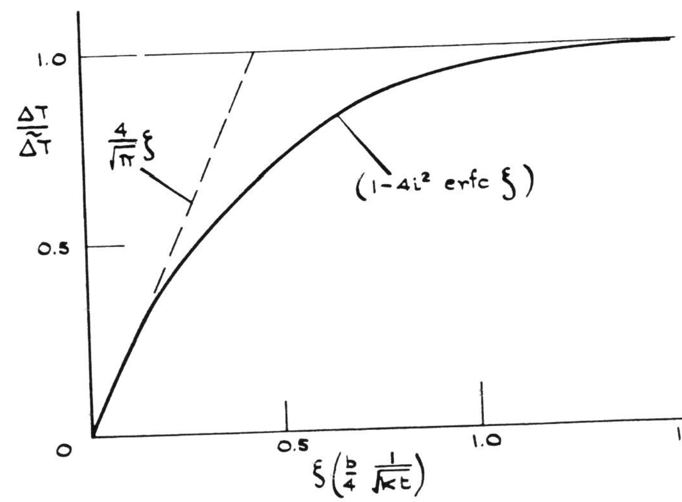


FIG.6 TEMPERATURE RISE IN THE CRACK TIP ZONE.

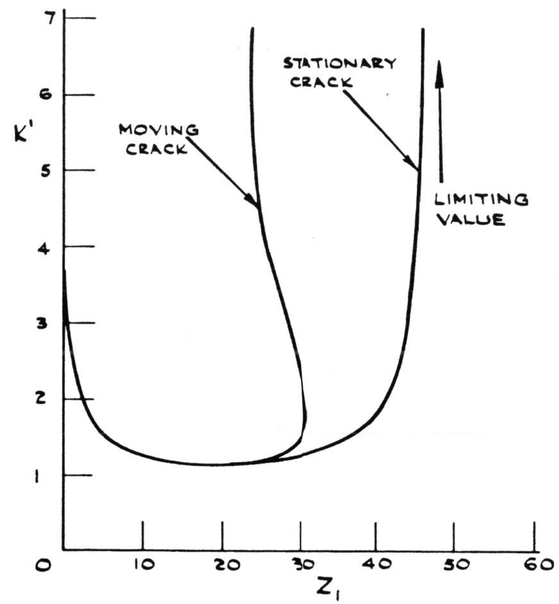


FIG. 7. K' VS. Z_1 FOR PMMA AT 20°C
$n = .06$, $N = 0.76$

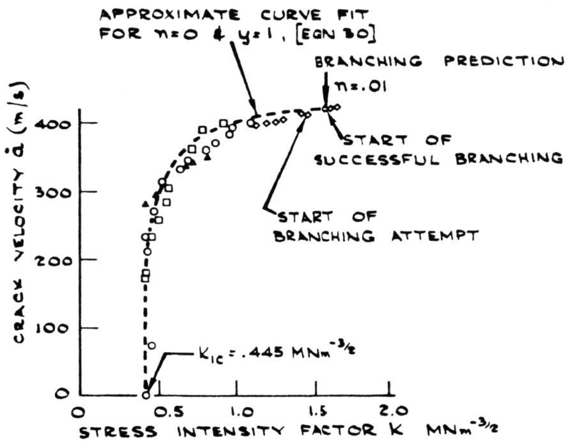


FIG. 8. CRACK VELOCITY $\dot{a}$ AS A FUNCTION OF STRESS - INTENSITY FACTOR K FOR HOMALITE 100 [3]

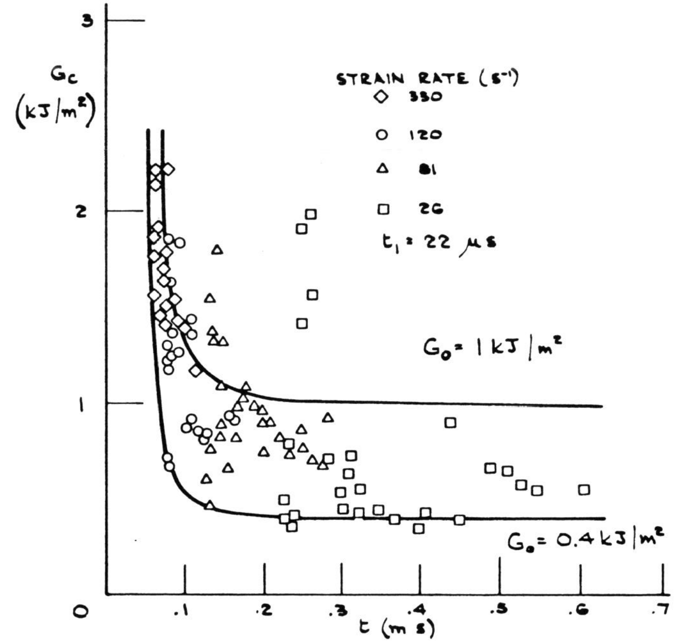


FIG. 9. G_c VS. LOADING TIME FOR PMMA AT 20°C - FROM IMPACT TESTS [5]