

FATIGUE FAILURE MECHANISMS IN COMPOSITE LAMINATES

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ABSTRACT

Failure mechanisms in composite laminates are quite different from those in homogeneous materials. While homogeneous materials under fatigue fail as a result of initiation and growth of a single dominant crack, composite laminates can sustain many cracks in the weak phase before ultimate failure. In unidirectional composites under longitudinal fatigue the subcritical failure can take the form of interfiber matrix cracking normal to the fibers. The matrix cracking is enhanced by the biaxial state of stress between fibers especially when applied stress is outside linear elastic range of the resin. The dominant modes of subcritical failure in multidirectional laminates are ply cracking and delamination. Ply cracking leads to a stress relaxation in the cracked ply and hence slows down with fatigue cycles. Even of the same stacking sequence a laminate with thicker plies shows earlier crack initiation but slower crack multiplication than a laminate with thinner plies. Delamination between plies can start not only at free edges but also from ply cracks. Both ply cracking and delamination reduce stored energy and hence stiffness. The energy release rates associated with ply cracking and delamination are independent of crack size, unlike the crack growth in homogeneous materials.

KEYWORDS

Fatigue; fracture; composites; ply cracking; delamination.

INTRODUCTION

Polymer composites reinforced with glass, graphite or aramid fibers now find many applications as structural materials because they offer high strength-to-weight ratio, excellent dimensional stability and good corrosion resistance. Since most structural applications involve fatigue environment, fatigue behavior of these relatively new materials must be fully known to insure structural integrity and safety.

Because of inherent heterogeneity and anisotropy, failure of composites entails many competing mechanisms. Whereas fatigue failure of homogeneous materials such as metals and unreinforced plastics is the result of initiation and propagation of a single dominant crack, fatigue failure of composites is characterized by initiation and multiplication of many cracks in the weak phase.

Although failure sequence in a given laminate depends on the type of material and the stacking sequence, the most common initial failure is the through-the-thickness cracking of off-axis plies along the fibers. Ply cracking rarely requires fiber fracture and runs through the matrix and interface.

In many laminates ply cracking is followed by delamination between plies especially at free edges. While ply cracks are not necessarily a precursor to delamination at free edges, they definitely are in regions far away therefrom.

As off-axis plies fail and delamination grows, load is increasingly carried by on-axis plies, i.e., plies with fibers in the loading direction. The ultimate failure of a laminate coincides with the failure of on-axis plies.

Since ply cracking and delamination do not immediately trigger ultimate failure, they constitute subcritical failure modes. Nevertheless, they can degrade structural performance by reducing stiffness, strength and environmental resistance. Therefore, a more efficient application of composite laminates calls for a better understanding and control of these subcritical failures, as well as critical failure of on-axis plies.

The present paper provides a review of the fatigue failure mechanisms in composite laminates with emphasis on subcritical failure under tension-tension fatigue. It is shown that matrix cracking is possible even in on-axis fatigue of unidirectional composites. The sudden nature of ply cracking and delamination are also discussed in the paper.

UNIDIRECTIONAL COMPOSITES

On-Axis Fatigue

Epoxy resins are viscoelastoplastic materials: they show a strain-rate dependence and are capable of plastic flow. The particular behavior of a given epoxy depends on its formulation as well as on the spatial arrangement of molecular chains and the crosslink density. On the other hand, glass, graphite and aramid fibers are much stronger and stiffer than resins and carry most of the load in properly designed composite laminates.

Static failure of most unidirectional composites is fiber-dominated as the matrix has higher strain capability than the fibers. When a weak fiber fails, the subsequent failure sequence depends on the behavior of the matrix and interface. If the matrix is brittle and the interface strong, the crack induced by the failure of the weak fiber is likely to propagate across the neighboring fibers, leading to ultimate failure of the composite. However, if the matrix and interface are weak, the fiber break is more likely to be followed by matrix/interface failure along the fiber, so that stress concentration on the adjacent fibers is reduced (Hahn, 1979).

Although dispersed fiber breaks can occur in composites with low fiber content, ultimate failure of real composites with fiber volume content in the range of 50 to 70% usually shows a stress concentration effect (Rosen, 1965; Owen, 1974).

Under cyclic loading, fatigue sensitivities of matrix and interface additionally affect fatigue response of composites. Figure 1 shows schematic strain-life relations for fiber, resin and composite where the fiber is assumed to be much more fatigue-resistant than the resin at low strains.

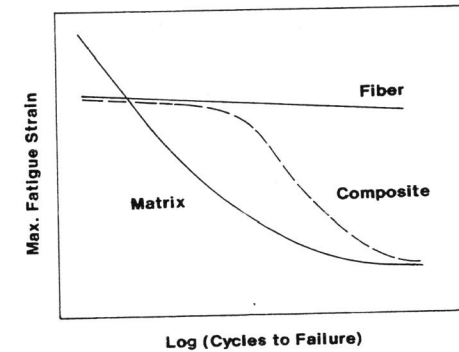


Fig. 1. Fatigue behavior of composite and constituent materials.

In the low-cycle region, life of the resin exceeds that of the fiber because of better strain capability of the former. As a result, the composite failure in low-cycle fatigue is due to failure of fibers as in static tension. However, in the high-cycle region, the fiber has higher fatigue strength than the resin. Consequently, high-cycle fatigue is expected to be associated with more damage in the matrix. The extent of fatigue damage will depend on fatigue strength of the fiber; i.e., the lower the fiber fatigue strength, the less damage in the matrix (Hahn, 1979).

The exact failure mechanisms in the high-cycle region still remain to be known. A glass/epoxy composite was found to exhibit only slightly higher fatigue limit strain than its matrix resin regardless of fiber volume content, thereby prompting a suggestion of a matrix-dominated fatigue failure (Dharan, 1975a, 1975b). However, a fiber-dominated fatigue was also proposed in light of the observation that resin-impregnated glass strands showed the same rate of fatigue degradation as dry bundles (Mandell, 1981). In any case, glass composites show some accumulation of fiber damage as evidenced by the reduction of axial stiffness (Dharan, 1975a; Hahn and others, 1982).

Graphite fibers such as AS and T300 have static failure strains which are close to fatigue limit strains of epoxy resins. Thus, the fatigue ratios of graphite/epoxy composites are much higher than those of glass/epoxy composites (Hahn, 1979). For high-modulus graphite composites with a failure strain of about 0.5%, the fatigue limit strain is near the lower tail end of the static strength distribution (Owen, 1974; Sturgeon, 1973). Also, graphite/epoxy composites fail abruptly with most damage occurring within a few loading cycles immediately before ultimate failure (Sturgeon, 1973; Awerbuch and Hahn, 1977). The lack of strength and stiffness reduction further confirms the absence of gradual damage growth in graphite composites.

Aramid fiber composites are different from glass or graphite composites in that both failure strain and fatigue limit are high (Hahn and Chin, 1982). These composites have higher fatigue limit strain than resins, indicating a fiber-controlled failure.

If fatigue failure is dominated by fibers, failure is likely to be sudden without much visible evidence of fiber breaks because fibers are load-carrying members. In the case of matrix-dominated failure, on the other hand, matrix cracking will be stable and matrix cracks should be visible before ultimate failure.

Matrix cracks were found on edges of E-glass/epoxy specimens (Dharan, 1975a). However, no such cracks could be seen on surfaces of S2-glass/epoxy specimens (Hahn and others, 1982). Possible reasons for matrix cracking also seem contradictory. Even under longitudinal tension the matrix is subjected to a multiaxial state of stress that favors cracking. However, the smallness of the matrix phase between fibers tends to retard cracking (Hahn, 1983a; Aveston and Kelly, 1973). Thus, the net effect will be a compromise between these two competing mechanisms.

A better understanding of failure mechanisms in composites requires an understanding of failure processes at the constituent level. Specimens containing a bundle of fibers in transparent epoxy allow for monitoring of failure sequence in the fibers and matrix (Lorenzo and Hahn, 1983; Lorenzo and Hahn, 1984). Results of this study are summarized below.

Figure 2 shows stress-life (S-N) relations for a ductile epoxy, Epon 815/Versamid 140 (60/40 by weight), when the epoxy contains a bundle of E-glass or T300 graphite fibers. The reduction in fatigue life in the presence of

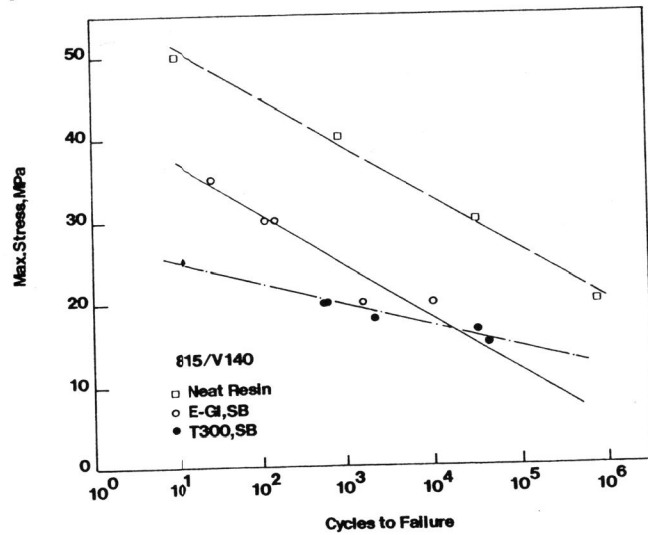
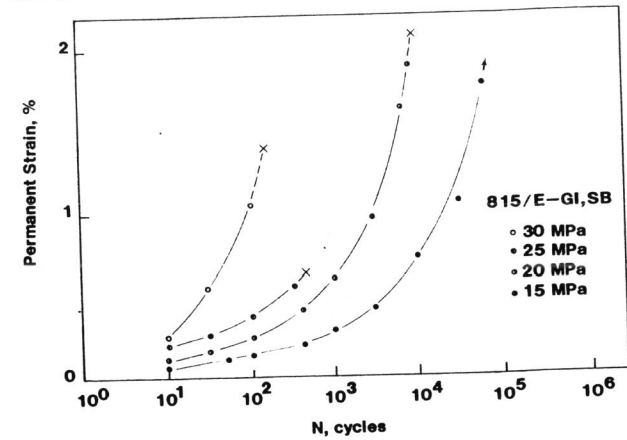
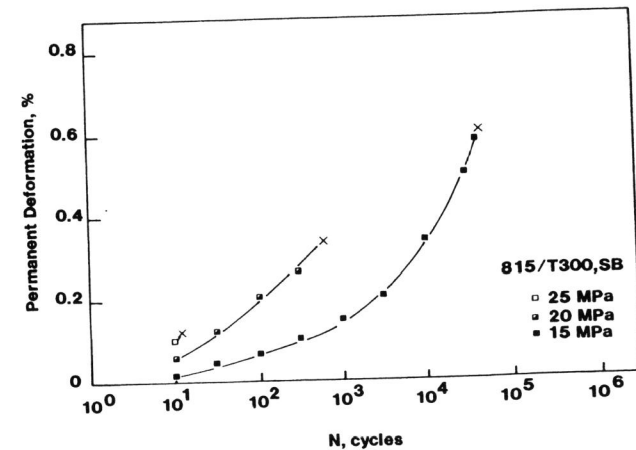


Fig. 2. S-N relations of ductile epoxy specimens.

E-glass bundle is the result of bundle failure accelerating crack growth in the matrix. The ductile epoxy exhibits a large cycle-dependent creep strain even at low fatigue stresses (Fig. 3). The total strain that represents the sum of creep and fatigue strains can thus exceed the failure strain of the fibers. The resulting bundle failure increases the stress in the resin, thereby accelerating initiation and growth of a crack. The final failure was always the result of a crack growth in the resin.



a



b

Fig. 3. Cycle-induced creep in ductile epoxy specimens: a. single glass bundle and b. single graphite bundle.

The lower fatigue strengths observed of T300 specimens in the high-cycle region is due to the lower strain capability of T300 fibers compared with E-glass fibers. Again, creep of the resin overloaded the graphite bundle whose failure then triggered final failure without any noticeable damage in the resin.

In fatigue E-glass fibers started to fail randomly along the bundle mainly because of creep of the resin, Fig. 4.

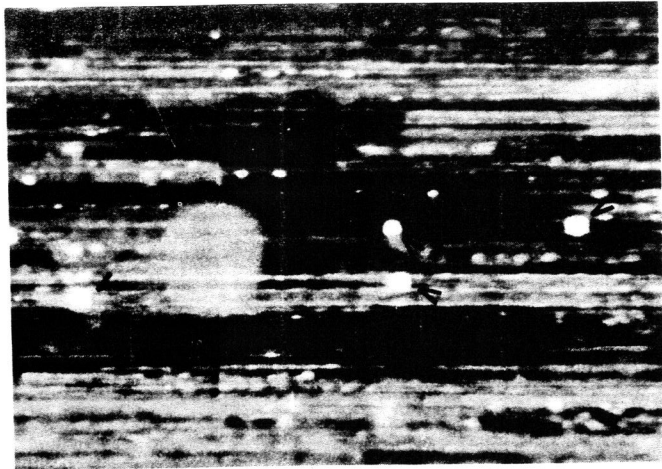


Fig. 4. Isolated fiber failure in a 815/E-G1 bundle specimen.

Only at a few places several fibers fractured on the same plane (Fig. 5). With further cycling complete bundle failure occurred at some of these clustered fiber breaks indicating a strong stress concentration effect.

Early in the life matrix cracks were detected between unbroken fibers while no crack growth was seen in the resin outside the bundle. These cracks were normal to the fibers and did not seem to grow around the fibers (Fig. 6). The appearance of matrix cracks indicates that the biaxial state of stress in the matrix within the bundle promotes cracking.

Graphite fibers also failed in fatigue but without much matrix cracking because these fibers had much lower strain capability than E-glass fibers. Isolated fiber breaks were scarce: when they appeared the surrounding fibers started to fracture due to stress concentration. The stress concentration

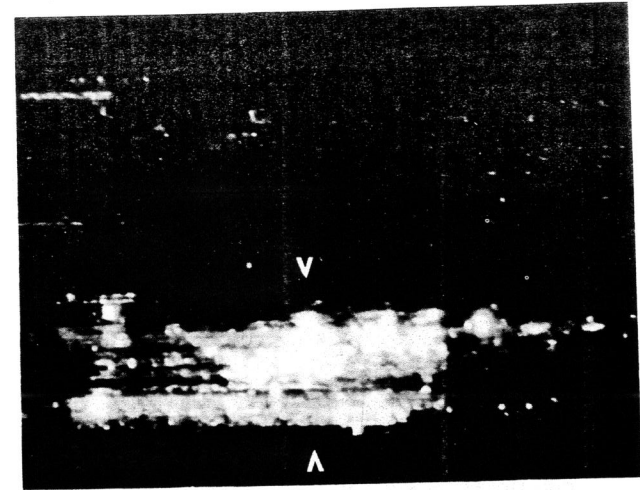


Fig. 5. Accumulation of fiber breaks due to stress concentration in a 815/E-G1 bundle specimen.

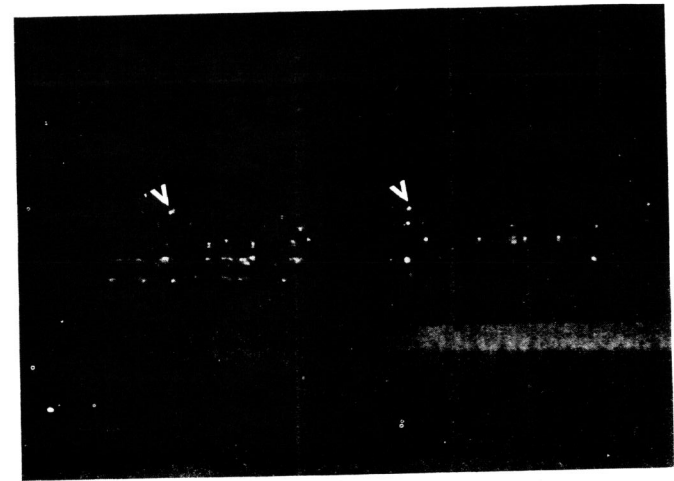


Fig. 6. Isolated matrix cracks between fibers in a 815/E-G1 bundle specimen.

effect was much stronger than in the glass bundle probably due to better interfacial bonding and higher fiber-to-matrix modulus ratio in the graphite bundle. Figure 7 shows an accumulation of fiber breaks with minimal matrix cracking between fibers. Most matrix cracks are seen to be confined to the regions of fiber breaks.

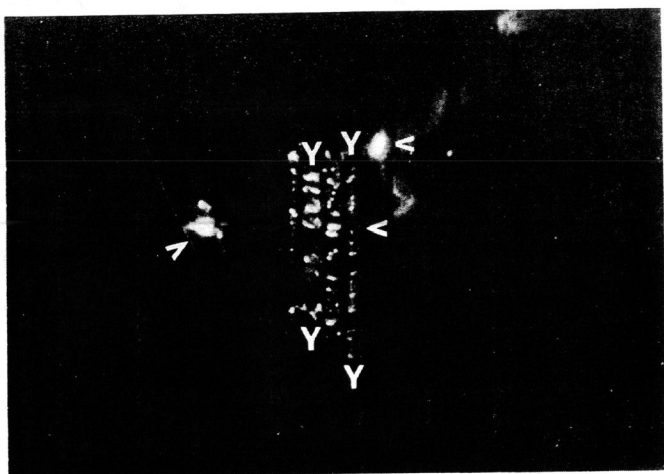


Fig. 7. Fiber breaks and matrix cracking in a 815/T300 bundle specimen. Fiber breaks are indicated by "v" and matrix cracks by "Y".

When a brittle resin, Epon 828/Z (80/20 by weight), was used in place of the ductile one, failure modes changed. The lack of any appreciable creep prevented fibers from being overloaded, and hence final failure of the specimen was always the results of resin failure regardless of fiber type. Such resin-controlled fatigue failure is in contrast to the fiber-controlled static failure.

The S-N relations in Fig. 8 reflect the observed failure modes in the brittle resin. The glass bundle has no effect on initiation and growth of a crack in the resin. The graphite bundle, however, reduces the stress in the resin and thus leads to a longer life. The difference between the effects of the glass and graphite bundles is mainly the result of the latter bundle having a larger number of stiffer filaments than the former (~3000 filaments versus ~200).

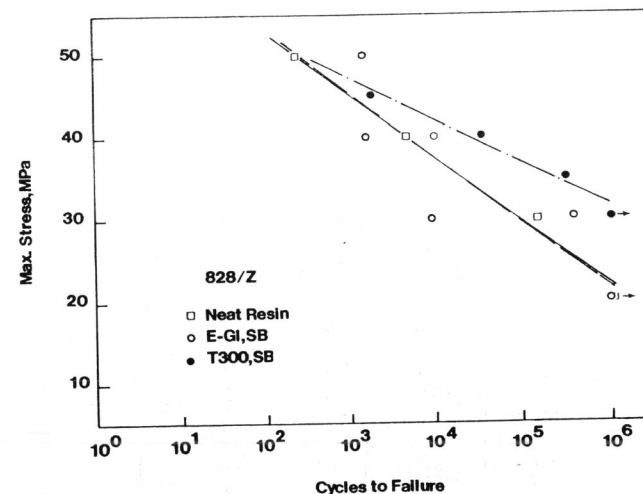


Fig. 8. S-N relations of brittle epoxy specimens.

In the low-cycle region, matrix cracking between fibers was quite extensive in the glass bundle, but it was absent in the graphite bundle because the matrix was subjected to a higher strain in the former than in the latter, (Fig. 9). In the high-cycle region, the same difference between the two types of fibers was observed. However, even in the glass bundle the matrix cracking was not so extensive as in the low-cycle region.

The study of fiber bundles embedded in epoxy has shown that interfiber matrix cracking can occur prior to fiber failure probably because of the biaxial state of stress. The matrix cracking is more pronounced when applied strain is outside the linear elastic range of the resin. Thus, matrix cracking is more likely in a composite reinforced with high-strain fibers.

In the high-cycle region, E-glass fibers are expected to have higher fatigue strength than most resins currently used in composites when strain is used for comparison. Yet, the results of Mandel (1981) indicate that the difference is not much. Thus, fatigue of E-glass/epoxy composites is concluded to be dominated equally by the fiber and matrix.

Off-Axis Fatigue

Uniaxial static strength decreases rapidly as the off-axis angle between applied load and fiber direction increases. Such a decrease corresponds to the transition of failure mode from a fiber-controlled to a matrix/interface-controlled fracture (Awerbuch and Hahn, 1981). As the off-axis angle increases, the resolved transverse and shear stresses increase while the fiber stress decreases. Since the transverse and shear strengths are lower than the longitudinal strength, failure is initiated in the matrix or at the interface. The mode of matrix/interface-controlled failure depends on the relative magnitudes of the resolved transverse and longitudinal shear stresses. When the off-axis angle is greater than 45 degrees, the transverse stress is

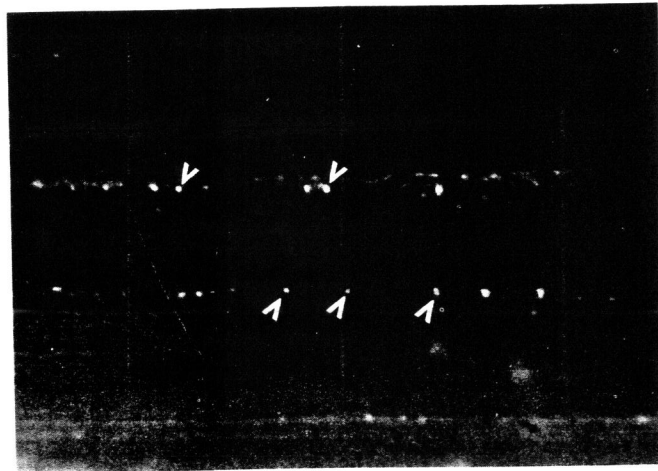


Fig. 9. Isolated matrix cracks between fibers in a 828/E-G1 bundle specimen.

greater than the shear stress, and vice versa. In the presence of the shear stress, matrix cracks form at an angle to the fibers (Fig. 10). These cracks are fairly normal to the average maximum principal stress in the matrix, indicating the applicability of the maximum stress fracture criterion to the matrix cracking (Hahn and Johannesson, 1983a). The resulting fracture surface thus shows many matrix serrations (Sinclair and Chamis, 1977; Awerbuck and Hahn, 1981; Hahn and Johannesson, 1983a).

When the resolved shear stress is low, a cleavage type of fracture is seen on the fracture surface. That is, the fracture surface is smooth and very few matrix serrations are found as a result of the matrix cracks being parallel to the fibers. Such behavior is expected from the state of stress.

Off-axis fatigue failure of graphite/epoxy composites is sudden and no reduction in strength is observed in run-out specimens. The fracture surfaces in fatigue failure do not differ much from those in static failure. Unlike in homogeneous materials, no region of crack initiation can be detected on fatigue fracture surfaces. Therefore, most of the fatigue life is believed to be spent on crack initiation.

The crack growth parallel to the fibers is very sensitive to the energy release rate G . For graphite/epoxy composites the exponent n in a crack

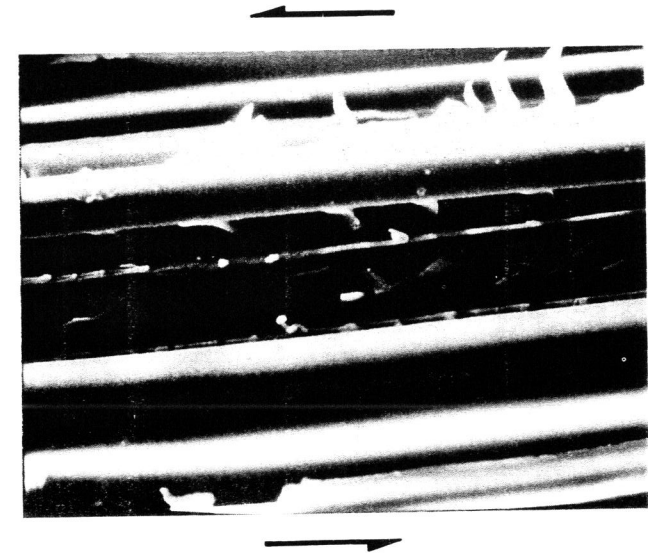


Fig. 10. Formation of matrix cracks in shear.

growth law of the form

$$\frac{da}{dn} \propto G^n \quad (1)$$

ranges from 20 to 30 for mode I and from 6 to 9 for mode II loading (Wilkins, 1982). These values are much higher than those for metals. Thus, a crack, once initiated, will accelerate rapidly leading to catastrophic failure.

MULTIDIRECTIONAL COMPOSITES

Failure Modes

Failure modes and failure sequence in fatigue of multidirectional laminates are similar to those in static tension. An analogy can be drawn between the two types of loading if the applied stress in static tension is replaced by the number of cycles endured in fatigue. Damage initiation takes the form of longitudinal fracture of the weakest plies. As fatigue proceeds further, cracks begin to appear in the next weakest plies while the weakest ply undergoes additional cracking. Since the ply stress in a ply is reduced by multiple cracking, further cracking becomes more difficult. Also, the existing cracks start growing along the interfaces, producing delamination and thereby reducing stress concentration on the neighboring plies. Ultimate failure of the laminate occurs when the fibers in the loading direction fail.

A comparison in damage growth between composite laminates and homogeneous materials under constant-amplitude fatigue is schematically shown in Fig. 11. Here damage represents crack density while it stands for crack length in the latter. The damage ratio is then the current damage normalized with respect to the damage at ultimate failure. While damage in composite laminates is seen to accelerate and then decelerate with fatigue cycles, it accelerates monotonically in homogeneous materials.

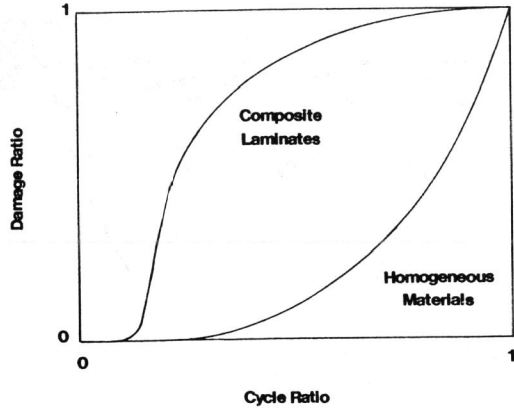


Fig. 11. A schematic view of fatigue damage accumulation in composites and homogeneous materials.

Decelerating damage ratio in laminates is the result of the ply stress in the cracked ply decreasing with increasing crack density. If this ply stress reaches a fatigue limit before 0-degree plies fail, an equilibrium crack density will be obtained. Otherwise, the crack density will still be on the increase when the laminate fails.

The load carrying capability of a ply diminishes when cracks appear. Also, the interruption of interlaminar load transfer by delamination impairs the synergistic effect of lamination. Therefore, both ply cracking and delamination in a laminate reduce stiffness and strength (Broutman and Sahu, 1969; Hahn and Kim, 1976; Highsmith and Reifsnider, 1982; Stinchcomb and Reifsnider, 1983; Ryder and Crossman, 1983; O'Brien, 1982).

Although ply angle may be chosen arbitrarily, the most frequently used ones are 0, ±45 and 90 degrees. Therefore, the following discussion will be limited to (±45) angle-ply laminates, (0/90) cross-ply laminates and finally (0/±45/90) laminates. Failure mechanisms of any other laminate can be fairly well understood from those of the aforementioned laminates.

In angle-ply laminates, the first ply-failure occurs almost simultaneously in both plies at their weakest points. The resulting cracks do not lead to immediate failure of the laminate because further crack propagation through the thickness is prevented by the neighboring plies with opposite fiber direction. As fatigue proceeds further, more cracks appear in the plies until delamination connects cracks in different plies and final fracture occurs.

The ability of angle-ply laminates to contain ply cracks manifests itself in a low scatter in fatigue life (Yang and Jones, 1978). Since no plies have fibers in the loading direction, the ply cracks add to the apparent creep of the laminate. Therefore, angle-ply laminates behave like a ductile material.

In cross-ply laminates 90-degree plies fail much before 0-degree plies. Although ply cracking is quite extensive in 90-degree plies, it can also occur in 0-degree plies because of a relatively high transverse tensile stress resulting from mismatch in Poisson's ratio between 0- and 90-degree plies (Broutman and Sahu, 1969; Ryder and Crossman, 1983). The transverse stress σ_T in the 0-degree plies of a $[0/90]_S$ laminate subjected to a stress σ is given by the laminated plate theory as

$$\frac{\sigma_T}{\sigma} = 2 \frac{\nu_{LT}(E_L E_T - E_T^2)}{(E_L + E_T)^2 - 4\nu_{LT}^2 E_T^2} \quad (2)$$

where E_L and E_T are the longitudinal and transverse moduli, respectively, and ν_{LT} is the major Poisson's ratio. For graphite/epoxy composites the ratio $\nu_{LT} \sigma_T / \sigma$ is about 0.03. However, for glass/epoxy the ratio can be as high as 0.06. Therefore, if the corresponding fatigue strength ratio is lower than the calculated stress ratio, even the 0-degree plies will crack along the fibers.

Failure modes in (0/±45/90) family of laminates are combinations of those in angle-ply and cross-ply laminates. Figure 12 shows normalized S-N relations for glass/epoxy (Hahn and Kim, 1976) and graphite/epoxy (Ryder and Walter, 1976) quasi-isotropic laminates. The graphite laminate is seen to be more fatigue resistant than the glass laminate because of better fatigue resistance of graphite fibers.

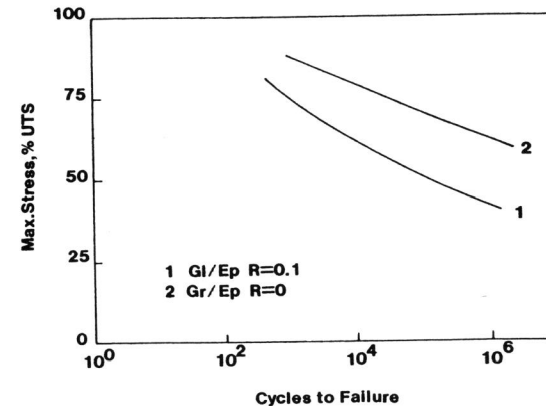


Fig. 12. S-N relations of quasi-isotropic glass/epoxy and graphite/epoxy laminates.

The initial failure mode in multidirectional laminates is the cracking of 90-degree plies. The 90-degree ply failure is then followed by cracking of off-axis plies and interlaminar delamination. Ultimate failure of the laminate coincides with failure of 0-degree plies. A typical sequence of failure observed on an edge is schematically shown in Fig. 13 (Stinchcomb and Reifsnider, 1985).

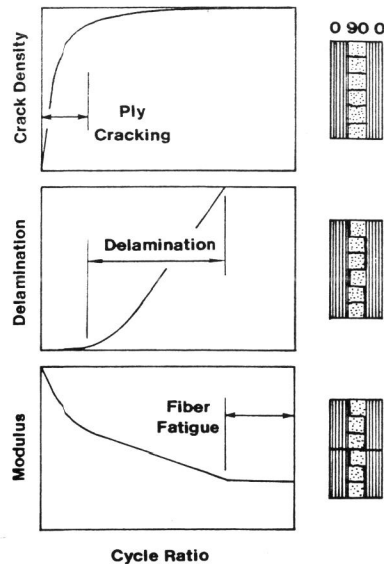


Fig. 13. Schematic changes of crack density, delamination and modulus in composite laminates under fatigue.

While ply cracking is the result of low strain capability of constituent plies in the transverse direction (Hahn and Tsai, 1974), delamination between plies is caused by interlaminar stresses. Interlaminar stresses can be quite high at free edges or at ply cracks (Pipes and Pagano, 1970; Wang and Crossman, 1977). The free-edge interlaminar stresses are induced to make ply deformation compatible with one another.

Ply cracks and delamination reduce modulus and strength of a laminate. When a ply cracks, its load carrying capability decreases and hence more load is carried by 0-degree plies. Consequently, the laminate stiffness is reduced. Also, since 0-degree plies have to carry more load, they will fail at a lower level of applied stress. In the limiting case of extensive ply cracking, the only load carrying plies will be 0-degree plies. Therefore, the resulting stiffness and strength of the laminate will be equal to those of the surviving 0-degree plies adjusted for appropriate cross-sectional area.

Cracked plies also become completely ineffective when delamination separates those plies from 0-degree plies. Even in the absence of ply cracking, delamination eliminates the synergistic effect of lamination in load sharing amongst plies. For example, the laminated plate theory predicts a modulus of 55 GPa for a AS/3501 [0/90/±45] laminate (Hahn, 1982). When all plies are completely separated from one another, the resulting modulus is 45 GPa.

Furthermore, if all off-axis plies crack, the laminate modulus is reduced again to only 35 GPa.

In most laminates, ply cracks do not seem to have much deleterious effect on the neighboring 0-degree plies, probably because of delamination at tips of ply cracks. Yet there is some evidence of increased fiber failure due to stress concentration at ply cracks (Reifsnider and others, 1983).

The damage-induced reduction in modulus is schematically shown in Fig. 13. In a delamination-prone laminate, such as a narrow $[0_2/\pm 45/90]_S$ coupon, the delamination period as well as the ply cracking period is much shorter than the fiber fatigue period (Hwang, 1982). Therefore, most of the modulus reduction takes place in the early stage of fatigue, and the stiffness remains constant thereafter until failure. However, when delamination is retarded by using a wide $[0/90/\pm 45]_S$ laminate, the delamination period covers most of the life, and hence a monotonically decreasing modulus is observed (Kim, 1980).

Most laminates ultimately fail before the laminate modulus fully decreases to the value predicted from the initial longitudinal modulus of 0-degree plies alone (Hahn and Kim 1976; Kim, 1980; Highsmith and Reifsnider, 1982). One reason is that no complete delamination takes place before ultimate failure. Yet such incomplete reduction of modulus also indicates that 0-degree plies retain most of their original longitudinal modulus until ultimate failure.

Strength reduction in fatigue is not likely unless fatigue damage exceeds the damage at static failure. If damage growth in fatigue is the same as that in static tension up to the maximum static damage state, residual strength will remain equal to static strength until the modulus in fatigue falls below the secant modulus at static failure. Thereafter, residual strength can be obtained as a product of the laminate modulus and the longitudinal static failure strain of 0-degree plies because 0-degree plies do not show much degradation in longitudinal properties (Ryder and Crossman, 1985; Reifsnider and Stinchcomb, 1985). However, this hypothesis has yet to be proven conclusively.

Analysis of Ply Cracking and Delamination

The extent of ply cracking and delamination in a laminate depends on its stacking sequence and ply thickness. Increasing ply thickness, while holding the same stacking sequence, facilitates both ply cracking and delamination. Thus, any reasonable analysis method should be able to describe this so-called thickness effect.

There are two generic approaches to the prediction of the initiation of ply cracking and delamination. The first method uses a statistical failure criterion based on stress state, whereas the second method is based on linear elastic fracture mechanics employing energy release rate.

Prediction of first ply-failure using the stress method starts with calculation of the stresses in the weakest ply in terms of the applied laminate stress. The probability of failure of the weakest ply at the applied stress is then obtained by substituting the ply stresses into an appropriate statistical failure criterion.

For laminates containing contiguous 90-degree plies of thickness h_{90} , the analysis can be simplified if the rule of mixtures can be used for the

laminate modulus, and the maximum stress criterion for failure. Suppose the laminate is subjected to uniaxial tension in the 0-degree direction. Then, the average laminate strain ϵ_{FPF} at failure of 90-degree plies can be shown to be (Hahn, 1982)

$$\epsilon_{FPF} \propto h_{90}^{-1/\alpha} \quad (3)$$

Here, the transverse tensile strength has been assumed to have a two-parameter Weibull distribution and α is the corresponding shape parameter. However, Eq. (3) underestimates the thickness dependence experimentally observed by Bader and others (1979).

To predict delamination, the interlaminar normal stress σ_z is determined in terms of the laminate stress. The Weibull theory of volume effect is then used over the free-edge region of tensile σ_z (Rodini and Eisenmann, 1978). While this method provided a good experimental correlation for various graphite/epoxy laminates, a deterministic method based on average of σ_z over the plane of maximum σ_z has also been found to be equally successful (Kim and Soni, 1984).

In the energy method applied to ply cracking, an inherent crack is assumed to exist through the thickness of the 90-degree ply to be analyzed. This crack is further assumed to be longer than the ply thickness. The associated energy release rate G is then independent of the crack length and given by (Hahn and Johannesson, 1983a)

$$G = \frac{1}{h_{90}} \Delta U \quad (4)$$

where ΔU is the difference between stored energies per unit width of the laminate before and after full cracking of the 90-degree ply. Thus, full cracking of the 90-degree ply is expected to occur when G equals the energy absorption rate G_c , that is,

$$\Delta U = h_{90} G_c \quad (5)$$

Equation (5) was successfully used by Parvizi, Garret and Bailey (1978), Bader and others (1979), and Nuismer and Tan (1982) to explain the thickness dependence of ply cracking.

The energy difference ΔU can also be calculated for multiple cracking. In general, ΔU decreases with increasing crack density so that the predicted rate of increase in crack density decreases with increasing applied stress. Experimental observations bear out the predicted behavior.

When the energy method is used, choosing a proper inherent flaw is important. If an inherent crack is assumed to be entirely through the ply width but only partially through the ply thickness, a different model results (Wang and Crossman, 1980; Crossman and others, 1980). However, this through-the-width flaw is more difficult to justify than the through-the-thickness flaw, and introduces computational complexity.

The energy release rate associated with free-edge delamination becomes independent of delamination size once delamination grows longer than one ply thickness or two (O'Brien, 1982; Wang and Choi, 1985). Furthermore, this steady-state energy release rate G can be estimated from the difference

between stored energies before and after full delamination as (O'Brien, 1982)

$$G = \frac{\epsilon^2 h}{2} (E_o - E_{Del}) \quad (6)$$

Here ϵ is the applied strain, h the laminate thickness, and E_o and E_{Del} are the laminate moduli before and after full delamination, respectively. If the inherent delamination is larger than the critical size, the strain ϵ_{Del} at the onset of delamination is given by

$$\epsilon_{Del} = \left[\frac{2G_c}{h(E_o - E_{Del})} \right]^{1/2} \quad (7)$$

Note that ϵ_{Del} is inversely proportional to the square root of the thickness h .

The effect of ply cracks on overall laminate stiffness reduction has been predicted by Nuismer and Tan (1982), and Dvorak, Laws and Hejazi (1985). On the other hand, the laminate modulus E after a partial delamination can be estimated by a rule of mixtures equation (O'Brien, 1982),

$$E = E_o - (E_o - E_{Del}) \frac{a}{w} \quad (8)$$

where a and w are the delamination and specimen widths, respectively. Since E_{Del} is less than E_o , Eq. (8) correctly predicts a decrease of laminate modulus with increasing delamination.

Both a ply crack and delamination grow under a constant energy release rate independent of their size. Therefore, their growth should be more stable than a crack growth in homogeneous materials.

Delamination growth is similar to a crack growth except that, in reality, the delamination surface is neither well defined nor smooth on a microscopic scale. Therefore, the same type of growth analysis as for homogeneous materials can be applied at least on a macroscopic scale. However, a new approach should be developed to analyze multiplication of ply cracks (Wang, 1983).

As discussed earlier, both ply cracks and delamination may be present in fatigue. Therefore, their interaction should be taken into account in an improved analysis. Further, ply crack growth and delamination in general are mixed-mode fracture processes, and hence call for a suitable mixed-mode fracture criterion (Hahn, 1985b).

CONCLUSIONS

Fatigue failure of composite laminates has been extensively studied during the past decade. As a result, a good understanding of failure modes of these materials exists today.

Because of the inherent heterogeneity and anisotropy, failure mechanisms in composites are varied and compete with one another. Although most of the key failure modes have been identified, their temporal sequence and spatial interaction remain to be elucidated in order to develop simple analysis

models. For example, if delamination does not start until ply cracking is almost complete, analysis of the latter needs not take delamination into account.

Once a damage state is known, there are available analysis methods, both sophisticated and simple, that can predict the consequent mechanical properties. Yet a synthesis of all different effects of damage including damage growth modeling is needed.

Since ultimate failure of laminates is controlled by on-axis plies, longitudinal fatigue of unidirectional composites should be fully understood. Especially important is an understanding of the roles of constituent materials in fatigue so that guidelines for material improvement can be more specifically set forth. The use of model composites can be useful in delineating failure mechanisms.

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