THE SAFETY OF ISOTROPIC FLYWHEELS

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ABSTRACT

A probabilistic safety criterion for isotropic flywheel rotors is established based on the tolerated noncontainment failure rates of commercial aircraft turbojet engine rotors. A technique is developed combining reliability with fracture mechanics, and a sample calculation provided, to show the energy-storage levels that isotropic flywheel rotors could achieve within the constraints of this safety criterion.

KEYWORDS

Flywheel safety; probabilistic fracture mechanics; mechanical energy storage; structural reliability; flywheel rotor damage.

The need to conserve petroleum has stimulated the search for new concepts in automobile propulsion that are either independent of gasoline or use it more efficiently. One of these concepts involves the use of a flywheel to conserve the energy that is normally dissipated in braking for later use in accelerating the vehicle from rest. This scheme allows the use of smaller, more efficient engines that need only have sufficient power for steady state road loads rather than for the higher power requirements for acceleration. Flywheels may also be used to improve the acceleration capability of electric vehicles while at the same time load leveling their batteries for increased range.

The practical application of flywheels requires that they be as small as possible and yet be capable of storing sufficient kinetic energy to perform their function. Their energy storage capacity per unit of mass increases with speed but is limited by the strength of the rotor material. Naturally, the higher the state of stress in the material the higher will be its energy storage density. On the other hand, the closer the maximum design stress is to the failure governing strength of the material, the lower will be its structural reliability. Thus, the design of a flywheel involves a trade-off between achievable energy storage density and reliability.

While it is important from the standpoint of cost that the energy density of the flywheel be maximized, it is equally, if not more, important that the flywheel be highly reliable since an uncontained rotor failure could result in severe property damage or injury to persons in the immediate vicinity. The use of a conservative safety factor is impractical since it may unduly penalize the performance of a flywheel. In addition, safety factors are usually based upon subjective perceptions of structural adequacy which may be too inaccurate for high speed flywheels used in automobiles. Clearly a more rational approach is needed if flywheel performance, cost, and reliability are to be optimized.

The approach selected is based upon structural reliability methods which postulate maximum acceptable risks as failure criteria. While acceptable risk is also a highly subjective concept, it is possible to specify it objectively relative to an implicitly sanctioned level of risk associated with a familiar component long in use. In the case of isotropic flywheel rotors, its risk of failure could be related to that of aircraft turbine rotors. While uncontained turbine rotor failures are comparatively rare, they have occurred with marked regularity and have caused damage to aircraft, injury to passengers and, in fact, some fatalities. Despite this the aircraft industry continues to flourish. It seems that the risk of injury from turbine rotor failures is low enough to be considered acceptable by the flying public. Of course, those concerned with the design of turbo-jet engines are continually trying to reduce the frequency of uncontained rotor failures but they do so in an atmosphere of tacit acceptance of the existing level of risk and with the knowledge that their efforts can never reduce it to absolute zero. Our goal, then, is to design flywheels such that their safety is equivalent to that of aircraft turbine rotors. Doing so implies that there may be an uncontained flywheel rotor failure, but these occurrences can be made so rare that public confidence in flywheel augmented automobile propulsion systems is not discouraged.

Since 1964 the noncontainment rate of all types of turbojet engine failures has averaged about 1 per 10^6 engine hours per year (Witmer, 1977). Noncontainments due to rotor burst have run about 1 per 2×10^6 engine hours per year (National Transportation Safety Board, 1974). Table 1 shows both the types and engine hours of all U.S. turbo-engine aircraft for 1975 (Shoaka, Loebl and Patterson, 1977). Using the total of approximately 2×10^7 engine hours per year gives an average turbine rotor burst noncontainment rate of about 10 per year. This statistic does not pose a threat to the viability of the air transportation system. Thus we may conclude that a noncontainment rate of 10 per year for an entire transportation system constitutes an acceptable level of risk.

Ten flywheel-rotor noncontainments per year would be acceptable for the automobile transportation system, provided it were spread over approximately 10^8 vehicles in use throughout the year. Automobiles, on the average, are usually assumed to travel 10,000 miles per year, 5,000 of which are in an urban traffic pattern such as the SAE J227D driving cycle shown in Fig. 1. The total number of rotor stress cycles per year, assuming that one rotor cycle of stress corresponds to one acceleration/braking cycle, would be

$$10^8 \text{ vehicles} \times 5 \times 10^3 \frac{\text{urban mile}}{\text{vehicle year}} \times 1 \frac{\text{stress cycle}}{\text{mi}} = 5 \times 10^{11} \frac{\text{stress cycles}}{\text{year}}$$

For a noncontainment rotor failure rate of 10 per year, the corresponding probability of failure is

$$\frac{10 \text{ noncontainments/year}}{5 \times 10^{11} \text{ stress cycles/year}} = 2 \times 10^{-11} \frac{\text{noncontainments}}{\text{stress cycle}}$$

This is well below the maximum failure risk of 2 to 6×10^{-5} that other studies (Bowdi, 1979) indicate the public will tolerate.

The criterion for an acceptable level of safety of isotropic flywheel rotors, then, is that the number of noncontainment failures be limited to 10/year. Since fracture is the relevant failure mode and fatigue-crack propagation is the corresponding damage mechanism, the techniques of fracture mechanics are applicable. Consequently, the failure criterion is the critical crack size, i.e., the crack size at which the rotor material reaches its critical stress intensity at the maximum design stress.

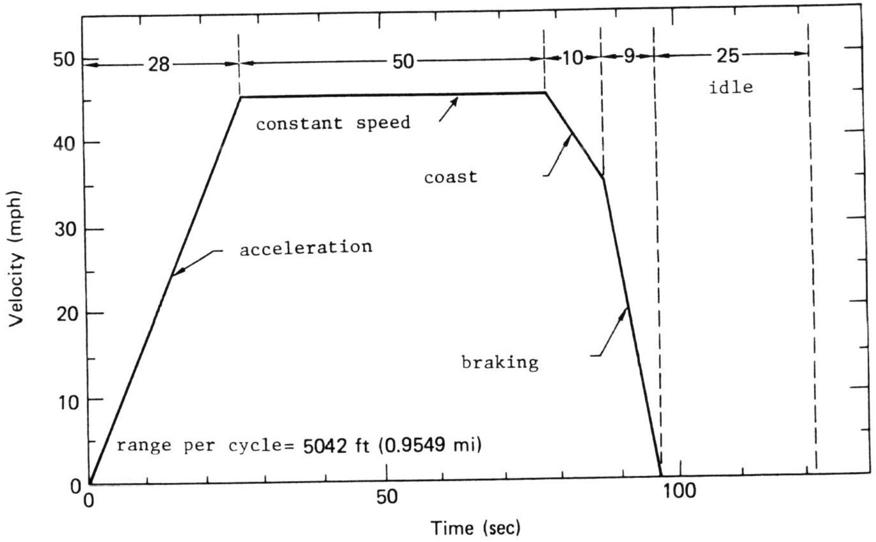

Fig. 1. The Society of Automotive Engineers J227a(D) driving cycle

Since the fracture-mechanics approach postulates that all structures contain unavoidable flaws, it is assumed that the flywheel rotor of each new car contains an initial flaw or crack of size "a." This crack will grow while the vehicle is in use, increasing the risk of failure with time. Thus at any given time the entire vehicle fleet will present a mix of reliability levels depending on the age distribution of the vehicles. A typical age distribution is shown in the first column of Table 2 for vehicles 10 years and younger. Limiting the useful life of the flywheel rotor to 10 years allows this distribution to be normalized over the total vehicle population of 10^8 vehicles as shown in the last column. The total number of noncontainment rotor failures per year would then be

$$N_F = \sum_{i=0}^{i=10} P_{Fi} N_{Vi} \qquad (1)$$

where P_{Fi} is the failure probability of a rotor of age i and N_{Vi} is the number of vehicles of this age in the vehicle fleet. The failure probability, P_F, of a flywheel rotor as a function of age now needs to be determined.

TABLE 1 Annual Engine Hours of U.S. Turbine-Powered Aircraft in 1975

Aircraft Type		Aircraft Hours	Engine Hours
Turbojet	4 engine	1,770,203	7,080,812
	3 engine	2,464,000	7,394,019
	2 engine	1,263,805	2,527,610
Turboprop	4 engine	454,701	1,818,804
	2 engine	337,889	675,778
Total		6,290,598	19,497,023

TABLE 2 Vehicle Age Distribution

Year	Percent of all all vehicles[a]	Percent of all vehicles (normalized)	Number of vehicles, N_{Vi} (normalized)
Under 1	6.6	7.2	7,200,000
1-2	7.9	9.5	9,500,000
2-3	10.0	12.0	12,000,000
3-4	11.4	13.71	13,700,000
4-5	10.1	12.2	12,200,000
5-6	8.4	10.1	10,000,000
6-7	8.1	9.8	9,800,000
7-8	8.0	9.6	9,600,000
8-9	7.0	8.4	8,400,000
9-10	5.5	6.6	6,600,000
	83.0	100.0	100,000,000

[a]See (Shoaka, Loebl, and Patterson, 1977)

Structural reliability is defined as the probability that the strength of a component exceeds the applied stress. If the probability density distributions of both the strength and stress are Gaussian, then structural reliability is expressed as a normally distributed bivariate function by (Shigley, 1972)

$$R = \frac{1}{2\pi^{1/2}} \int_{-K_p}^{\infty} e^{-(z^2/2)}\, dz \qquad (2)$$

$$\text{where } K_p = \frac{\bar{K}_{IC} - \bar{K}_I}{[\sigma^2(K_{IC}) + \sigma^2(K_I)]^{1/2}} \qquad (3)$$

$\bar{K}_{IC}$ = the nominal critical stress intensity of the rotor material,

$\bar{K}_I$ = the nominal stress intensity in the flawed rotor,

$\sigma(K_{IC})$ = the standard deviation of the critical stress intensity

$\sigma(K_I)$ = the standard deviation of the stress intensity.

If we let $\sigma(K_{IC}) = C_v\bar{K}_{IC}$ and $\sigma(K_I) = C_v\bar{K}_I$,

where C_v is the coefficient of variation of the stress and strength parameters and, for simplicity, we assume it is about the same for both, then from Eq. (3)

$$K_p = \frac{\bar{K}_{IC} - \bar{K}_I}{C_v(K_{IC}^2 + K_I^2)^{1/2}} \qquad (4)$$

Values of the reliability R corresponding to K_p can be found using the normal probability tables. The probability of failure P_F is the complement of the reliability defined by

$$P_F = 1 - R. \qquad (5)$$

The critical stress-intensity factor K_{IC} is a material property that is a measure of its toughness when flaws are present. In isotropic flywheel rotors this toughness requirement is mandatory, and values of K_{IC} should not be less than 100 ksi-in.$^{1/2}$. The coefficient of variation C_v reflects the variability of the data used to determine both stress and strength. The reliability of flywheel rotors is particularly sensitive to this coefficient, which should be reduced to as small a quantity as possible within the constraints of time and cost. To evaluate K_I we use an expression that is applicable for surface cracks (ASTM Committee on FTHSSM, Materials Research Standards, 1964)

$$K_I = 1.1\pi^{1/2}\, \sigma\left(\frac{a}{Q}\right)^{1/2} \qquad (6)$$

where σ is the maximum design stress, a is the crack depth, and Q is a function of crack geometry. A crack propagation law that is applicable to a variety of high-strength steels is (Barsom, Imhof, and Rolfe, 1971)

$$\frac{da}{dn} = 0.46 \times 10^{-8} (\Delta K_I)^{2.25} \text{ in./cycle,} \qquad (7)$$

where n = number of stress cycles.

The range of cyclic stress intensity is ΔK_I, where $\Delta K_I = K_{Imax} - K_{Imin}$, so that

$$\Delta K_I = 1.1\pi^{1/2} \Delta\sigma\left(\frac{a}{Q}\right)^{1/2}. \qquad (8)$$

Substituting Eq. (8) into Eq. (7) and combining constants gives

$$\frac{da}{dn} = 2.07 \times 10^{-8}(\Delta\sigma)^{2.25} \left(\frac{a}{Q}\right)^{1.125}.$$

Integrating this expression gives

$$\int_{a_i}^{a} \frac{da}{a^{1.125}} = \frac{2.07 \times 10^{-8}(\Delta\sigma)^{2.25}n}{Q^{1.125}} \int_0^N dn$$

which results in

$$a = \left[\frac{1}{\frac{1}{a_i^{0.125}} - \frac{2.07 \times 10^{-8}(\Delta\sigma)^{2.25}N}{8Q^{1.125}}}\right]^8 \qquad (9)$$

and from Eq. (6)

$$K_I = \frac{1.1\sigma\pi^{1/2}}{Q^{1/2}} \left[\frac{1}{\frac{1}{a_i^{0.125}} - \frac{2.07 \times 10^{-8}(\Delta\sigma)^{2.25}N}{8Q^{1.125}}}\right]^4 \qquad (10)$$

This general expression for K_I can be simplified if we make the following assumptions:

1. Each rotor sees 5000 stress cycles per year, corresponding to an average of 5000 miles per year of urban driving on a SAEJ227D cycle. Thus N = 5000 t, where t is the number of years of flywheel service.

2. The yield strength of the material is 180 ksi.

3. The flaw-shape factor Q corresponds to an a/2c of 0.5; $\sigma/\sigma_y < 0.5$ and Q is therefore 2.4.

4. The stress range $\Delta\sigma$ is governed by the minimum speed of the flywheel rotor, which is usually 50% of the maximum design speed of rotation. Since the stress varies with the square of the speed, the minimum stress will be $\sigma/4$ and the stress range $\Delta\sigma = 3\sigma/4$.

Applying these assumptions to Eq. (10) gives

$$K_I = \left[\frac{1.059\sigma^{0.25}}{\frac{1}{a_i^{0.125}} - 2.5 \times 10^{-6} \sigma^{2.25} t}\right]^4 \text{ ksi-in.}^{1/2} \qquad (11)$$

The steps in the solution of Eq. (1) are summarized in Table 3 for the particular

values listed at the bottom of the table. It is apparent that for the design stress level assumed, the expected number of failures is only significant for the population of 10-year-old vehicles.

The flywheel energy density, e_f, may be related to the assumed design stress level by

$$e_f = k_1 k_2 \frac{\sigma}{\rho}, \qquad (12)$$

where k_1 is the fraction of available energy from the flywheel corresponding to its minimum speed; k_2 is the shape factor of the flywheel which, for a solid steel disk of uniform thickness, is about 0.6; σ is the design stress, and ρ is the material density.

It is impractical to solve explicitly for the stress level and, consequently, the maximum energy density of the flywheel corresponding to 10 non-containment failures per year. Instead we compute the number of non-containment failures for a range of flywheel stress levels and plot them against the energy density associated with each stress level as shown in Fig. 2. These data are plotted for two values of critical stress intensity. The energy densities associated with 10 non-containment failures per year in a population of 10^8 vehicles is 7.7 watt-hours/kg for K_{IC} = 100 ksi-in$^{1/2}$ and 8.3 watt-hrs/kg for K_{IC} 140 kis-in$^{1/2}$. These energy densities are, of course, related to the assumptions of initial flaw size and the coefficient of variation used to illustrate the methodology.

TABLE 3 Sample Computation for the Number of Noncontainment Rotor Failures in a Population of 10^8 Vehicles of Various Ages.[a]

Year	Cycles	K_I	$K_{IC}-K_I$	K_p	P_F	N_V	N_F	N_F
1	5,000	16.25	83.75	8.249	8×10^{-17}	7.2×10^6	8×10^{-10}	5×10^{-10}
2	10,000	18.12	81.88	8.057	4×10^{-16}	9.5×10^6	3.8×10^{-9}	4.3×10^{-9}
3	15,000	20.19	79.81	7.823	2.6×10^{-15}	12.0×10^6	3.2×10^{-8}	3.6×10^{-8}
4	20,000	22.57	77.43	7.553	2.1×10^{-14}	13.7×10^6	2.9×10^{-7}	3.3×10^{-7}
5	25,000	25.31	74.69	7.241	2.2×10^{-13}	12.2×10^6	2.7×10^{-6}	3×10^{-6}
6	30,000	28.47	71.53	6.880	3×10^{12}	10.1×10^6	3.1×10^{-5}	3.4×10^{-5}
7	35,000	32.15	67.85	6.460	5×10^{-9}	9.8×10^6	5.2×10^{-4}	5.5×10^{-4}
8	40,000	36.43	63.57	5.973	1.2×10^{-9}	9.6×10^6	1.2×10^{-2}	1.2×10^{-2}
9	45,000	41.46	58.54	5.408	3.2×10^{-8}	8.9×10^6	2.7×10^{-1}	2.8×10^{-1}
10	50,000	47.39	52.51	4.755	9.9×10^{-7}	6.6×10^6	6.53	6.85

[a]Based on the following assumptions

Table 3 continued :

K_{IC} = 100 ksi - in.$^{1/2}$
a_i = 0.025 in.
σ_{max} = 74,000 psi
σ_{min} = 18,500 psi
σ_y = 180,000 psi
a/2c = 0.5
C_V = 0.10
$\Delta\sigma$ = 67,500

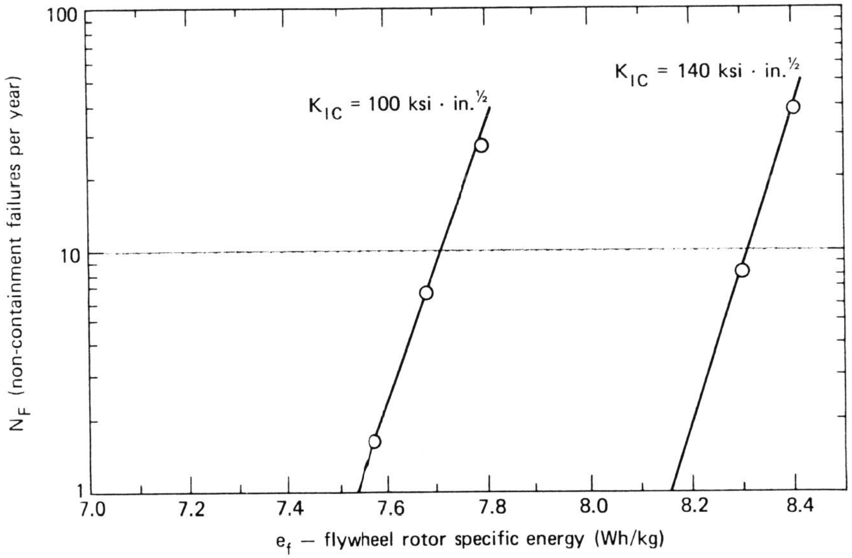

Fig. 2. Variation of rotor noncontainment failures per year vs. flywheel-rotor specific energy for two levels of critical stress intensity

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