RESIDUAL STRESS EFFECTS ON FRACTURE TOUGHNESS MEASUREMENTS

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ABSTRACT

Macroscopic residual stresses in fracture toughness test specimens can erroneously affect the fracture toughness measurement. However, most current fracture toughness test methods do not measure nor correct for residual stress effects. A theory is presented which leads to a test method for short rod fracture toughness specimens in which the effects of macroscopic residual stresses are measured and corrected from the data. The theory is applied to fracture toughness measurements of 18 different groups of tungsten carbide specimens. In some cases, the correction for residual stress effects is more than 20 percent. Large reductions in data scatter as compared to uncorrected data indicate the validity of the theory.

KEYWORDS

Residual stresses; fracture toughness; short rod specimen; test methods; tungsten carbides.

INTRODUCTION

Residual stresses in materials can be divided into two categories - microscopic and macroscopic. Microscopic residual stresses fluctuate from maximum compression to maximum tension over distances which are of the same order as the microstructure of the material. Thus, although microscopic residual stresses may well affect the fracture toughness, the toughness measured on such a specimen can be considered as a material property inasmuch as the residual stresses are a homogeneous characteristic of the material from a macroscopic fracture toughness viewpoint.

Macroscopic residual stresses, on the other hand, can affect the measurement of fracture toughness by causing an entire crack front of macroscopic dimensions to be in a state of tension or compression *before the application of any external load.* This paper is concerned with the effects of macroscopic residual stresses on fracture toughness measurements.

It has long been known that macroscopic residual stresses can have a profound effect on fracture. Shot peening retards the formation of surface cracks in metals, for example (Elber, 1974), and cracks in glass often slowly enlarge because of a combination of residual stress and stress corrosion cracking. Elber (1974), Bucci (1980) and Underwood and Throop (1980) have noted strong residual stress effects on crack initiation and fatigue crack propagation in metals. Since fracture toughness is considered to be a material property, and since macroscopic residual stress in a specimen definitely is *not* a material property, any apparent toughness variation which results from macroscopic residual stresses must be considered as erroneous.

Standard fracture toughness test methods do not address the problem of residual stresses. This paper considers the problem of macroscopic residual stresses in the short rod fracture toughness test specimen suggested by Barker (1977). It is shown that residual stress effects in these specimens can be detected in the toughness test, and that a relatively simple theory leads to a data analysis procedure which corrects the toughness measurement for residual stress effects.

THEORY

In previous derivations of the equations for measuring the fracture toughness from linear elastic fracture mechanics (LEFM) principles, such as in Irwin and Kies' classic paper of 1954, it has been tacitly assumed that there are no macroscopic residual stresses in the test specimen. However, a relatively simple generalization of the derivation suggests a test method and a data analysis technique which provide correct fracture toughness measurements in spite of the presence of modest residual macroscopic stresses in the test specimen. The derivation is presented below with reference to the short rod specimen configuration, which is easily tested in the manner suggested by the theoretical development. Previous short rod derivations have been given by Barker and Leslie (1977) and Barker (1979). Residual stress fields in which the periphery of the specimen is in longitudinal tension or longitudinal compression affect the short rod fracture toughness measurements the most. It is these which are treated in the following theory.

The short rod test configuration is illustrated in Fig. 1a. The specimen is tested by slowly increasing the load, F, on the specimen until a crack initiates at the point of the "V". The crack growth in the short rod specimen tends to be initially stable because of the constant widening of the crack front (dimension b in Fig. 1a) as the crack propagates. However, the load goes through a smooth maximum when the crack reaches approximately the location shown in Fig. 1a, and thereafter the crack-advancing load decreases with further crack growth.

The equation for the plane-strain critical stress intensity factor, K_{IcSR}[1] is derived using the familiar compliance approach. Plasticity effects in the specimen are assumed negligible. It is assumed that the energy required to advance a steady-state crack a small distance, Δa (Fig. 1b, 1c), is

$$\Delta W = G_{Ic}\, \bar{b}\, \Delta a, \qquad (1)$$

[1] In keeping with the ASTM definition of the symbol K_{Ic} as the plane-strain critical stress intensity factor as measured by applying the method of ASTM E 399 (1978), the symbol K_{IcSR} is used in this paper to denote the plane-strain critical stress intensity factor as measured by the short rod specimen.

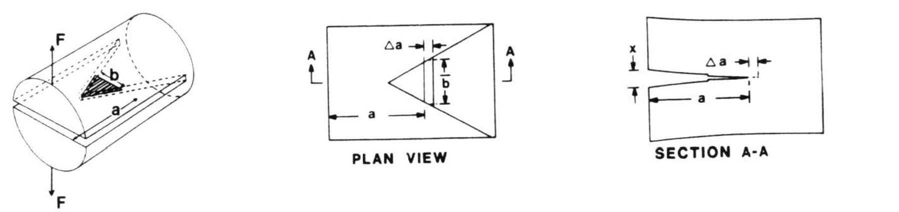

Fig. 1. Schematics of the short rod specimen.

where $\bar{b}$ is the average width of the crack front between crack lengths a and $a+\Delta a$, and where G_{Ic} is a characteristic material property that denotes the energy required per unit area swept out by the crack front. Most of the energy, W, comes from the work done on the specimen during the course of the test. Fig. 2 represents a test record of the load applied to the specimen, F, versus the opening of the mouth of the specimen, x. From 0 to A, the mouth of the specimen is being flexed open before any crack initiation at the point of the "V". At A, where the curve first deviates from linearity, the crack starts growing, and continues to grow as the curve proceeds through BCD.

The total work done on the specimen at any point in the test is simply $\int F dx$ from $x = 0$ out to the current mouth opening. However, part of the total work is recoverable in an elastic release, as from B to E, or C to F. Since the formation of the crack is an irreversible process, it is only the irrecoverable part of the work done on the specimen which can contribute to the work of crack formation, ΔW, in Equation 1. Assume that the specimen has been loaded along OAB, and that at point B, the crack length is a. Assume also that a release of the load from point B would produce the linear unloading path BE. If it were not for residual stresses, our assumption of negligible plasticity would require that the crack would tend to close completely upon release of the external load, i.e., the release path would lead directly toward the origin, O. However, if a tensile longitudinal residual stress field is present near the periphery of the specimen, the mouth will remain partly open even after the external load is completely removed, as indicated in Fig. 3. Hence, the release path may lead to point E rather than O.[2]

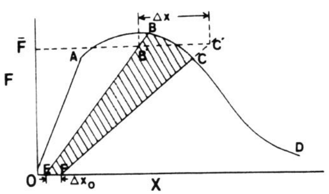

Fig. 2. Schematic of a short rod load-displacement test record.

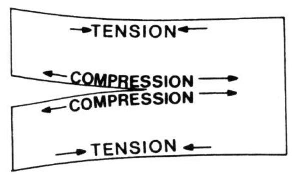

Fig. 3. Illustration of longitudinal residual stresses which affect the fracture toughness measurement.

[2] A compressive longitudinal residual stress field near the periphery would cause the mouth to tend to over-close, i.e., points E and F of Fig. 2 would lie to the left of the origin. This is discussed later.

Now let the test be continued from B to C, resulting in an additional crack growth Δa, and an additional crack surface area $\bar{b}\,\Delta a$ (Fig. 1b). If a release of the load from point C would produce the release path CF, it is clear that the additional irrecoverable work done on the specimen in going from B to C is represented by the cross-hatched area of Fig. 2. This work must contribute to ΔW.

One can obtain an analytical expression for the cross-hatched area by approximating it by the trapezoid EB'C'F, where the height of the trapezoid, $\bar{F}$, is the average load applied to the specimen as the test record advances from B to C. If the distance B'C' is called Δx, and the distance EF is called Δx_0, then the cross-hatched area is equal to $\frac{1}{2}\bar{F}(\Delta x+\Delta x_0)$, and we can write

$$\Delta W = \tfrac{1}{2}\bar{F}(\Delta x+\Delta x_0) + \Delta W', \qquad (2)$$

where $\Delta W'$ is the contribution to ΔW from sources other than the irrecoverable work done on the specimen during the test.

The only source of energy for $\Delta W'$ is the internal strain energy which is already in the specimen because of its residual macroscopic stresses. For the purposes of this derivation, it is assumed that $\Delta W'$ is negligibly small compared to $\frac{1}{2}\bar{F}(\Delta x+\Delta x_0)$, and that $\Delta W'$ can therefore be dropped from Equation 2. The application of the results of this derivation must therefore be restricted to those cases for which the above assumption is true. Thus, we can re-write Equation 2 as

$$\Delta W = \tfrac{1}{2}\bar{F}\ \Delta x(1+p), \qquad (3)$$

where $p \equiv \Delta x_0/\Delta x$. The parameter p can be viewed as a measure of the magnitude and direction of the residual stress effect. If $\Delta x_0 = 0$, the two unloading load-displacement paths point toward the same spot on the zero-load axis, indicating no longitudinal residual stress effect exists between crack lengths a and $a+\Delta a$. A positive value of p suggests tensile longitudinal residual stresses near the periphery of the specimen. A negative value of p occurs when Δx_0 is negative, i.e., when the elastic unloading paths, if extended linearly to the zero-load axis, would cross before they got there, as indicated in Fig. 4c. Equation 3 is valid for either positive or negative p values.

In a paper by Barker (1979), the same operational definition of p was used to denote the plasticity exhibited by a short rod specimen during a fracture toughness test. In that derivation, it was assumed that there were no macroscopic residual stresses in the specimen. The present derivation, on the other hand, assumes the presence of residual stresses, but no plasticity, and arrives at the same operational definition of p. Thus, in a more general sense, p might be thought of as a measure of the degree to which LEFM assumptions are violated in a particular test. In this analysis, however, we shall continue to regard p as a measure of the magnitude of residual stress effects in the fracture toughness measurement.

Once Equation 3 is obtained, the balance of the derivation follows the same steps as those of Barker (1979), and arrives at

$$K_{IcSR} = A\ F_c(1+p)/B^{3/2}\ , \qquad (4)$$

in which A is the dimensionless specimen calibration constant defined by Barker (1979), F_c is the load as the crack passes through the critical crack length, a_c, and $(1+p)$ is used as the small-p approximation of the expression $[(1+p)/(1-p)]^{\frac{1}{2}}$ which results from the derivation.

The evaluation of p is simple from an experimental viewpoint (Fig. 2). It requires no calibration of the specimen mouth opening displacement transducer, because the calibration constant cancels out on taking the ratio $\Delta x_0/\Delta x$. The two unloading-reloading cycles which determine p can also be used to define the point on the load-displacement path at which the crack passed through the critical crack length, a_c. The slope of each elastic unloading path determines the crack length through the compliance versus crack length calibration of the specimen geometry. The load-displacement point at which the crack length was a_c was found by interpolation between the known crack lengths where the unloading-reloading cycles were initiated. The load at this point is F_c.

EXPERIMENTS

Short rod fracture toughness tests were done on 18 different groups of tungsten carbide specimens of various grades and from several manufacturers. The tests were done on a service basis, and thus the details of specimen composition, grain size, and hardness are largely unknown. The cobalt content range among the different grades was at least as large as 6 percent to 16 percent. All specimens were 12.7 mm diameter by 19.05 mm long. They were tested on the Fractometer I fracture toughness test machine (Barker, 1978), and plots of load versus specimen mouth opening displacement were made for each test, including at least two unloading-reloading cycles for evaluation of p-factors.

The load-displacement plots gave p-factors which ranged in value from -0.29 to +0.20. Fig. 4a shows one of the cases in which p was zero, i.e., the residual macroscopic stresses were negligible in the longitudinal direction, and the test conformed well to LEFM principles. Fig. 4b shows a test record in which p was positive, while Fig. 4c shows a case in which it was negative. Notice that the test record is flattened in the positive p case, and that it is more peaked in the negative p case. Fig. 4d shows a case in which p was initially about zero, but then went negative, undoubtedly due to a residual stress field which varied along the length of the short rod specimen. The changing p-value is reflected in the skewed shape of the load-displacement test record, which tends to become peaked rather late in the test where p is quite negative.

In the data analysis, the fracture toughness was obtained first according to LEFM principles, i.e., assuming no residual stresses in the test specimen. The toughness value so obtained was given the symbol K_{QSR}. The measured residual stress effect was then taken into account by calculating

$$K_{IcSR} = (1+p)\ K_{QSR} \qquad (5)$$

The standard deviations of the K_{QSR}'s were 3.0 percent or larger in 9 of the 18 groups of specimens. The data for these groups are presented in Table I, where p_{min} and p_{max} represent the minimum and maximum values of p observed within the group. Notice that the standard deviations of the K_{IcSR} values are always sub-

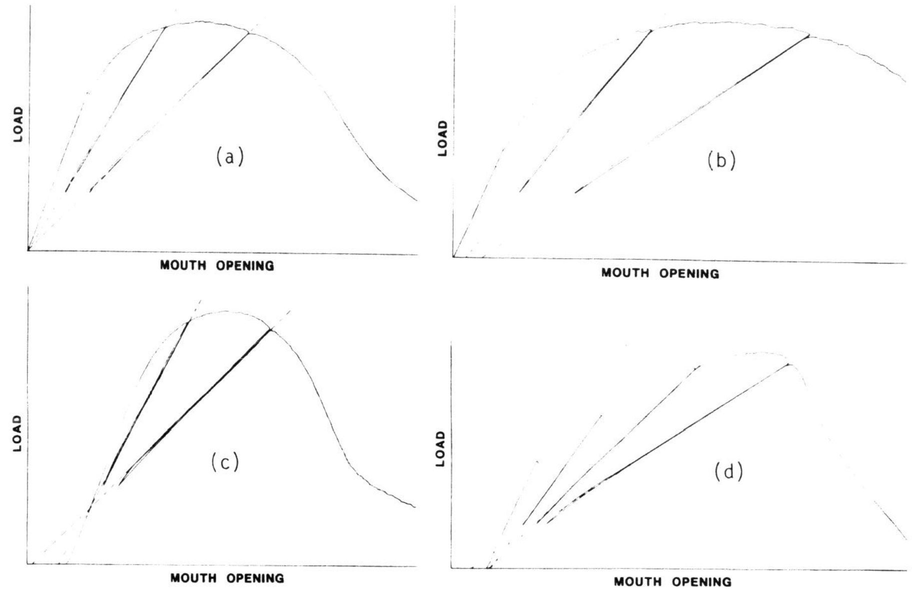

Fig. 4. Four load-displacement test records from the test series.

stantially less than the standard deviations of the K_{QSR}'s, and that 6 of the 9 K_{IcSR} standard deviations are less than 3.0 percent.

The corresponding data for the 9 remaining groups of specimens with K_{QSR} standard deviations of less than 3.0 percent are shown in Table II. In these cases, five of the K_{IcSR} standard deviations are smaller, two are unchanged, and two are larger than the corresponding K_{QSR} standard deviations.

TABLE I Tungsten Carbide Toughness Test Results

No. of Spec.	P_{min}	P_{max}	Av. K_{QSR} MPa$\sqrt{m}$	Av. K_{IcSR} MPa$\sqrt{m}$	Std. Dev. of K_{QSR}'s	Std. Dev. of K_{IcSR}'s
11	-.07	.00	19.5	18.9	3.0%	1.1%
5	-.04	.05	16.8	16.5	3.8%	3.3%
3	.05	.09	14.1	15.2	3.0%	2.3%
3	-.29	-.20	12.2	9.1	4.9%	2.1%
3	-.04	.08	11.4	11.4	4.5%	1.7%
3	-.12	.20	10.2	10.0	22.5%	8.5%
3	-.03	.09	13.0	13.5	4.5%	1.6%
3	-.07	.03	13.3	13.2	9.5%	3.4%
3	-.03	.00	15.6	15.5	3.3%	1.8%
				Average:	6.6%	2.9%

TABLE II More Tungsten Carbide Toughness Test Results

No. of Spec.	P_{min}	P_{max}	Av. K_{QSR} MPa$\sqrt{m}$	Av. K_{IcSR} MPa$\sqrt{m}$	Std. Dev. of K_{QSR}'s	Std. Dev. of K_{IcSR}'s
11	.07	.10	7.6	8.2	1.9%	1.5%
5	-.08	-.02	15.2	14.3	2.5%	1.6%
3	-.09	-.06	14.5	13.5	1.6%	0.0%
3	-.07	-.04	15.3	14.4	2.1%	1.1%
3	-.08	-.08	15.4	14.1	1.5%	1.5%
3	-.06	-.05	15.6	14.8	0.9%	0.5%
3	-.06	-.06	16.2	15.2	2.5%	2.5%
3	-.12	-.07	15.8	14.4	0.7%	2.3%
3	-.05	.03	14.9	14.3	1.3%	2.1%
				Average:	1.7%	1.5%

DISCUSSION

There can be little doubt that the parameter p is a measure of the effects of residual stresses in the tungsten carbide specimens of this study. It constitutes a direct measure of the residual bending of the specimen halves which develops as the crack advances through the specimen. The bending cannot result from plastic deformation of the specimen during the test for two reasons: (1) the only non-elastic zone in these specimens during the test is at the crack tip, where it is much too small to account for the observed effect; and (2) most of the p values were negative, which signifies residual bending in the opposite direction from that to be expected from plastic deformation. In tests of certain steel specimens, relatively large negative p values have correlated well with independent observations of curvature of the specimen halves after the test.

Since p is a measure of residual stress effects, one would expect a successful theory to predict no residual stress correction to the measured toughness whenever $p = 0$. This is the case in the theory of this paper, since setting $p = 0$ in Equation 4 leads to the completely LEFM equation for K_{IcSR} (Barker, 1979). Furthermore, if several specimens are essentially identical except for their residual stresses, one would expect some scatter in their K_{QSR} values because of residual stress differences, but much less scatter in the K_{IcSR} values if the corrections for residual stress are valid. The data in Table I demonstrate that this is the case. Finally, in a group of specimens in which some had essentially no residual stresses and others had varying degrees of residual stress, the theory should correct the K_{QSR}'s of the specimens with residual stresses to agree with the K_{QSR}'s of the specimens with no residual stress effects, and the latter should not be changed by the theory in calculating the K_{IcSR} value. Again, this was the case for a number of the specimen groups in Table I. Thus, the experimental data of Table I provides conclusive evidence of the value of the theory and the data analysis technique.

Table II shows the results of all the groups for which the standard deviations of the K_{QSR} values were less than 3 percent. In this case, the application of the residual stress theory resulted in a much smaller improvement in the standard deviations: 1.7 percent for the K_{QSR}'s and 1.5 percent for the K_{IcSR}'s. The smaller reduction in standard deviations is to be expected, however, because some finite standard deviations due to causes other than residual stresses must be expected. When the standard deviations of the K_{QSR}'s are already close to those to be expected from non-residual stress causes, correcting for the residual stress effect cannot decrease the standard deviations very much. However, this does not diminish the *validity nor the importance* of the corrections for residual stress effects. The average percent difference between the K_{QSR} and K_{IcSR} values in Table II is 7 percent, in spite of only 1.7 percent average standard deviation of the K_{QSR} values.

One of the assumptions made in the derivation of the residual stress equation for K_{IcSR} was that the residual stresses in the specimen are relatively small, i.e., p should be small. No theoretical value for the maximum allowable p has been determined, however, and the data of the present paper do not appear to define a precise limiting value for p. At the present time, it may be prudent to regard the toughness measurement as suspect if $|p| > .20$ or even $|p| > .15$. Constraining $|p|$ to be less than .15 for a test validity requirement should not pose any great difficulties.

Finally, although the rationale is slightly different between the elastic-plastic and the residual stress derivations, the resulting equations and data analysis procedures for the two cases are identical. In the general case in which the specimen contains both residual stresses and elastic-plastic effects, it is easily shown that precisely the same K_{IcSR} equation and data analysis method simultaneously treats both residual stress and elastic-plastic effects. The fracture toughness test data alone do not supply enough information to separate the two effects.

CONCLUSIONS

Although macroscopic residual stresses in fracture toughness test specimens can erroneously affect fracture toughness measurements, most current test methods neither detect nor correct for residual stress effects. However, a relatively simple theory leads to a data analysis procedure which both measures and corrects for residual stress effects in short rod fracture toughness specimens. The application of the measurement technique to 18 groups of tungsten carbide specimens of various grades has indicated the validity of the theory. Whenever standard deviations were larger than normal, the application of the theory greatly reduced the data scatter. The corrections for residual stresses were usually important even when the uncorrected data scatter was small. The average residual stress effect in the experiments of this study was about 7 percent, but the effect was more than 20 percent in some specimens.

The test and data analysis procedure for treating residual stresses turns out to be the same as that for handling modest elastic-plastic effects. In fact, the one method simultaneously treats both effects. The ability to account for residual stress and elastic-plastic effects in short rod tests greatly enhances the value of the short rod method of fracture toughness measurement.

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