PLASTIC EFFECTS IN DYNAMIC CRACK PROPAGATION

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ABSTRACT

In order to investigate rapid crack growth at large-scale yielding a crack propagation experiment on a highly loaded, single edge-notched rectangular sheet is performed. The experiment is analysed by the aid of a finite element program allowing for dynamic elastic-plastic calculations. An elastic linearly strain-hardening plastic material model is used. For comparison the experiment is also analysed assuming purely linear elastic material behaviour. The energy flow to the crack-tip region is found to be significantly influenced by the introduction of plastic effects in the analysis. The values obtained in the elastic-plastic analysis are only about half of the corresponding values obtained in the purely elastic analysis. Finally, some comments for further research concerning crack-tip models, both the one used here and others, are made.

KEYWORDS

Crack propagation; numerical analysis; fracturing; velocity; dynamic tests.

INTRODUCTION

Most of the work done in the field of dynamic fracture mechanics has been based on the assumption of linear elastic fracture mechanics (LEFM). In the linear theory it is implied that for a given material a one-to-one unique relationship exists between crack-propagation toughness K_{pc} and instantaneous crack-tip velocity $\dot{a}$ or, equivalently, between critical energy-release rate G_{pc} and instantaneous crack-tip velocity. One can expect this to be true for a purely elastic material, but to be valid only to a limited extent for structural materials.

Several authors have used methods based on purely linear elastic material behaviour to evaluate rapid crack propagation experiments; see for instance Dahlberg, Nilsson, and Brickstad (1980); Hahn and others (1978); Kalthoff, Beinert, and Winkler (1978); Nilsson (1974). In most of these investigations the $G_{pc} - \dot{a}$ relationship, obtained from experiments on moderately to highly loaded specimens, is found to be dependent on the specimen geometry. That is, no unique $G_{pc} - \dot{a}$ relationship is found from experiments on specimens where extensive nonelastic deformation occurs. Investigations on several different specimen geometries indicate that a geometry condition for the limit of dynamic LEFM of the same type as the well-known ASTM-condition for static

LEFM (ASTM, 1973) can be applied (Dahlberg, Nilsson, and Brickstad, 1980). This condition is found to be more restrictive in the dynamic case than in the static case. Therefore, nonelastic effects must probably be explicitly included in the analysis of fast fracture in many engineering problems.

In this investigation plastic effects are explicitly included in the evaluation of a rapid crack propagation experiment on a highly loaded specimen. A finite element method (FEM) allowing for dynamic elastic-plastic calculations is used to analyse the experiment. The investigation is a first rough attempt to include plastic effects. No strain-rate or thermal effects are for instance included in the analysis.

CRACK-TIP MODEL

To analyse rapid crack propagation in elastic bodies several different methods to model the crack-tip region have been proposed and used. One promising method, it seems, is to use a cohesive zone model where a law is prescribed which relates the cohesive force to the separation of the crack surfaces. This model has been used in FEM-simulations of rapid crack growth by Keegstra, Head, and Turner (1977). Singular crack-tip elements, which have proven efficient for static calculations, have also been proposed for modelling of running cracks by for instance Aberson, Anderson, and King (1977). A third method to simulate crack growth employing a particular procedure for gradual node relaxation (Rydholm, Fredriksson, and Nilsson, 1978) has also proven useful. This method has been used for instance by Dahlberg, Nilsson, and Brickstad (1980).

Achenbach and Kanninen (1978) have given an asymptotic solution to the very near crack-tip region of a crack propagating in antiplane strain, mode III, in an elastic-plastic material. Dynamic effects are also included in their analysis. They considered the elastic, linearly strain-hardening plastic material model and found that the power of the strain singularity depends on the strain-hardening modulus. If their result is substituted into an integral expression for the energy flow to a moving singular crack tip given by Strifors (1977), it can be readily shown that only purely elastic material behaviour results in nontrivial contributions to the integral if thermal effects are neglected. In other words, energy flow to the crack-tip region is obtained only if the material responds elastically in this region. The integral expression derived by Strifors (1977) gives the energy flow through a control surface in the shape of a narrow tube surrounding the singular crack border as the diameter of the tube approached zero. Unfortunately, the asymptotic solution to the near crack-tip region of a propagating mode I crack in elastic-plastic material is not yet available in the literature. The same restriction as in the mode III case regarding the material behaviour in the near crack-tip region may, however, very well be required in the mode I case as well. If so, this result supports the use of a crack-tip model of the same type as has been used in elastic analysis.

In this investigation the experiment is analysed using the method developed and implemented into the FEM-program system NONSAP by Rydholm, Fredriksson, and Nilsson (1978). In this method the upper half of the symmetric plane body is modelled by finite elements. The nodes along the crack plane in front of the crack tip are constrained against motion perpendicular to the crack plane. Crack growth is simulated in the following way: When the crack tip reaches a node, the force in the node is calculated and the constraint against motion is replaced by the force. As the crack tip advances to the next node, this force is gradually relaxed to zero. When the crack tip reaches the next node, the same procedure is repeated and so on until the crack has grown through the body. The output of the program is values of the total energy flow to the crack-tip region during crack growth between neighbouring nodal points. An average of the energy flow to the crack-tip region during crack growth between two neighbouring nodal points is then calculated by dividing the obtained total energy flow to the crack-tip region during this growth by the area of the interjacent element side. This method for crack growth simulation has been generalized to allow for elastic-plastic material behaviour.

ENERGY BALANCE CONSIDERATIONS

If thermal effects are neglected, the energy balance for a plane body of unit thickness with one propagating crack tip under mode I conditions can be written

$$\frac{\partial}{\partial a}(W-U-T-D)=\frac{\partial E_f}{\partial a} \tag{1}$$

where a is the crack length, W the energy input to the body, U the strain energy, T the kinetic energy, D the dissipated energy, and $\partial E_f/\partial a$ the energy flow to the crack-tip region.

To be able to predict crack growth a fracture criterion is necessary. With the crack-tip model adopted in this investigation, an energy criterion is inherent and can be given the following form

$$\begin{aligned} \frac{\partial E_f}{\partial a} &= \gamma_f \qquad \dot{a}>0 \\ \frac{\partial E_f}{\partial a} &< \gamma_f \qquad \dot{a}=0 \end{aligned} \tag{2}$$

where γ_f is the specific fracture energy.

If this description of crack growth shall be meaningful, the specific fracture energy must be a material property, possibly dependent on the instantaneous crack-tip velocity. So far this is a hypothesis.

In dynamic LEFM a purely linear elastic analysis is performed. The energy balance then takes the form

$$\frac{\partial}{\partial a}(W-U-T)=\frac{\partial E_s}{\partial a} \tag{3}$$

where $\partial E_s/\partial a$ is the energy flow to the crack-tip region.

The concept energy release rate G is introduced and defined

$$G=\frac{\partial}{\partial a}(W-U-T) \tag{4}$$

Substitution into the energy balance yields

$$G=\frac{\partial E_s}{\partial a} \tag{5}$$

The crack growth criterion is in this theory often formulated

$$\begin{aligned} G &= G_{pc} \qquad \dot{a}>0 \\ G &< G_{pc} \qquad \dot{a}=0 \end{aligned} \tag{6}$$

For a given material, G_{pc} is assumed to be a unique function of the instantaneous crack-tip velocity only. This assumption can only be fulfilled if nonelastic deformations occurring in the body are associated with crack growth and controlled solely by the stress-intensity factor. As mentioned in the introduction, experimental results show that this assumption is not fulfilled and, accordingly, dynamic LEFM is invalid if extensive nonelastic deformation occurs in the body.

EXPERIMENTAL PROCEDURE

The experiment considered was carried out using a single edge-notched rectangular specimen with dimensions according to Fig. 1. The notch length was chosen short, a_o = 3 mm, in order to obtain crack propagation in a highly loaded specimen. The notch root was sharpened by a special saw blade giving a root radius of about 0.1 mm. The specimen was made of a sheet of a cold rolled and hardened carbon steel with a thickness of 0.5 mm, thus giving plane stress conditions. The mechanical properties of the material are given in Table 1.

TABLE 1 Mechanical Properties of the Material

Density	ρ = 7870 kg/m^3
Poisson's ratio	ν = 0.3
Young's modulus	E = 205·10^3 MN/m^2
Strain-hardening modulus	E_t = 10.4·10^3 MN/m^2
Yield strength	σ_{ys} = 1530 MN/m^2
Static fracture toughness	K_c = 125 MN/m$^{3/2}$

The static fracture toughness given in the table is the value at initiation of slow, stable crack growth.

The specimen was given a uniform displacement, see Fig. 1, in a special testing machine (Dahlberg, 1978) where fixed grip conditions can be realized. The boundary displacement was slowly increased until fracture occurred. The actual boundary displacement at fracture v_o was determined by the aid of three strain gauges glued to the specimen; see Fig. 1. A continuous recording of crack length as a function of time was made by use of an impedance method developed by Carlsson (1962).

EVALUATION PROCEDURE

The FEM-analysis was performed for conditions of plane stress. Bilinear rectangular four-node elements were used. The mesh contained 342 elements and 651 degrees of freedom; see Fig. 2. An elastic, linearly strain-hardening plastic material model was used which well describes the behaviour of the specimen material at quasi-static loading. That is, no strain-rate effects were included in the constitutive relationship. Due to the large extension of the crack-tip model used in this analysis, the highest strain rate observed had a magnitude of less than 10^3 s^{-1}. This is thought to have limited influence on yielding and can probably be neglected without introducing any large errors in the analysis. The dissipated energy gives cause to a temperature rise in the body which can be considerable in the vicinity of the propagating crack tip, see for instance measurements and analyses by Weichert and Schönert (1978). But for simplicity, thermal effects were neglected in this investigation. von Mises' yield criterion and the associated Prandtl-Reuss' incremental flow rule were used. Isotropic strain-hardening was assumed. For comparison the experiment was also analysed assuming purely linear elastic material behaviour.

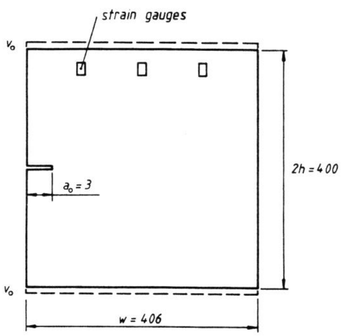

Fig. 1. Specimen geometry and loading conditions.

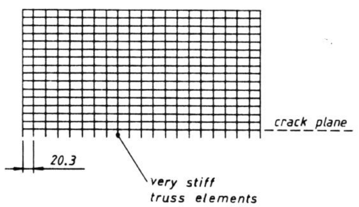

Fig. 2. FEM-mesh used in the calculations.

As input to the FEM-program, the times and the actual loads when the crack tip passes the nodal points along the crack plane are required. The program then assumes a constant crack-tip velocity between neighbouring nodal points and makes a linear interpolation of the load. The load was given to the FEM-program in the form of a constant boundary displacement v_o measured at initiation of crack growth. This displacement was kept constant during the whole FEM-simulation of the experiment. The calculated average of the energy flow to the crack-tip region between two neighbouring nodal points is referred to the midpoint of the interjacent element side, and the same is done for the crack-tip velocity $\dot{a}$. To obtain an estimate of the velocity $\dot{a}$ in a certain midpoint, a third-degree polynomial was fitted to the passing times of the four neighbouring nodal points, two on each side of the midpoint. The derivative of this interpolation polynomial with respect to time then gave the velocity $\dot{a}$ at the considered midpoint.

RESULTS AND DISCUSSION

The experiment was analysed both assuming elastic-plastic material behaviour as described above and linear elastic material behaviour according to dynamic LEFM.

Figures 3 and 4 show the total energy balance obtained from the elastic analysis and from the elastic-plastic analysis, respectively. In both cases the energy balance is satisfied within the expected accuracy of the computations. The kinetic energy turned out to be very small in both cases. A comparison of the energy flow to the crack-tip region for the two cases shows that the introduction of plasticity in the analysis has a significant influence upon the result. The energy flow to the crack-tip region is reduced to approximately half the value in the elastic-plastic analysis as compared to the elastic analysis. The differing amount of energy is mainly consumed in dissipative processes. Due to the crack growth simulation technique adopted, some energy flow to the crack-tip region is obtained also immediately after the crack has grown through the body. Since the boundary conditions were constant displacements or zero tractions, the energy input to the body during crack growth was zero.

Instantaneous crack-tip velocity $\dot{a}$, critical energy-release rate G_{pc}, and specific fracture energy γ_f are plotted as functions of relative crack length a/w in Fig. 5. Due to shortcomings in the measuring equipment, the crack-tip velocity could only be evaluated in the interval $0.225 \leq a/w \leq 0.875$. As seen in Fig. 5, $\dot{a}$ is decreasing in the evaluated interval and both G_{pc} and γ_f show, as well, a decreasing tendency in the same interval. Superposed on this decreasing trend is an oscillation in both $\dot{a}$, G_{pc}, and γ_f. This oscillation in the different curves is almost synchronous and has a period indicating that it is due to elastic wave reflections from the clamped specimen boundaries.

Figure 6 shows the obtained G_{pc}- and γ_f-values as functions of $\dot{a}$. In the figure is also given the one-to-one unique G_{pc}-$\dot{a}$ relationship as obtained by Dahlberg, Nilsson, and Brickstad (1980) from experiments on several different specimen geometries made of the same material as was used here. The G_{pc}-values from this investigation well

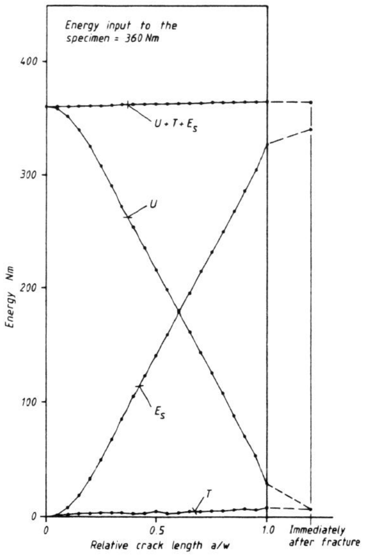

Fig. 3. Strain energy U, kinetic energy T, and accumulated energy flow to the crack-tip region E_s versus dimensionless crack length a/w as obtained from the elastic analysis.

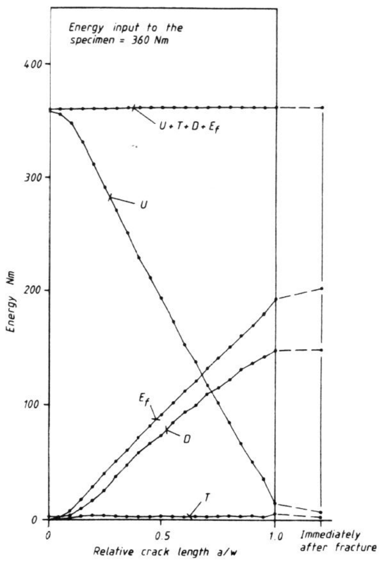

Fig. 4. Strain energy U, kinetic energy T, dissipated energy D, and accumulated energy flow to the crack-tip region E_f versus dimensionless crack length a/w as obtained from the elastic-plastic analysis.

exceeds the unique G_{pc}-curve. This is not surprising in view of the large nominal tensile stresses occurring in the specimen. The nominal stress in the crack plane was about 90 per cent of the yield strength of the material. A check of the geometry condition for dynamic LEFM suggested by Dahlberg, Nilsson, and Brickstad (1980) for tensile loaded sheets also shows that the condition is not fulfilled in this experiment. If we now turn our attention to the γ_f-values, it is seen in Fig. 6 that they have only about half the magnitude of the G_{pc}-values and are gathered around the G_{pc}-curve. A large part of the γ_f-values in Fig. 6 is, however, situated well above the G_{pc}-curve, and since the G_{pc}-curve is obtained from elastic analyses of experiments where exactly the same crack-tip model as adopted here was used, a geometry independent γ_f-value cannot be expected to be obtained with this size of the crack-tip region. With the crack-tip model used, the size of the crack-tip region is varying between five and ten per cent of the specimen width w. The size of the crack-tip region and accordingly the distance between the nodes in the crack plane must be decreased if a geometry independent γ_f-value is to be expected.

The shape of the plastic zones during crack growth are shown in Fig. 7. Three-point integration order in the Gauss' quadrature formula was used in these computations, and to each integration point is associated one ninth of the volume of the element. Due to the high nominal stress state in the specimen, plastic deformation occurs already at loading of the uncracked specimen. It is seen that the plastic zone is moderate for small crack lengths but increases to considerable size for larger crack lengths. Consequently, the values of G_{pc} obtained for small crack lengths, when the plastic zone size is moderate, also correspond to the circles in Fig. 6 which best agree with an extrapolation of the G_{pc}-curve. Due to elastic wave propagation effects and the discretus behaviour inherent in the crack growth simula-

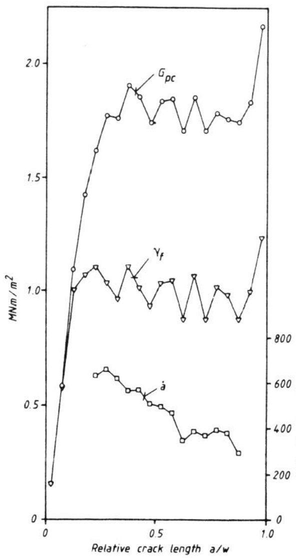

Fig. 5. Critical energy release rate G_{pc}, specific fracture energy γ_f, and crack-tip velocity $\dot{a}$ versus dimensionless crack length a/w.

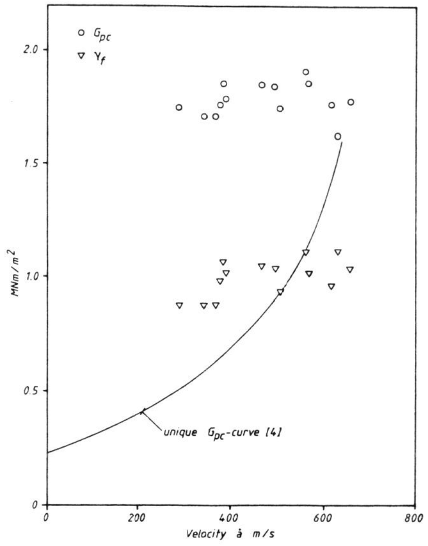

Fig. 6. Critical energy release rate G_{pc} and specific fracture energy γ_f versus crack-tip velocity $\dot{a}$.

tion technique, sometimes unloaded, previously yielded, material exists ahead of the crack tip.

CONCLUDING REMARK

A characteristic length measure, namely, the distance between the nodal points in the crack plane, has tacitly been introduced in this analysis. The necessity to introduce such a concept can be avoided, if the crack-tip region is instead modelled by for instance a singular crack-tip element. A suitable choice of the constitutive relation would be an elastic visco-plastic behaviour (Nilsson, 1980). The element will then also meet the requirement of elastic crack-tip behaviour, since such materials respond elastically at the unbounded strain rates which occur in the singular region. This is, however, a more elaborate matter and is left for future studies.

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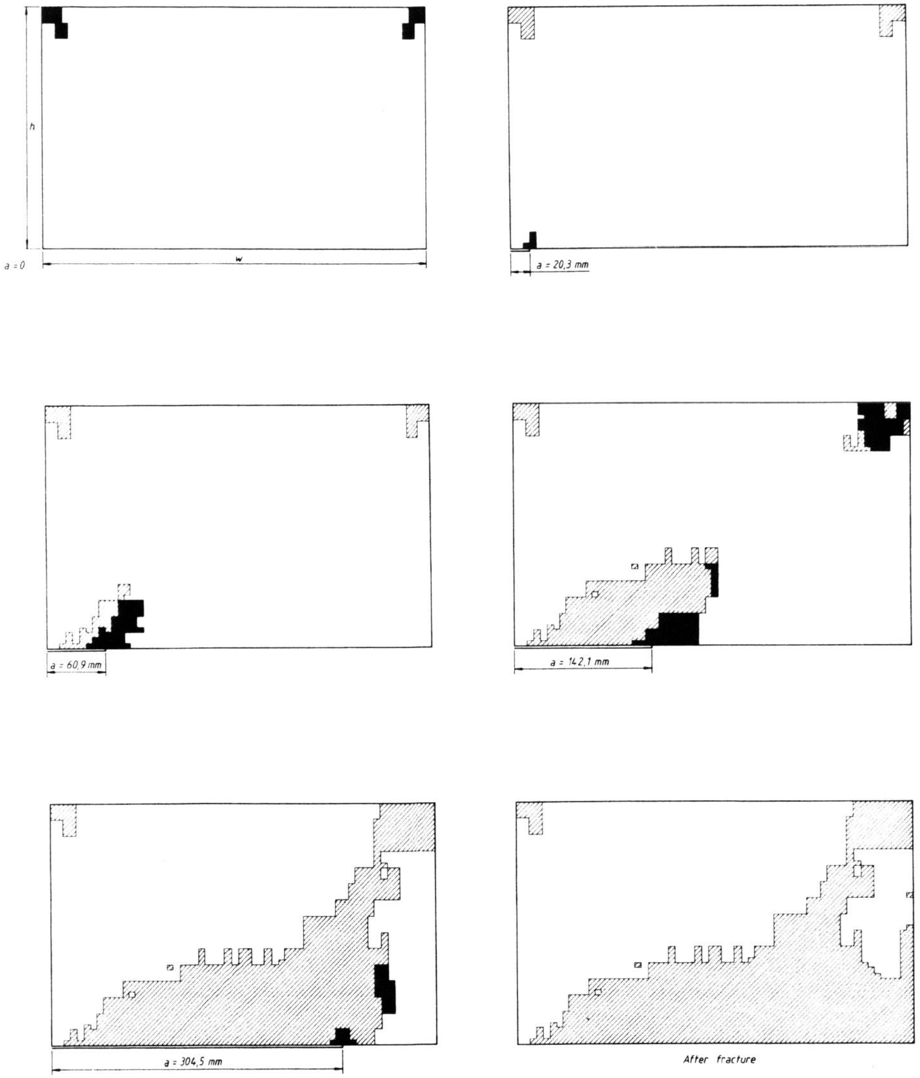

Fig. 7. Shape of the plastic zones at different crack lengths. Black area shows currently yielding material and dashed area unloaded, previously yielded, material.

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