MICROCRACKS, DEFORMATION AND PHYSICAL PROPERTIES OF PLAIN CONCRETE

K. Kato*

*Dept. of Civil Eng., The National Defense Academy, Yokosuka, Japan

ABSTRACT

This paper deals with the relationships between the microcracking and the deformation of the concrete subjected to the compressive loading. The meanings of the singular points of the proportional limit, the critical stress and the flow stress on the deformation curve are discussed. Especially, "the triple epsilon G method" for measuring the volumetric strain directly was newly developed. These singular points correspond well to the characteristics of the acoustic emission. A new stress-strain curve including only the physical constants of the concrete, and the generalized equations of the Poisson's ratio and the volumetric strain are derived.

KEYWORDS

Concrete, Microcracking, Shell crack, Volume change, Critical stress, Flow stress, Acoustic emission, Stress-strain curve, Resisting moment, Poisson's ratio.

INTRODUCTION

Almost all concrete structures in the civil and architectural engineering are always subjected to the sustained load in a broad sense, including not only external forces but also dead loads, and therefore the creep deformation occurs in them. Price (1951) showed that the long-term strength was only 70 per cent of the short-term strength. The cause of such a strength reduction is considered to be dependent on the extension of microcracks in the concrete. The specific strength, so-called, the creep limit that can permanently sustain the load is called "Permanent strength", "Critical strength", "True ultimate strength" and so on. The volumetric deformation of the concrete is valid as the quantitative index to show the inelastic behavior and the fracture, and depends on the cumulative effect of microcracks to be latent in the concrete and developed in it in accordance with loading. The structural defects in the concrete include the spherical air and water voids in the mortar matrix, the separations of the matrix from the aggregates, the shell cracks in air voids (Kato, 1968), the mortar cracks and the aggregate cracks. Generally, the transverse strain against the longitudinal strain is apt to receive the effects of the local variation of the concrete system, and therefore the deformation apparently shows the anisotropy. The author developed a new procedure to obtain directly the anisotropic volumetric strain, considering the strain component towards the third axial direction. The flow stress was defined for the first time, showing the hazardous stress level near the ultimate failure. The proportional limit, that is, the another true strength, can permanently withstand the repeated fatigue loading. This was determined by the modified Tuckerman's method of difference to avoid the personal error (Kato, 1971, 1972). These singular points of deformation were observed by the acoustic emission technique. A new stress-strain equation excepting the experimental assumptions and constants was derived, being constituted by the physical constants of the concrete only. Thus, the ultimate resisting moment of the singly reinforced concrete beam computed by the author's equation is compared with other. The relations of the stress-Poisson's ratio, and -volumetric strain are formularized.

NEW STRESS-STRAIN EQUATION AND ITS APPLICATION

It stands to reason that the latent structural defects and the internal microcracking according to loading may be evaluated by the logarithmic decrement. The author made it clear experimentally to be able to show the relationship between the logarithmic decrement δ and the longitudinal compressive strain ε_C of the concrete specimen subjected to the compressive loading generally as follows.

$$\delta = K_1 + K_2\varepsilon_C^2 \quad (1)$$

where K_1 and K_2 = constants.
And also, the relationship between the logarithmic decrement and the tangent modulus of elasticity E_t of the concrete can be expressed as follows.

$$E_t = a + b\,\delta \quad (2)$$

where a and b = constants. The stress-strain relation can be obtained as follows.

$$\sigma = \int_0^{\varepsilon} E_t\, d\varepsilon$$

$$= \frac{3}{2}\{\, 1 - \frac{1}{3}\,(\varepsilon_C/\varepsilon_{CB})^2\,\}\, E_S\varepsilon_C \quad (3a)$$

or

$$\alpha = \frac{3}{2}\,(\, 1 - \frac{\beta^2}{3}\,)\,\beta \quad (3b)$$

where σ and ε_C = any stress and strain, respectively, ε_{CB} = the strain at the ultimate strength σ_{CB}, approximately 2100×10^{-6}, E_S = the secant modulus of elasticity at the ultimate strength σ_{CB}, α = the stress level σ/σ_{CB}, and β = the strain level $\varepsilon_C/\varepsilon_{CB}$.
It is remarkably characteristic that Eq. (3a) or Eq. (3b) can be given by the non-destructive physical constants only because the secant modulus of elasticity E_S (kgf/cm^2) is given by the longitudinal ultrasonic pulse velocity $V_{\ell o}$ (m/sec) as follows.

$$E_S = 0.262V_{\ell o} - 932 \qquad (ton/cm^2) \quad (4)$$

Desayi and Krishnan (1964) proposed a relatively simple stress-strain equation including some assumptions. The shape of the descending branch in the case of the constant strain rate in comparison with the constant stress rate is very rapid (Swamy, 1970). The shapes of the Kato,s curve and the Desayi and Krishnan's one, as it were, belong to the cases of the constant strain rate and the constant stress rate, respectively. Therefore, the computation of the ultimate resisting moment of the concrete beam subjected to bending lies practically inside the safe zone to the general structure when the loading type is unexpected. Fig. 1(a) shows the generalized stress-strain diagram obtained by Eq. (3a) or Eq. (3b). Referring to Fig. 1(a), an example of the singly reinforced concrete section is computed by these equations in Table 1 on the basis of the data assumed in the same table. In the calculations of the area between the curve and the strain axis, and the moment of this area about the stress axis as shown in Table 1, Desayi and Krishnan's equation requires the logarithm and the arctangent. On the contrary, the author's equation only requires the polynomial and then is very easy to be computable.

NEW STRESS-POISSON'S RATIO EQUATION

The generalized stress-Poisson's ratio equation made from the point of the statistical view within the strength range of 150 to 600 kgf/cm^2 gained from the 113 specimens is given by Eqs (4a) and (4b) and shown in Fig. 1(b).

$$\nu = P_1(\alpha) = 0.155422 + 0.118320\alpha + 0.096744\alpha^2 + 0.086538\alpha^3 \qquad (0 \leqq \alpha \leqq 0.8) \quad (4a)$$

$$\nu = P_2(\alpha) = 0.20^{-10}\sum_{m=0}^{10} {}_{10}C_m\, \nu_m\,(\alpha - 0.8)^m(1.0 - \alpha)^{10-m} \qquad (0.8 \leqq \alpha \leqq 1.0) \quad (4b)$$

where ν = the Poisson's ratio at any stress level, α = the stress level, ν_m = the experimental values given in Table 2, m = 0, 1, 2, ·····, 10, and $P_1(\alpha)$ and $P_2(\alpha)$ = the functions of α.

NEW STRESS-VOLUMETRIC STRAIN EQUATION

The volumetric strain ε_V on the assumption of the isotrpy on the basis of the elastic theory is expressed by Eq. (5).

$$\varepsilon_V = \varepsilon_C - 2\varepsilon_T \quad (5)$$

where ε_C = the longitudinal compressive strain and ε_T = the transverse tensile strain.

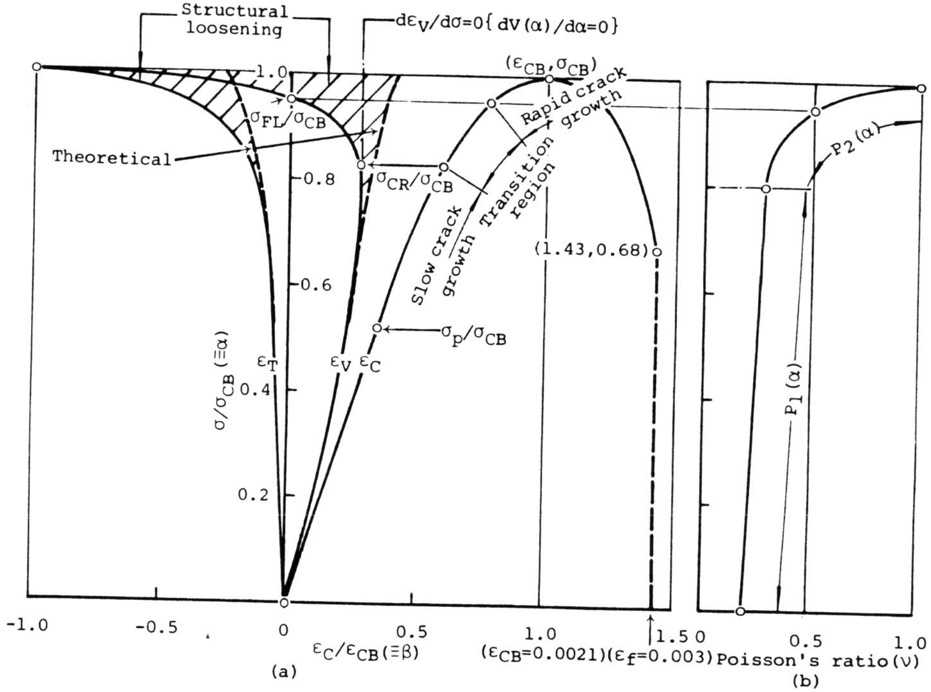

Fig. 1. Stress-strain and -Poisson's ratio curves

TABLE 1 Calculation of Singly Reinforced Concrete

	Equations	Desayi & Krishnan's curve*	Kato's curve
	Data	$\sigma_f = (7/8)\sigma_{CB}$ $\varepsilon_f = 0.003$ $\varepsilon_s = 0.00133$	$\varepsilon_{CB} = 0.0021$ $\varepsilon_f = 0.003$ $\varepsilon_s = 0.00133$
Results	ε_{CB}	0.00177	(0.0021)
	Area under stress-strain curve $= \int_0^{\varepsilon} \sigma \, d\varepsilon$	$0.800\sigma_{CB}\varepsilon_f$	$0.707\sigma_{CB}\varepsilon_f$
	Moment of the area $= \int_0^{\varepsilon} \sigma \varepsilon_c \, d\varepsilon$	$0.458\sigma_{CB}\varepsilon_f^2$	$0.423\sigma_{CB}\varepsilon_f^2$
	Height of neutral axis = x	0.692d	0.693d
	Total compression = C	$0.554\sigma_{CB}bd$	$0.490\sigma_{CB}bd$
	Lever arm = z	0.704d	0.721d
	Ultimate resisting moment = M_R	$0.392\sigma_{CB}bd^2$	$0.353\sigma_{CB}bd^2$
	Ultimate balanced steel ratio = p	$0.554\sigma_{CB} / \sigma_{sy}$	$0.490\sigma_{CB} / \sigma_{sy}$
	Notations**	d = Effective depth b = Width of section σ_{sy} = Yielding stress of steel ε_s = Steel strain at σ_{sy} ε_{CB} = Strain at the ultimate strength of concrete σ_f = Stress at the failure of concrete ε_f = Maximum strain at σ_f ε_c = Any strain at any stress σ E = Constant $\equiv 2\sigma_{CB} / \varepsilon_{CB}$ A_s = Area of reinforcement	

* $\sigma = E\varepsilon / \{ 1 + (\varepsilon_c/\varepsilon_{CB})^2 \}$ **

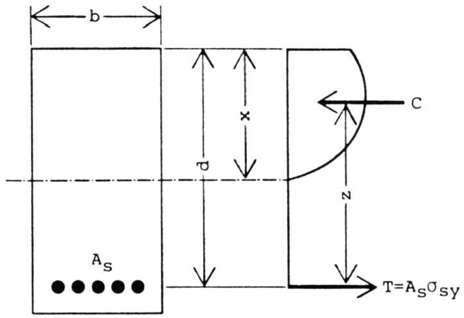

TABLE 2 Relationship Between m, α and ν_m

m	α	ν_m
0	0.80	0.232
1	0.82	0.236
2	0.84	0.243
3	0.86	0.254
4	0.88	0.271
5	0.90	0.300
6	0.92	0.342
7	0.94	0.430
8	0.96	0.615
9	0.98	0.825
10	1.00	1.000

The critical stress in which Eq. (5) satisfies Eq. (6) is only an approximate value of the concrete as the composite material.

$$d\varepsilon_V / d\sigma = 0 \tag{6}$$

Thus, the volumetric strain ε_V from the orthotropic viewpoint can be given by Eq. (7), called "the triple epsilon G method" being more practical than the procedure to use the dilatometer (Spooner, 1973).

$$\varepsilon_V = \varepsilon_C - \varepsilon_{T1} - \varepsilon_{T2} \equiv 3\varepsilon_G \tag{7}$$

where ε_C = the longitudinal compressive strain along the y-axis, ε_{T1} and ε_{T2} = the transverse tensile strains along the x-axis and the z-axis, respectively, and ε_G = the reading of the strain indicator in connecting the three sheets of strain gages in the series and assuming the whole as the one active gage of the Wheatstone bridge method.
This method can measure the average transverse strain and the separate strains, by using the strain selector (Kato, 1976).
The critical stress level is given as the function of the strength σ_{CB} (150 to 600 kgf/cm^2) as shown in Eq. (8)

$$\sigma_{CR} / \sigma_{CB} = 0.722 + 0.287 \times 10^{-3}\sigma_{CB} \tag{8}$$

And also, the critical volumetric strain $\varepsilon_{V,CR}$ at the critical stress is given by Eq. (9).

$$\varepsilon_{V,CR} = 453 + 0.67\sigma_{CB} \quad (\times 10^{-6}) \tag{9}$$

The average critical volumetric strain is approximately 700×10^{-6}. That the volumetric strain is zero, namely $\varepsilon_V = 0$ means that the material is theoretically incompressible, that is , fluid. This stress named the flow stress σ_{FL} is the very important singular point of deformation warning the approach of the decisive ultimate strength σ_{CB} and is given by Eq. (10).

$$\sigma_{FL} / \sigma_{CB} = 0.904 + 0.123 \times 10^{-3}\sigma_{CB} \tag{10}$$

The average flow stress level is approximately 0.95, and therefore the ultimate failure may be forecast at earlier as much as 5 per cent stress level than the ultimate strength.
The general volumetric strain curve observed from the point of the statistical view can be expressed from Eqs (3), (4) and (7), by using the average transverse strain in place of the two transverse strains in Eq. (7), as follows.

$$\varepsilon_V = \varepsilon_{CB} \, f(\alpha) \, \{ 1 - 2P(\alpha) \} = \varepsilon_{CB} V(\alpha) \tag{11}$$

where $V(\alpha) \equiv f(\alpha)\{1 - 2P(\alpha)\}$.
The author named $V(\alpha)$ "the unit volumetric strain function", and the curve of $V(\alpha)$ is shown in Fig. 1(a).
It is natural that the concrete itself does not expand above the critical stress level but its expansion results from the apparent volumetric change including the internal interstices accompanied by the occurence and the development of the microcracks, what is called, the structural loosening as shown in Fig. 1(a). And also, Fig. 1(a) shows that the slow crack growth, the crack growth of transition and the rapid crack growth can be defined by the stress ranges below the critical stress, between the critical stress and the flow stress and above the flow stress, respectively, by the acoustic emission observation (Kato, 1977).

CONCLUSION

The microcracks occur and develop in the concrete subjected to loading in addition to the latent cracks and it stands to reason that the inelasticity on the dynamical behavior results from those microcracks. The apparent upper limit of the linearity corresponds to the proportional limit, which is the true strength for the repeated fatigue. The critical stress to give the minimum volume on the volumetric strain curve corresponds to the true strength for the sustained load. The flow stress on this curve is the very valid singular point to forecast the ultimate failure. Especially, a new and handy measuring technique using the wire strain gages for the volumetric strain was developed. The characteristics of the RMS waves and the counter curve of the acoustic emission correspond well to these singular points. A new stress-strain curve including only the physical constants of the concrete derived from evaluating the quantity of the internal structural defects by the logarithmic decrement was proposed, and the ultimate resisting moment and so on of the singly reinforced concrete beam subjected to bending were calculated and compared with other. The author's equation consists of the physical properties of concrete itself, is easy to be computable, and keeps the safety against the unexpected loading. The curves of the Poisson's ratio and the volumetric strain were shown for the first time.

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