

INTERACTION OF DAMAGE MECHANISMS DURING HIGH TEMPERATURE  
FATIGUE

A. Plumtree\* and J. Lemaitre\*\*

\*Department of Mechanical Engineering, University of  
Waterloo, Waterloo, Ontario

\*\* LABORATOIRE DE MECANIQUE ET TECHNOLOGIE  
UNIVERSITE PARIS VI - ENSET. CACHAN (FRANCE)

ABSTRACT

Alloy 800 (Sandvik-Sanicro 31) has been subjected to fully reversed push-pull strain controlled tests at 600°C. High and low strain rates were imposed by controlling the test frequency. The conditions for these strain rates were chosen on the basis of fracture analysis. The failure at the low strain rate was entirely intercrystalline whereas that at the high strain rate was completely transcrystalline and striated. Tests were carried out by mixing sequences of cyclic creep (intercrystalline) and fatigue damage. On considering the accumulation of damage in a non-linear manner, it was found that this approach gave a unified explanation to the observed results. It is shown that interaction occurs between the cyclic creep and fatigue mechanisms after a certain amount of prior damage has been imposed.

KEYWORDS

Creep-fatigue interaction, high temperature fatigue, life prediction.

INTRODUCTION

One of several phenomenological models which may be used to predict failure of materials at high temperature is the application of the concept of non-linear damage accumulation as shown by Lemaitre and Chaboche (1974). Using this approach Plumtree (1977) and Lemaitre and Plumtree (1979) have recently modified existing constitutive equations for damage accumulation during creep-fatigue interaction and presented a unified approach for high temperature fatigue analysis.

Equations may be written in terms of stress for one-dimensional rupture problems. For a nominal stress of  $\sigma$ , the damage parameter,  $D$ , is zero for the material containing no cracks and unity when rupture takes place, also  $\sigma/(1-D)$  is an "effective" stress, taking into account the weakness of the material due to the presence of voids or micro-cracks. Hence it must be emphasized that this damage concept is concerned with the formation of new surfaces (either internal or external).

Kachanov (1958) derived a differential equation of evolution of intergranular creep cracking using thermodynamic principles. Later, Lemaitre and Chaboche (1974) modified it to take into account non-linear damage cumulation:

$$\frac{dD}{dt} = \left\{ \frac{|\sigma|}{A + [1-D]} \right\}^r [1-D]^{-q} \quad (1)$$

where  $t$  is the time and  $A$ ,  $r$ ,  $q$ , are material characteristics ( $A_+$  for tension,  $A_-$  for compression, if damage is anisotropic). These are obtained from the relation of viscoplastic strain behaviour coupled with damage. Using the boundary conditions such that damage is negligible during secondary creep and that damage is unity when  $t = t_c$ , then integration of Equation 1 for the evolution of creep damage at constant stress yields

$$D = 1 - \{1 - t/t_c\}^{\frac{1}{r+q+1}} \quad (2)$$

The derivation of this equation and those for the evolution of fatigue damage at constant stress are given by Lemaitre and Plumtree (1979).

Although the thermodynamic approach using damage as an internal variable directly leads to differential equations of evolution with stress as the controlling variable, in many practical applications, tests are carried out under strain control. Damage equations involving strain may be derived using constitutive equations relating stress with strain.

For a perfectly visco-plastic material which has attained saturation under cyclic strain conditions, the stress response is constant. It is then possible to integrate Equation 1 over one cycle in order to obtain the evolution of damage. As shown by Lemaitre and Plumtree (1979) the differential damage equation may be written

$$\frac{\delta D_c}{\delta N} = \frac{(1-D)^{-q'}}{(q+1)N_c} \quad (3)$$

where  $q'$  is a material constant and  $N_c$  is the number of cycles to failure.

Integration of Equation 3, using  $D(0) = 0$  for a constant strain range gives the damage evolution,  $D(N)$

$$D_c = 1 - \left\{1 - \frac{N}{N_c}\right\}^{q'+1} \quad (4)$$

Pure fatigue occurs at high frequencies resulting in time (and frequency) independent plasticity. Under these conditions the damage accumulation per cycle may be expressed (Lemaitre and Chaboche 1974) as:

$$\frac{\delta D_F}{\delta N} = \left[\frac{\Delta\sigma}{B(1-D)}\right]^\beta [1-D]^{-P} \quad (5)$$

For constant strain:

$$\frac{\delta D_F}{\delta N} = \frac{(1-D)^{-P}}{(p+1)N_F} \quad (6)$$

where  $\Delta\sigma$  is the stress range and  $B$ ,  $\beta$  and  $p$  are material constants. Integration of Equation 6 gives the evolution of damage at constant strain amplitude:

$$D_F = 1 - \left\{1 - \frac{N}{N_F}\right\}^{1/(p+1)} \quad (7)$$

Interaction of time dependent and time independent cyclic plasticity requires that Equations 3 and 6 be integrated to determine the history under cyclic strain conditions and to predict when failure will occur. If interaction of these two damage mechanisms occurs only after a certain incubation period ( $N^*$ ) then for a given number of high frequency,  $n_F$ , (time dependent plasticity) or low frequency cycles,  $n_c$ , (time dependent plasticity) which is less than  $N^*$  then the damage in each case is zero. However, when the number of high frequency or low frequency cycles exceeds  $N^*$  (i.e.  $n_c$  or  $n_F > N^*$ ) the two mechanisms can interact either simultaneously or sequentially.

In the first case, the damage mechanisms interact within the same low frequency cycle which may include a hold period. The total damage accumulation during one cycle is:

$$\frac{\delta D}{\delta N} = \frac{\delta D_c}{\delta N} + \frac{\delta D_F}{\delta N} \quad \text{and} \quad D = \int_0^N \frac{\delta D}{\delta N} \delta N \quad (8)$$

which has been considered by Lemaitre and Chaboche (1974).

In the second case where the two damage mechanisms interact sequentially, for  $n_c$  cycles of pure cyclic creep followed by  $n_F$  cycles of pure fatigue then,

$$\frac{\delta D}{\delta N} = \frac{\delta D_c}{\delta N} \left( \text{or } \frac{\delta D_F}{\delta N} \right), \quad D = \int_0^{n_c} \frac{\delta D_c}{\delta N} \delta N + \int_{n_c}^{n_c+n_F} \frac{\delta D_F}{\delta N} \delta N \quad (9)$$

The amount of damage required for this type of interaction will be considered in the present paper.

#### APPLICATION

Smooth fatigue specimens were machined from bars of Sanicro 31 (trade name of Sandvik AB), similar to Alloy 800. These specimens which were 5 mm in diameter had a gauge length of 7.5 mm. Prior to machining, the bars had been solution heat treated at 1120°C for 15 min and then water quenched.

Strain controlled fully reversed cyclic tests were carried out using a servo-controlled electro-hydraulic closed loop testing system. The temperature of testing, 600°C, was achieved using a resistance furnace. Details of the test procedure have been published elsewhere (Abdel-Raouf, Plumtree and Topper, 1973).

After failure, the surface damage and fracture surfaces were examined using the scanning electron microscope. Accompanying metallographic examination was carried out using optical microscopical techniques.

The conditions for the reference tests were selected on the basis of the results of cyclic tests carried out at various frequencies and after examining the fracture surfaces. The high frequency reference tests were chosen as those when the life was independent of frequency (as well as temperature) and for the low frequency reference tests, the life was time (and temperature) dependent.

The reference function  $N_F(\Delta\epsilon)$  for high strain rate cycling ( $\dot{\epsilon} = 2.64 \times 10^{-1} \text{ sec}^{-1}$ ) was established for frequencies in the range of 5 to 10 Hz. As expected, the life decreased with increase in total strain range. For a total strain range of 2.4% the fatigue life was 700 cycles (this was also the life for a frequency of 10 Hz at the same strain range). For a total strain range of 2.64%,  $N_F$  was 550. Under these high frequency strain cycle conditions, the fracture surface was completely striated. A record of the stress response throughout this test allowed the damage to be assessed for any particular number of cycles.

To obtain  $p$  (in Equation 7), the evolution of damage is required. Among different methods (Lemaitre and Duffailly, 1977), the simplest is to record the change of stress amplitude during the 5 or 10 different tests and to derive the damage evolution from the cyclic stress-strain equation including a damage term, i.e.

$$\Delta\sigma^* = (1-D_F) L \Delta\epsilon_p^{-M} \quad (10)$$

where L and M are material characteristics and  $\Delta\epsilon_p$  is the plastic strain range.

If  $\Delta\sigma^*$  is the value of  $\Delta\sigma$  at steady state or stabilized conditions when the damage is negligible then,

$$\Delta\sigma^* = L \Delta\epsilon_p^{-M} \quad (11)$$

Under constant strain conditions then,

$$D_F = 1 - \frac{\Delta\sigma}{\Delta\sigma^*} \quad (12)$$

Using Equation 7, the log-log plot of  $(1-D_F) [=(\Delta\sigma/\Delta\sigma^*)]$  from one test versus  $(1-N/N_F)$  gives the slope,  $1/p+1$ , from which the value of p was determined (=27).

The reference function  $N_C(\Delta\epsilon)$  for a low strain rate ( $\dot{\epsilon} = 1.32 \times 10^{-4} \text{ sec}^{-1}$ ) was established for frequencies in the range of 0.0025 to 0.0038 Hz. For the total strain range of 2.64%, the fatigue life  $N_C$  was 198 cycles. For all these low frequency tests, the fracture was intercrystalline. In a similar manner to the high frequency tests, the stress response for the test carried out at this total strain range of 2.64% was used to determine the damage  $D_C$  and then damage evolution according to Equation 4. The exponent,  $q'$ , was determined to be 15.

To study the effect of prior low frequency cycling on the high frequency life, block tests were performed. This involved strain cycling the specimen at  $\nu = 0.0025 \text{ Hz}$  ( $\Delta\epsilon = 2.64\%$ ) for a predetermined number of cycles then changing the frequency to  $\nu = 5 \text{ Hz}$  ( $\Delta\epsilon = 2.64\%$ ) and continuing the test to failure. The results are given in Table 1.

TABLE 1 Low (L.F.C.) and High Frequency Cycling (H.F.C.) Block Test Results

Test No.	Initial L.F.C.		H.F.C. to Failure		
	Cycles	$N/N_C$	Cycles	Obs. $N/N_F$	Predicted $N/N_F$
T1-B	100	0.51	216	0.39	0.29
T2-B	140	0.71	140	0.25	0.11
T3-B	50	0.25	365	0.66	0.60

Sequential mixing tests were carried out using a total strain range of 2.64% in order to apply the damage accumulation model to strain controlled situations and to assess its ability to predict failure in conditions where large amounts of different damage processes (i.e. high degree of mixing) were imposed. A block consisting of a given number of low frequency ( $\nu = 0.0025 \text{ Hz}$ ) strain cycles, followed by a given number of high frequency ( $\nu = 5 \text{ Hz}$ ) cycles was repeated until failure occurred and the number of sequences of fatigue and creep-type damage process was recorded. For instance, in Test No. T16-M, 2 cycles at low frequency were followed by 4 cycles at high frequency and this sequence was repeated 52 times until failure occurred halfway through the 53rd low frequency sequence. Under these conditions, the number of sequences was regarded as 104.5 (i.e. 52.5 low frequency and 52 high frequency). In all, a total of 6 such sequential mixing tests were carried out and the results are given in Table II.

TABLE II Sequential Low (L.F.C.) and High (H.F.C.) Frequency Cycling Mixing Test Results

Test No.	Cycles at		Observed No. of LFC + HFC Sequences to Failure	Predicted No. of Sequences to Failure
	LFC $n_C$	HFC $n_F$		
T2-M	100	216	2.3	1.7
T3-M	50	108	3.9	3.8
T4-M	25	54	8.7	7.9
T5-M	10	21	22.0	21.1
T6-M	5	11	35.6	41.2
T16-M	2	4	104.5	107.1

### PREDICTION

Life prediction is based upon the damage relations given in Equation 3 and 6 for cyclic creep and fatigue respectively, and on Equation 9 for sequential damage interaction. Consider  $n_C$  cycles of cyclic creep (= 2 for Test T16-M) in the first sequence of strain controlled cycles followed by  $n_F$  cycles of fatigue (= 4 for Test T16-M) in the next sequence. The whole process is then repeated. At the beginning of the  $i$ th sequence a certain amount of damage  $D_i$ , will have been inflicted and on completion of  $n_C$  cycles the new value of damage will be  $D_{i+1}$ . This may be determined by integration over the  $n_C$  cycles.

$$\int_{D_i}^{D_{i+1}} [1-D]^{q'} \delta D = \int_0^{n_C} \frac{dN}{[q'+1]N_C}$$

Hence

$$D_{i+1} = 1 - \{[1-D_i]^{q'+1} - \frac{n_C}{N_C} \frac{1}{q'+1}\} \quad (13)$$

This equation can be easily solved with the initial condition  $D_0 = 0$  in order to find the number of blocks  $i_R$  and the number of cycles  $n$  in the last block during which failure occurs:

$$i_R = \text{first } i, \text{ given } D > 1$$

$$D = 1 \rightarrow n = N_{(i_R-1)} [1 - D_{(i_R-1)}]^{p \text{ or } q'+1} \quad (14)$$

The predicted numbers of combined sequences are included in Table II and it is interesting to note that these predictions are very similar to the observed results.

For the block tests, the number of fatigue cycles to failure after a given cycle fraction at low frequency may be calculated by integration of the fatigue damage equation with  $D_C$  as an initial value of D:

$$\int_{D_C}^1 [1-D]^p \delta D = \int_0^N \frac{\delta N}{[p+1]N_F} \quad (15)$$

Since  $D_C$  is given in Equation 4, then

$$\frac{N}{N_F} = 1 - \{1 - \frac{N}{N_C}\}^{p+1} \quad (16)$$

These values are given in Table I and it will be seen that the predicted and observed results are again very similar.

#### DISCUSSION AND CONCLUSIONS

The results of the mixing tests indicate that interaction of the fatigue with the creep-type damage mechanism occurred after only two low frequency cycles had been imposed. The total number of the high frequency cycles in test T16-M (Table II) corresponded to 208 cycles (52 high frequency sequences consisting of 4 cycles in each) whereas failure occurred after 550 cycles when no mixing took place. Hence the two low frequency cycles were damaging to the subsequent high frequency cycles. This shows that  $N^* < 2/198 N_c$ ,  $\approx 0.01 N_c$ , giving a critical amount of damage,  $D_c^* \rightarrow F \sim 0.001$ .

Tests carried out by Plumtree and Douglas (1981) on Alloy 800 at 600°C which consisted of initial high frequency cycles followed by sequential mixing low and high frequency cycles to failure, showed that only after an initial high frequency sequence of half life ( $N > 0.5 N_F$ ) interaction with the following low frequency sequence occurred. This corresponded to a critical damage of  $D_F^* \rightarrow c \sim 0.03$ . Hence this sequence is not as deleterious as when creep-type damage is imposed first.

It has been shown that the non-linear damage accumulation model may be used with good accuracy to predict failure at 600°C of Alloy 800 which experiences interacting cyclic creep-fatigue damage.

#### ACKNOWLEDGEMENTS

This work was financed through a grant (A-2770) from the Natural Sciences and Engineering Research Council of Canada. The authors would like to express their sincere thanks to Mr. M.J. Douglas, Westinghouse Canada Ltd., for assistance with the experimental work and to Mr. Nils - Goran Persson, Sandvik AB, Sweden for providing the Sanicro 31 and for many valuable discussions. Thanks are also due to Miss Carmeline Fata for typing the manuscript.

#### REFERENCES

- Abdel-Raouf, H.A., A. Plumtree, and T.H. Topper (1973). Effects of temperature and deformation rate on cyclic strength and fracture of low-carbon steel. In Cyclic Stress-Strain Behaviour Analysis, Experimentation, and Failure Prediction, STP 519, ASTM, Philadelphia, pp. 28-57.
- Kachanov, L.M. (1958). On the time to failure under creep conditions. Izvest. Akad. Nauk. SSSR, OTN, 8, pp. 26-31.
- Lemaitre, J., and J-L. Chaboche (1974). A non-linear model of creep-fatigue damage accumulation and interaction. Proc. IUTAM Symposium Mech. of Viscoelastic Media and Bodies, Gothenburg. Office National D'Etudes et de Recherches Aérospatiales, Chatillon TT No. 1394.
- Lemaitre, J., and J. Duffailly (1977). Models and identification of plastic damage in metals. Proc. 3rd French Congress of Mechanics, Grenoble.
- Lemaitre, J., and A. Plumtree (1979). Application of damage concepts to predict creep-fatigue failures. Trans. ASME, J. Engineering Mat. and Technology, 101, 284-292.
- Plumtree, A. (1977). Creep/fatigue interaction in Type 304 stainless steel at elevated temperature. Met. Sci., 11, 425-431.
- Plumtree, A., and M.J. Douglas (1981). Initiation and growth of fatigue cracks at high temperature. In Proc. ICF5, The 5th Int. Conf. on Fracture, Cannes, France, Paper No. 151.