FRACTURE MECHANISM AND STRENGTH OF CONCRETE UNDER TRIAXIAL COMPRESSION

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ABSTRACT

It is shown that methods of fracture mechanics can be used to describe the behaviour of aggregative materials such as concrete. A model of concrete consisting of polygonial aggregate inclusions randomly distributed in a homogeneous matrix is introduced. This model takes into account pre-existing randomly inclined interface cracks between inclusions and matrix caused by shrinkage of hardened cement paste. It is shown that interface cracks start to propagate under triaxial external load (axial pressure $\sigma_3 < 0$ and confining pressure $\sigma_1 = \sigma_2 < 0$) according to Mode II irrespectively of the value of confining pressure. All the stages of crack propagation until the failure of a sample of concrete are investigated by means of a computer program (Monte Carlo method). A comparison with existing experimental results is given. On this basis some design criteria for building codes and design practice (including design of prestressed concrete reactor vessels) are formulated.

KEYWORDS

Fracture mechanics; concrete; triaxial compression; Monte Carlo method.

INTRODUCTION

Strength of concrete subjected to triaxial compression (axial compression σ_3 with confining lateral pressure $\sigma_1 = \sigma_2$ ($\sigma_{1,2,3} < 0$) can be described with help of coefficient of efficiency of confining lateral pressure:

$$K = \frac{|\sigma_3^*| - R_{cyl}}{|\sigma_2^*|} ,$$

where R_{cyl} is the uniaxial strength of a companion cylindrical specimen, and σ_2^* (σ_3^*) are the values of σ_2 (σ_3) by the failure of a specimen subjected to triaxial compression. Experimental values of K usually have a significant scatter, especially by low confined pressure (from K = 4 - 5 (Gvozdev, 1949) until 24.7 - 28.0 (Berg and Solomenzev, 1969), and the theoretical approach to the problem of triaxial strength of concrete is extremely desired.

This study is based on methods of fracture mechanics. It was shown by Zaitsev and Wittmann (1977) that these methods can be used to describe the behaviour of aggregative materials such as concrete. A model of concrete used in this study consist of polygonial aggregate inclusions randomly distributed in a homogeneous matrix (mortar). This model takes into account pre-existing randomly inclined interface cracks between inclusions and matrix caused by shrinkage of hardened cement paste (fig. 1,a).

Behaviour of a specimen having a shape of a cylinder and loaded with axial pressure σ_3 and confining pressure $\sigma_1 = \sigma_2$ will be analyzed. This problem can be approximately reduced to a plane problem when the behavior of a plate of thickness "1", cut out from the cylinder and loaded with axial stress $\sigma_y = q$ and confined pressure $\sigma_x = \eta q$ (η = const) will be analysed.

PROPAGATION OF INITIAL BOND CRACKS

It can be shown with help of methods of fracture mechanics (Panasjuk, 1968), that initial bond cracks between matrix and inclusions will propagate according to Mode II. An analogous result has been obtained with help of finite elements method by Palaniswamy and Shah (1975). The value of external load required for propagation of an initial crack of the length of $2l_1$ inclined by the angle of α to the y-axis is equal to

$$q^B_{II} = -\frac{K^B_{IIC}}{\sqrt{\pi l_1}\, D(\alpha,\rho,\eta)}$$

where K^B_{IIC} is critical value of stress intensity factor for bond cracks (Mode II), ρ - coefficient of friction between matrix- and aggregate materials

$$D(\alpha,\rho,\eta) = -k_\tau\,(1-\eta)\sin\alpha\cos\alpha - \rho\,(k'_\sigma\,\sin^2\alpha - \eta\,k''_\sigma\,\sin\alpha\cos\alpha)$$

k_τ, k'_σ, k''_σ - stress concentration factors on the inclusion boundary for respectively shear stress, normal stress from axial pressure, normal stress from confining pressure. By typical for concrete values of ρ (ρ = 0.7 - 0.9) the propagation of initial cracks is most likely on the contact surfaces inclined by the angle of $\alpha \cong 25°$ to the sample axis, independent from the confined pressure. At the same time, the value of q^*_{II} depends significantly on the value of confined pressure.

FORMATION OF CRACKS IN MATRIX

The next stage of crack propagation is connected with crack penetration into matrix. Cracks start to propagate approximately parallel to y-axis as cracks of Mode I (splitting cracks) by the value of axial pressure

$$q^M_I = -\frac{K^M_{IC}\sqrt{3/2}}{\sqrt{\pi L_1}\, D(\alpha,\rho,\eta)}$$

whereby K^M_{IC} is critical value of stress intensity factor for cracks in matrix (Mode I), $2L_1$ - length of the side of an inclusion, where the initial crack is situated. Predominant propagation of longitudinal (splitting) cracks on initial stages of loading of concrete sample is verified experimentally by Berg and Solomenzev (1969). Further stages of crack propagation can be described as follows

$$q = -\frac{K^M_{IC}\sqrt{\pi L_1}}{\frac{2}{\pi}\sqrt{\frac{L_1}{l_2}}\,A(\alpha,\rho,\eta) - \eta\sqrt{\frac{l_2}{L_1}}}$$

where l_2 is the length of the longitudinal part of the crack,

$$A(\alpha,\rho,\eta) = D(\alpha,\rho,\eta)\cdot\sin\alpha$$

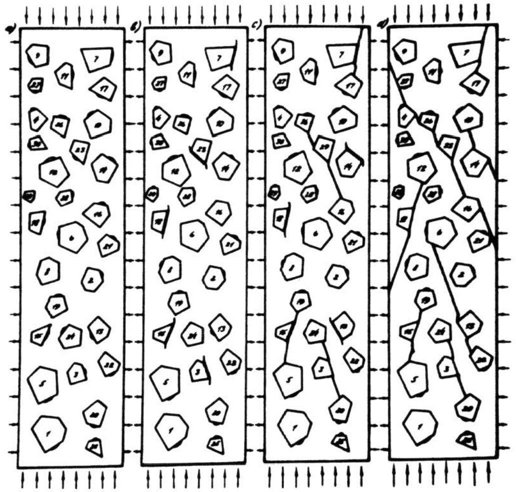

Fig. 1. One of realizations of Monte-Carlo method showing consequent stages of crack propagation in concrete.

Fig. 2 shows the relationship between related axial pressure q°_* and λ, where 2λ is the length of a crack ($2l_1$ or $2L_1 + 2l_2$). It was assumed that $\alpha = 30°$; $K^B_{IIC}/K^M_{IC} = 0.675$; $\eta = 0 - 0.2$. Line 1 gives values $q^\circ_* = |q^B_{II}|\sqrt{\pi L_1}/K^M_{IC}$, corresponding the propagation of initial bond cracks. For the case $\eta = 0.2$ line 2 is also shown. It gives values of $q^\circ_* = |q^M_{qI}|\sqrt{\pi L_1}/K^M_{IC}$, corresponding to penetration of an inclined crack into matrix. Initial bond cracks by load increase at first does not grow (vertical lines by $\lambda = \lambda_1$ on fig. 2). By load q_1 crack grows in unstable manner and reaches the length of $\lambda = 1$ (i.e. crack occupates the whole side of contact between matrix and inclusion).

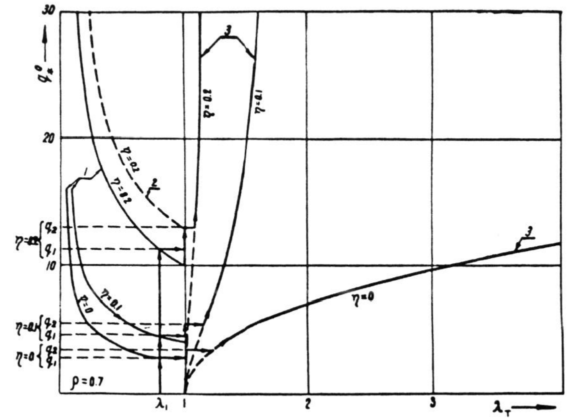

Fig. 2. Relationship between related axial stress q^{o}_{*} related crack length λ.

By the load value of q_2 crack "jumps" on one of lines 3 corresponding stable crack propagation in matrix. As can be seen from Fig. 2, confining pressure (especially by $\eta = 0.2$) leads to a significant increase of q_1 - and q_2 - values, i.e. stresses, required for initial crack propagation and penetration of crack into matrix. Still more is the influence of confined pressure on the propagation of longitudinal cracks in matrix: by $\eta = 0.1$ length of these cracks (line 3) is significantly less as compared to uniaxial tension ($\eta = 0$), and for the case $\eta = 0.2$ cracks formed in matrix, practically do not grow. Thus, the only way of fracture of concrete with the system of cracks described above is the propagation of shear cracks (Mode II). This stage of crack propagation can be described in an analogous way. The interaction of cracks can be approximatively estimated according to Panasjuk (1976).

SIMULATION OF CRACK PROPAGATION (MONTE CARLO METHOD)

Fig. 1(b-d) gives consequent stages of crack propagation (including fracture of a sample) as described above for one of realisations of Monte-Carlo method.

Fig. 3 (line "T", i.e. "Theory") the relationship between K-coefficient of efficiency of lateral pressure according Eq.(1) and related confined pressure $|\sigma_2|/R_{cyl}$ is given R_{cyl} is uniaxial strength of concrete.

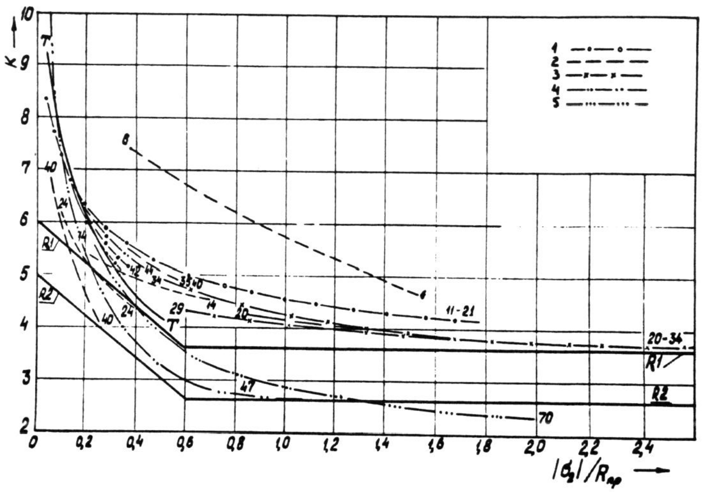

Fig. 3. Relationship between related value of confined pressure σ_2/R_{cyl} and coefficient of efficiency of confined pressure K. Lines 1-5 give experimental data according to Malashkin and oth. (1975), Malashkin and Tjablikov (1976), Krahl and oth. (1965), Berg and Colomenzev (1969), Bergues, Habib and Morlier (1971).

As can be seen from fig. 3, the value of K increases significantly by low values of lateral pressure. It can be explained by significant confined influence of lateral pressure on propagation of longitudinal splitting cracks.

By increase of lateral pressure the importance of splitting cracks (Mode I) in general picture of fracture will decrease, and final fracture will be caused by shear cracks (Mode II), for which the lateral pressure has lower confining effect.

Fig. 3. gives also experimental values of K, obtained in various investigations; figures near lines give uniaxial strength of concrete R_{cyl} in MPa. Experimental data confirm significant increase of K by low $|\sigma_2|/R_{cyl}$ values, found with help of method described above. To the other hand, these experimental data indicate on the influence of uniaxial strength R_{cyl} on K-values, which was also predicted by Monte-Carlo method.

Both features of behaviour of concrete subjected to triaxial compression can be taken into account by preparation of new Building codes. Fig. 3 shows proposed tentative recommendations (line R1) for low strength concretes, $R_{cyl} \leq 40$ MPa and line R2 for high strength concretes, $R_{cyl} > 40$ MPa). These recommendations can be used by design of various building structures, first of all by design of prestressed concrete reactor vessels.

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