FORECASTING THE FATIGUE LIFE OF WELDED JOINTS UNDER NARROW BAND RANDOM LOADING

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ABSTRACT

Using a fracture mechanics extension of Miner's rule, fatigue lives were forecast for three sets of narrow band random loading tests carried out on weldments. None of the forecasts were entirely satisfactory, although once the test results were known it was always possible to refine the calculations to give an improved prediction. It was concluded that, for narrow band random loading of welded joints at long endurances, some service simulation testing to representative endurances is necessary for accurate fatigue life determination.

KEYWORDS

Welded joints; fatigue; random loading; and life prediction.

NOTATION

a	Crack length
a_f	Final crack length
a_o	Initial crack length
b	Exponent in stress intensity factor expression
C, m	Constants in equation (1)
C_1	Constant in equation (4)
K_I	Opening mode stress intensity factor
ΔK_I	Range of K_I in fatigue cycle
ΔK_{Ip}	Prior value of ΔK_I
ΔK_{th}	Threshold value of ΔK_I

$\Delta K_{th,o}$	Value of ΔK_{th} without prior loading
N	Number of cycles
N_i	Number of cycles for failure at the i-th load level
n_i	Number of cycles at the i-th load level
$p(S/\sigma)$	Probability density of a peak of S/σ
R	Relative damage
$r(S/\sigma)$	Damage density of a peak of S/σ
S	Stress
S_a	Alternating stress
S_m	Mean stress
S_T/σ	Clipping ratio
S_{th}/σ	Threshold value of S/σ
ΔS	Range of stress in fatigue cycle
W	Thickness
α	Geometric correction factor
α_o	Reference value of α
γ	Exponent in equation (9)
σ	Root mean square value of S

INTRODUCTION

Ever since Miner's rule was first proposed, the problem of 'cumulative damage' under variable amplitude loading has been extensively discussed in the literature; see for example a recent review by Schütz (1979).

Although the basic mechanisms of fatigue failure are relatively well understood and documented (Frost, Marsh and Pook, 1974), the prediction of the fatigue life of a structure in service is difficult because of the amount of detailed information required. The interaction effects which occur between different load levels under variable amplitude loading are a particular problem. The difficulties account for the extensive use of service simulation testing (Marsh, 1979), which in principle can provide fatigue life information. Nevertheless theoretical methods are needed to extrapolate test data to broadly similar situations. In particular there is a need to predict fatigue behaviour under variable amplitude loading at long ($\sim 10^8$ cycles) lifetime.

In a welded joint virtually the whole life is occupied by crack propagation from pre-existing flaws so a fracture mechanics approach is appropriate. Back calculation from constant amplitude data leads to a fracture mechanics extension of Miner's rule. With interaction effects ignored, this appears to work quite well on

data taken from the literature for mild steel welds tested under narrow band random (NBR) loading (Pook, 1974). It is obviously desirable to check prediction methods by making forecasts of the results of laboratory tests before the results of the tests are known to the forecaster. Forecasts were therefore made for three sets of NBR tests on unstress-relieved weldments carried out in air at NEL. A particular aim was to see what could be learnt without the detailed information which is necessary for a classical fracture mechanics analysis (Tiffany and Masters, 1965).

FRACTURE MECHANICS EXTENSION TO MINER'S RULE

The method of extending Miner's rule is outlined below, full details are given by Pook (1974, 1977). Behaviour of the material was assumed to be linear elastic, a reasonable assumption for many fatigue problems (Frost, Marsh and Pook, 1974).

Constant Amplitude Loading

Fatigue crack growth data can for many purposes be conveniently represented by the Paris equation

$$\frac{da}{dN} = C(\Delta K_I)^m \qquad (1)$$

where a is the crack length, N the number of cycles, C a material constant, ΔK_I the range of opening mode stress intensity factor K_I during the fatigue cycle, but neglecting any compressive stress, and m an exponent, which for structural steels can usually be taken as 3.

The stress intensity factor K_I is given by

$$K_I = S(\pi a)^{\frac{1}{2}}\alpha \qquad (2)$$

where S is the applied stress, and α a geometric correction factor of the order of one.

If ΔK_I is required, S is replaced by ΔS, the stress range during a fatigue cycle but neglecting any compressive stresses, which to a first approximation simply close the crack. Crack growth does not take place unless ΔK_I exceeds a threshold value ΔK_{th}. Unlike crack growth rates, which for steels are largely independent of mean stress, ΔK_{th} is roughly proportional to $(S_a/S_m)^{1/3}$, where S_m is mean stress and S_a alternating stress.

If α is assumed constant, then the fatigue crack growth life N is given by

$$N = \frac{a_o^{1-\frac{1}{2}m} - a_f^{1-\frac{1}{2}m}}{C\Delta S^m \pi^{\frac{1}{2}m}(\frac{1}{2}m - 1)\alpha^m} \qquad (3)$$

where a_o is the initial crack length, and a_f the final crack length at which the specimen fails by brittle fracture or tensile overload.

Usually a_f is large compared with a_o and therefore has little effect on N. If a_f is neglected or assumed proportional to a_o, equation (3) can be written

$$N = \frac{C_1}{a_o^{(\frac{1}{2}m-1)}\Delta S^m}. \qquad (4)$$

The well-known result that the negative inverse slope of the S/N curve for a cracked specimen, plotted on logarithmic coordinates, equals the exponent m in the crack growth equation follows from equation (4).

Various other relationships are easily derived, but are only approximations where residual stresses are present, as they will be in unstress-relieved welds. Indeed Gurney and Maddox (1972) have argued that in such joints the effective stress cycle is from yield stress downwards, irrespective of the applied mean stress.

Variable Amplitude Loading

Under NBR loading (Pook, 1976), individual sinusoidal cycles whose frequency corresponds to the resonant frequency of the system may be distinguished but they have a slowly varying random amplitude. The probability density function for the occurrence of a positive-going peak approaches the Rayleigh distribution

$$p\left(\frac{S}{\sigma}\right) = \frac{S}{\sigma}\exp\left(\frac{-S^2}{2\sigma^2}\right) \tag{5}$$

where $p(S/\sigma)$ is the probability density of a peak of S/σ, and σ is the root mean square (r.m.s) stress amplitude of the loading.

The process is symmetrical about its mean value from which S is measured, and corresponding negative-going peaks occur. The Rayleigh distribution has the form shown in Fig. 1 and theoretically extends to infinite stress. However, for various practical reasons, peaks do not exceed a cut-off value of S/σ known as the clipping ratio (S_T/σ), which does not usually exceed 5.

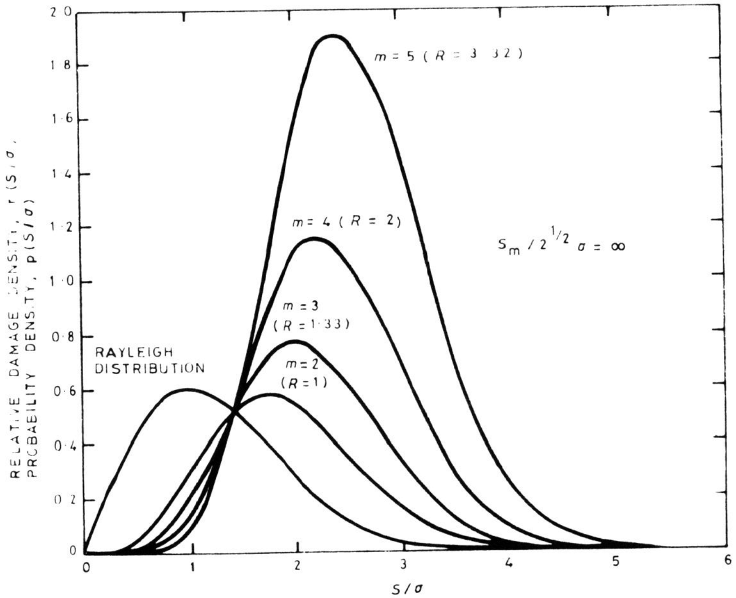

Fig. 1. Probability Density Function for the Rayleigh Distribution and Relative Damage Density Function for Various Values of m

The amount of crack growth due to variable amplitude loading can be estimated if it is assumed that each cycle causes the same amount of growth as if it were applied as part of a sequence of loads of constant amplitude. This approach ignores the interaction effects. In NBR loading, a cycle does not differ greatly from its predecessor; this reduces interaction effects, which in any case are relatively unimportant in low strength steels.

For calculations it is convenient to define a relative damage density function, $r(S/\sigma)$, which gives the probability density of relative crack growth due to a peak of S/σ compared to that for constant amplitude loading of the same r.m.s. stress amplitude. For NBR loading and the crack growth law given by equation (1).

$$r\left(\frac{S}{\sigma}\right) = \left(\frac{S}{2^{\frac{1}{2}}\sigma}\right)^m \frac{S}{\sigma}\exp\left(\frac{-S^2}{2\sigma^2}\right). \tag{6}$$

Damage density curves for various values of m are shown in Fig. 1. The area under the damage density curve gives the relative damage, R, which is the ratio of the amount of crack growth caused by a variable amplitude loading to that caused by a constant amplitude loading of the same r.m.s. stress amplitude. Some values of R are given in Fig. 1.

According to circumstances, various corrections may be needed. At the upper end the damage density curve is limited by the clipping ratio. When negative-going peaks fall below zero, the damage density is reduced. A fatigue crack of a given length will not grow unless the applied stress exceeds a critical value. Peaks of less than a threshold value of S/σ, S_{th}/σ, are therefore non-damaging, and this effectively truncates the damage density curve at its lower end, thereby reducing R. As a crack grows S_{th}/σ falls and R increases, and this must be taken into account in the estimation of total life.

Because R is a function of crack length, numerical methods are needed to calculate the fatigue crack growth life (Pook, 1974). In outline, arbitrary values assigned to a_o and a_f are used in conjunction with the constant amplitude results to calculate fictitious values for crack growth constants. These fictitious values are then used in the calculation of the NBR loading life. This implicitly assumes that the initial flaw size and the shape and pattern of crack growth are the same in all specimens.

The results obtained depend on the ratio assigned to a_f/a_o and also on the assumption that each specimen contains an identical flaw which leads to failure, but detailed information on the size, shape and location of the flaw is not required. The value of a_f/a_o has the greatest effect in the knee region, which is much less pronounced than for constant amplitude loading. The curve for $a_f/a_o \to \infty$ is close to that for $a_f/a_o = 10$, making this a convenient value for calculations. As $a_f/a_o \to 1$, the procedure becomes equivalent to a Miner's rule summation, that is that failure takes place when

$$\sum \frac{n_i}{N_i} = 1 \tag{7}$$

where n_i is the number of cycles at the i-th load level, and N_i is the number of cycles for failure at the i-th load level.

An advantage of basing predictions on constant amplitude data is that factors such as the effect of residual stress, not easily allowed for explicitly, are automatically taken into account.

Lap Welded Specimens

As a check on the utility of the method it was used (Pook, 1974) to forecast the lives of mild steel lap-welded joints, of the type shown in Fig. 2, tested under NBR loading at high mean stress ($S_m/2^{\frac{1}{2}}\sigma = 2$) (Marsh, Martin and McGregor, 1975). Nominal stresses were all below general yield. The results obtained are shown in Fig. 3. The slope of the constant amplitude S/N curve corresponded to m = 3.17, close to the expected value for mild steel, and a_f/a_o was taken as 10. The forecast coincided with a Miner's rule summation at short endurances and agreement with the experimental results was good. At longer endurances the curves diverge. The Miner's rule prediction is less conservative, but agrees better with the experimental results.

With the benefit of hindsight the method was modified and better agreement was obtained at long endurances (Pook, 1977).

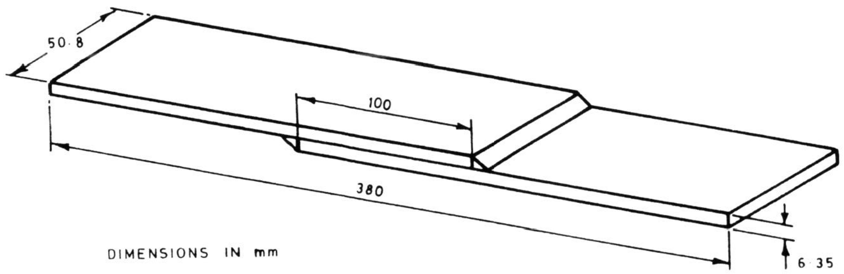

Fig. 2. Lap-welded Specimen

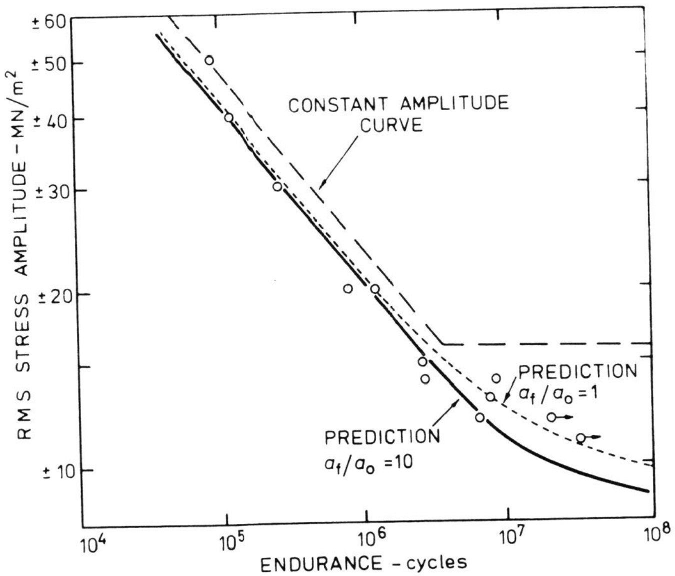

Fig. 3. Narrow Band Random Loading Results for Mild Steel Lap-welded Specimens

MODIFICATIONS TO THE METHOD

Cracks in Weldments

An outline of the modifications to the method is given below; full details are given by Pook (1977).

In welded joints the assumption that a remains constant as a crack grows is perhaps an over-simplification. Failure at a weld toe is usually from a small crack-like defect. Such defects are typically about 0.15 mm deep (Maddox, 1975). The effects of variations in α with crack length can conveniently be examined by writing $\alpha = \alpha_o(a/W)^b$, where $(a/W)^b$ is a non-dimensional factor and W is the thickness (or some other convenient characteristic dimension) (Pook, 1977; Det Norske Veritas, 1977). For toe cracks this provides a reasonable fit to stress intensity factors. Depending on precise geometry (Gurney, 1975), b ranges from -0.1 to -0.34 with a typical value of $-\frac{1}{4}$. Examination of stress intensity factors for various configurations (Rooke and Cartwright, 1976) suggests that b does not usually exceed $\frac{1}{4}$. Det Norske Veritas (1977) suggest values from -0.20 to 0.16.

Equation (3) when $\alpha_o(a/W)^b$ is substituted for α becomes

$$N = W^{mb}\frac{\left\{a_o^{\,1-m(\frac{1}{2}+b)} - a_f^{\,1-m(\frac{1}{2}+b)}\right\}}{C\Delta S^m\pi^{\frac{1}{2}m}\{m(\frac{1}{2}+b)-1\}\alpha_o^{\,m}}. \qquad (8)$$

In general, equation (8) indicates that behaviour is strongly dependent on the relationship between m and b. In particular the value of a_f becomes more significant as b is reduced.

The effect of variable amplitude loading can be further examined by again considering the behaviour of a welded joint subjected to NBR loading. Of particular interest is the effect of changes in α as a crack grows on the predictions made from constant amplitude data. Assuming $b = \frac{1}{4}$ or $-\frac{1}{4}$ and $a_f/a_o = 10$ causes a slight increase in life at long endurances, but the difference is only of the order of the line thickness on a graph. However, the lower the value of b, the more slowly will relative damage increase with crack length, making results more sensitive to the value of a_f/a_o. The previous conclusion that a_f/a_o could conveniently be taken as 10 does not necessarily apply for b < 0.

The assumption that the final crack length is independent of the applied load can be an over-simplification. In particular a_f is likely to change markedly with applied load in specimens where the net sectional area reduces rapidly as a crack grows. When this happens the slope of the S/N is less than the slope expected from the value of m. The difference between the S/N curve and a line having the expected slope can be thought of as a measure of a_f, which can be used in the estimation of fatigue life under variable amplitude loading (Pook, 1977). The calculations are similar to the hypothetical S/N curve method first suggested by Corten and Dolan (1956), and explain the success of that method. Making a correction of this type considerably improved the prediction for the lap welds shown in Fig. 2 (Pook, 1977).

Welded Tubular T-joints

Tests were carried out at NEL at zero mean load on welded tubular mild steel T-joints of the type shown in Fig. 4 (Martin and McGregor, 1977). Load was applied axially along the brace and failure took place in the parent material of the chord. As is conventional for tubular joints, results were plotted in terms of the 'hot spot' stress, which is the actual rather than the nominal stress at the stress concentration at which failure originates. In this type of joint, failure initiates at the weld toe and grows around the weld as indicated in Fig. 4, so that a_f, measured on the surface, is much longer than a_o. The actual pattern of crack growth in a tubular joint is complicated (Martin and MacGregor, 1977) and crack growth tends to be through the thickness, but it has been pointed out (Pan and Plummer, 1976) that crack lengths measured on the surface can be used as the basis of a meaningful fracture mechanics analysis. As failure originates at a toe crack, b can be taken as $-\frac{1}{4}$ while the crack is fairly short, and limited information for a similar type of joint (Pan and Plummer, 1976) suggests that this value is also appropriate at longer lengths.

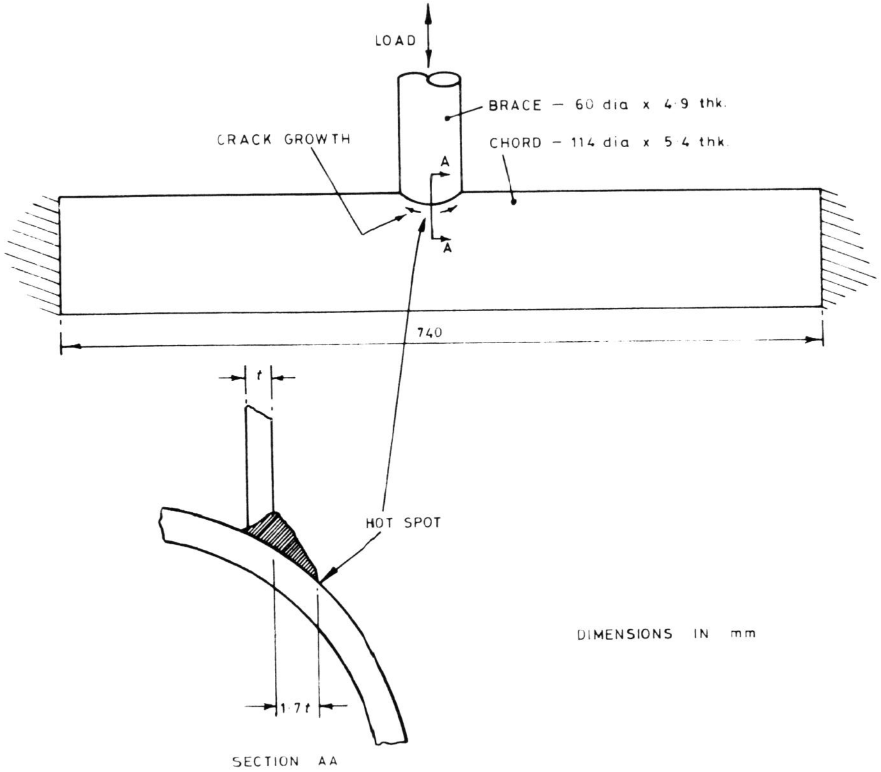

Fig. 4. Welded Tubular T-joint

Fig. 5 shows the NBR results for the tubular T-joints compared with various predictions based on constant amplitude results. The initial forecast, made before the results were known and marked 'no interaction', was based on the line through constant amplitude data corresponding to m = 3, with a_f/a_o taken at 100 and b as $-\frac{1}{4}$. This prediction is fairly reasonable at short endurances, but grossly underestimates the fatigue strength at long endurances. After the results were known further predictions were made in an attempt to find the source of the discrepancy without success so the predictions for these data were not discussed by Pook (1977) and Martin and McGregor (1977). From a practical viewpoint this discrepancy was not serious because the simple Miner summation, shown in Fig. 5, was conservative.

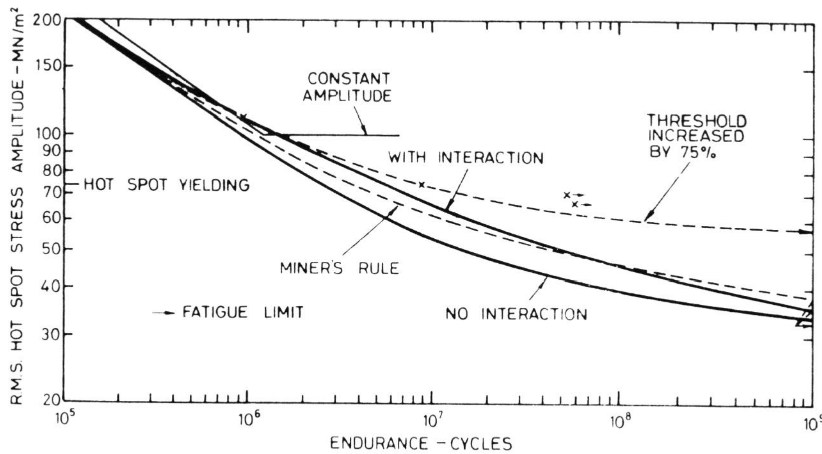

Fig. 5. Narrow Band Random Loading Results for Welded Tubular Mild Steel T-joints

EFFECT OF INTERACTIONS ON THE FATIGUE-CRACK GROWTH THRESHOLD

Prior loading can have a significant effect on the fatigue-crack growth threshold of mild steel. Clearly, all possible consequences for the high cycle region should be considered. For a range of steels tested at zero mean stress Klesnil and Lukáš (1972) found that

$$\Delta K_{th} = \Delta K_{th,o}\left(\frac{\Delta K_{Ip}}{\Delta K_{th,o}}\right)^{\gamma} \tag{9}$$

where $\Delta K_{th,o}$ is the threshold without prior loading, ΔK_{Ip} is the value of ΔK_I immediately before determination of the threshold, and γ is an exponent whose value varies with the tensile strength of the steel. For mild steel γ is 0.45.

In an attempt to understand the behaviour of welded joints at very long lives some NBR tests were carried out on edge-cracked mild steel plates (Pook and Greenan, 1979). All tests were at zero mean stress, and the initial cracks were approximately 4 mm deep. Specimens were stress relieved after precracking. Calculations showed that assuming that ΔK_{th} was elevated according to equation (9) gave an improved fit at long endurances.

In the light of these results two further predictions were carried out for the welded tubular T-joint results shown in Fig. 5. These were both modified versions of the forecast described above. In the first, marked 'with interaction', it was assumed that interaction effects increased the threshold according to equation (9). This still substantially underestimates the fatigue strength at long endurances. The second used a value of ΔK_{th} arbitrarily increased by 75 per cent, as suggested by Priddle (1975), and gave a much better fit at long endurances, but it can only be regarded as a curve-fitting procedure.

APPLICATION TO NON-STATIONARY NARROW BAND RANDOM LOADING

As part of the United Kingdom Offshore Steels Research Project (UK Department of

Energy, 1974), constant amplitude and non-stationary NBR loading tests are being carried out on cruciform welded, full penetration joints of the types shown in Fig. 6 (Holmes, 1978). All specimens were manufactured from medium strength structural steel plate to BS 4360 grade 50D (modified). Axial specimens were cut from 25 mm thick plate, tensile strength 536 MN/m^2, 0.2 per cent proof stress 383 MN/m^2, and tested under axial loading. Bend specimens were cut from 38 mm thick plate, tensile strength 538 MN/m^2, 0.2 per cent proof stress 370 MN/m^2, and tested in four-point bending. All tests reported here were carried out at zero mean stress. Full details of the specimens and test technique are given by Holmes (1978). The constant amplitude tests gave essentially the same results for both types of loading. The slope of the finite life portion of the S/N curve corresponds to m = 3.

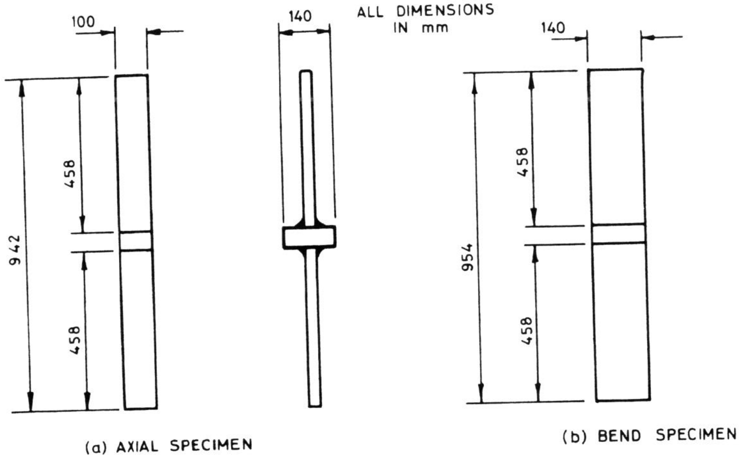

Fig. 6. Cruciform Welded Specimens

The constant amplitude axial loading test results were used to forecast the results of tests carried out using the load history shown in Fig. 7, which is intended (Pook, 1978) to be representative of those experienced by offshore structures. The forecast was carried out taking b = 0, a_f/a_o = 10, S_T/σ = 8 and γ = 0.5 and is shown marked 'with interaction' in Fig. 8. For comparison a Miner's rule summation and a prediction with no interaction are also shown.

All the predictions overestimate the lives obtained, with the no-interaction curve being nearest the test results. Use of the actual, rather than the nominal clipping ratios, reduces the predicted lives slightly. The constant amplitude S/N curve used was based on the axial results as only these were available at the time the forecasts were made. A revised Miner's rule summation, based on an S/N curve drawn through all the constant amplitude data, and a more accurate assessment of the load history used gave an improved prediction. Further analysis will be carried out.

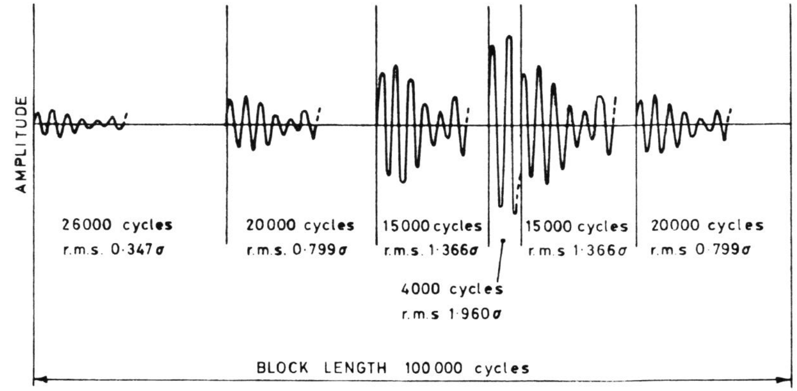

Fig. 7. Non-stationary Narrow Band Load History

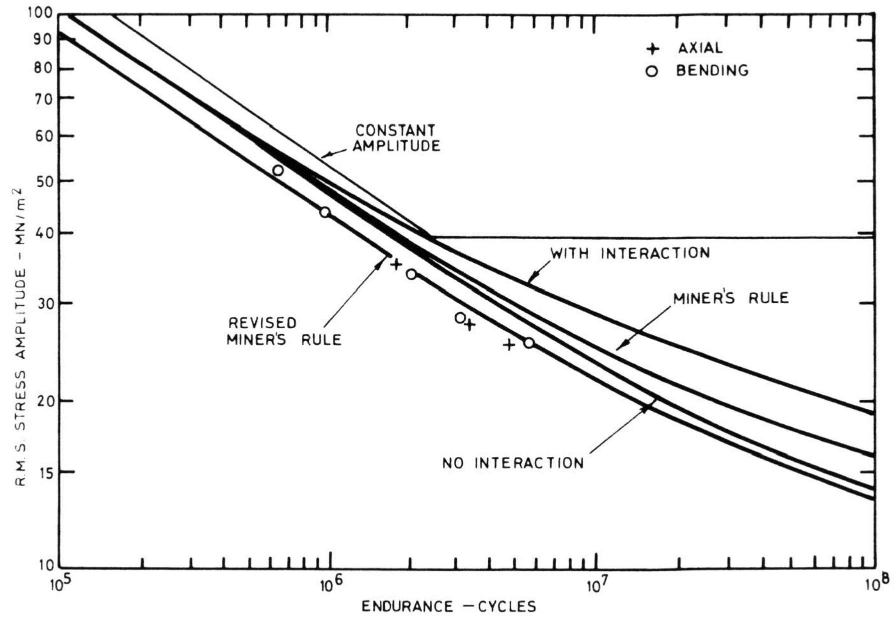

Fig. 8. Non-stationary Narrow Band Random Loading Results for Cruciform Welded Specimens, $S_T/\sigma \sim 7$ (axial) ~ 6(bending)

DISCUSSION AND CONCLUSIONS

None of the forecasts of fatigue life were entirely satisfactory even though they were for carefully controlled laboratory tests, and the method used automatically took a number of variables into account, such as the residual stresses which must have been present in the unstress-relieved joints tested. With the benefit of hindsight it was always possible to refine the calculations to give an improved prediction, so that it should be possible to make reasonable predictions for broadly similar situations. However it is clear that, for NBR loading of welded joints, service simulation tests carried out to representative endurances are essential if the fatigue strength at long endurance is to be determined accurately.

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