CRACK PROPAGATION AND FRACTURE OF COMPOSITE MATERIALS SUCH AS CONCRETE

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ABSTRACT

It is shown that the structure of concrete may be subdivided into three different levels, i.e. hardened cement paste, mortar, and coarse aggregates in a mortar matrix. Crack propagation in concrete is studied on these different levels. First of all crack propagation in a porous homogeneous material is investigated. Then crack extension as occurring in a precracked matrix is treated and finally crack theory is applied to a composite material. Using the derived formulae crack formation in normal and high strength concrete is simulated by means of a computer experiment. Results are compared with experimental findings. Furthermore the theoretical concept outlined in this contribution is used to predict the lifetime of hardened cement paste and concrete under high sustained load.

KEYWORDS

Concrete, mortar, hardened cement paste, structure, composite material, crack propagation, micromechanics, lifetime.

INTRODUCTION

There are several approaches to describe deformation and failure processes of composite materials such as concrete by means of mechanical models. Usually the complex structure of the material has to be modelled in a simplifying way. Results of all micromechanical methods of investigation depend essentially on realistic assumptions concerning the structure.

A stochastic theory for concrete fracture has been published recently (Mihashi and Izumi, 1977). In this approach a concrete specimen is supposed to consist of many elements and each element may crack independently. The tensile stress in the material varies locally and is taken into consideration by a probability density function.

The stochastic model indicates the probability of failure of hardened cement paste and concrete. In this way the influence of rate of loading, temperature, and size effect on the mean value of strength can be described. In addition to this the variance of strength can be directly linked with the structure of the material. In a comprehensive report experimental results verifying this theoretical concept will be described in detail (Wittmann and Mihashi, 1980).

Based on the fact that in composite materials such as concrete many microcracks are distributed all over the loaded material far below failure, Bazant and Cedolin have suggested the method of blunt crack band propagation (Bazant and Cedolin, 1979). Instead of investigating individual crack formation and crack growth a smeared crack band with a blunt front is introduced. This approach turned out to be highly effective in finite element analysis. Cracks are smeared over a finite zone. This zone retains only the capability of transmitting stresses parallel to the crack direction. It is possible to link this analysis with classical fracture mechanics.

Another method which combines fracture mechanics and finite element analysis has been developed by a Swedish group (Hillerborg and co-workers, 1976 and 1978). Similar to the model of Barenblatt they assume that near a crack tip a plastic zone is created. Within this plastic zone stress can be transferred to some extent. The load bearing capacity of the plastic zone decreases as the crack opens. The actual materials behaviour may then be characterized by choosing an appropriate function for the variation of stress with crack width.

Mainly for the investigation of temperature induced cracks the effect of stress gradients has been studied (Rösli and Harnik, 1974). In a similar approach, stress relaxation has been taken into consideration (Wittmann and co-workers, 1978 and 1979). Podvalny (1973) studied crack formation in a mortar layer surrounding a coarse aggregate particle. His analysis is especially suitable to study crack formation as a consequence of differential thermal expansion and due to differential shrinkage. Other authors applied finite element method to calculate stresses and crack formation around an aggregate in a loaded matrix (Modéer, 1979). In this way the special properties of the interface can be taken into consideration separately.

Stresses in a given porous structure depend on pore geometry and pore size. In case the porous structure is completely known, the stress distribution can be calculated. In most real porous structures, however, the probability of occurence of different pore shapes and the pore size distribution have to be determined or estimated. Then a "random pore" characterizing a given material can be generated (Zaitsev and Wittmann, 1973). By means of an analysis of stress exceedance, crack formation and failure can be studied.

In a great number of papers the description of the observed behaviour are purely phenomenological. These investigations, however bear little significance on the present context and they shall therefore not be discussed any further.

This is by no means a complete list of micromechanical approaches. But some trends in recent development are touched. In this contribution, the application of crack theory to porous and composite materials will be dealt with. Finally theoretical predictions will be compared with experimental findings.

SOME REMARKS ON THE STRUCTURE

With respect to crack formation and crack propagation the structure of concrete may be subdivided into different levels. Hardened cement paste is a porous material

with a high internal surface. This system may be described in terms of micro-level. A characteristic hydraulic radius of the pore system is of the order of magnitude of 20 Å. These micropores may be neglected in micromechanics of fracture of concrete because there are plenty of larger pores in the material. Therefore hardened cement paste may be looked upon to consist of a homogeneous material containing capillary pores. Capillary pores are at least ten times larger than gel pores and their density as well as their characteristic pore radius depends on the water-cement ratio and on the age of a specimen. A typical pore size distribution is shown in Figure 1. Many data on pore size distribution of hardened cement paste are to be found in the literature (see f.e. Diamond, 1971).

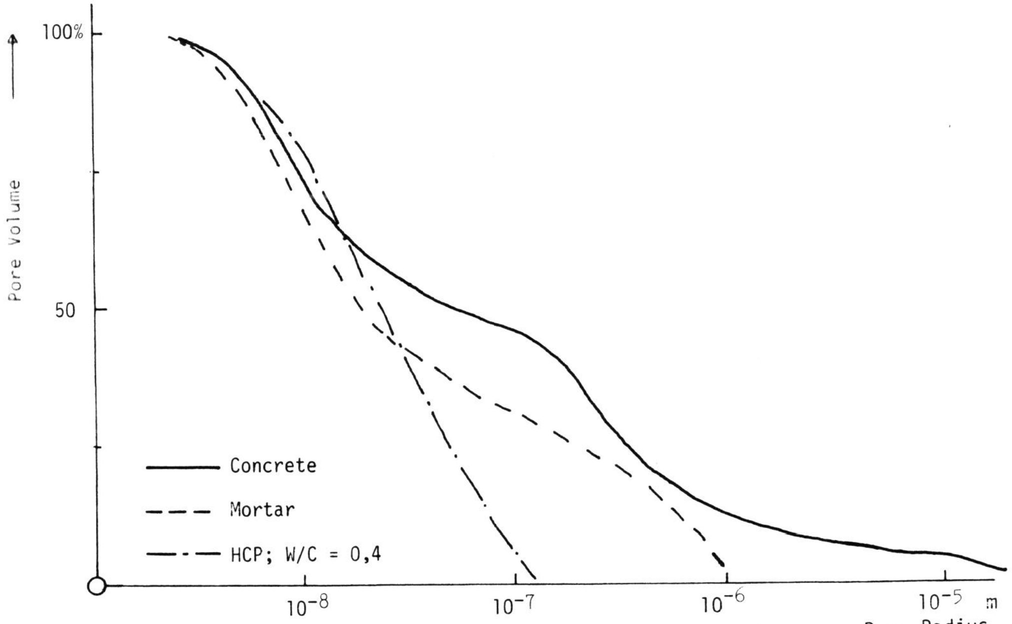

Fig. 1. Typical pore size distribution of concrete (solid line). For comparison corresponding distribution functions of mortar and hardened cement paste (HCP) are shown.

Fine aggregates in mortar are bound together by hardened cement paste. In this way a structure similar to natural sand stones is formed. This structure will be dealt with as mezzo-level. The amount of coarser pores depends essentially on the cement content. A corresponding typical pore size distribution is again plotted in Figure 1. It is obvious that mortar has a biporous system i.e. capillary pores of the binding hardened cement paste and pores originated by the grain assemblage (Rostasy and co-workers, 1980). By means of micrographs the biporous system of mortar can be analysed too.

Besides a characteristic pore size distribution crack arresting by aggregates may occur. But because of the small particles the influence with respect to micromechanics is of minor importance. The material, however, becomes much more tough.

Finally compaction pores and cracks formed by bleeding, capillary shrinkage and drying shrinkage have to be taken into consideration on the macro-level. Crack formation and crack propagation on the macro-level is dominated by the interaction of cracks with aggregates. We shall deal with crack formation on the three different levels just mentioned consecutively.

In Figure 1 a typical pore size distribution of normal concrete is shown by means of a solid line. The capillary pores of hardened cement paste can be easily recognized in the total pore size distribution of concrete. At higher radii the pores of the mortar structure (mezzo-level) can be seen and finally compaction pores of concrete appear at a radius of about 10^{-5} m and above.

CRACK PROPAGATION IN A POROUS MATERIAL

We shall start to describe crack propagation through a porous material with the easiest case i.e. one circular hole in a homogeneous infinite plate. The stress distribution if an external uniaxial compressive stress is applied is well-known. At the two poles where maximum tensile stress is observed a crack may develop as soon as critical load value is reached. If a tensile load were applied catastrophic crack propagation would occur. Under compression, however, stable crack growth takes place. The crack length increases as the load is increased. It is useful to relate crack length l to the radius of the pore r :

$$\lambda = \frac{l}{r} \qquad (1)$$

It can be shown (Wittmann and Zaitsev, 1974; Zaitsev 1971) that the related crack length λ is dependent on the load q by the following equation :

$$q = \sqrt{\frac{\pi E \gamma}{r}} \sqrt{\frac{(1+\lambda)^7}{2\left[(1+\lambda)^2-1\right]}} \qquad (2)$$

If we consider an idealized material with one single pore the first root in equation (2) becomes a material constant :

$$C = \sqrt{\frac{\pi E \gamma}{r}} \qquad (3)$$

Now the load q may be related to the materials properties C and one gets the following implicit expression for related crack length as function of related load q* :

$$q^* = \frac{q}{C} = \sqrt{\frac{(1+\lambda)^7}{2\left[(1+\lambda)^2-1\right]}} \qquad (4)$$

Relation (4) is graphically shown in Figure 2. It is obvious that from relation (4) no failure criterion may be derived. There is no critical length. It may be seen, however, from equation (2) that with increasing pore radius the necessary load to create a crack with a given length decreases.

In a real porous material pores are randomly distributed. The pore size may be approximated by an extreme value distribution function and the distance may be looked upon to be normally distributed. If we neglect for a moment the pore size distribution we may simulate a porous structure in a simplified way by randomly distributed pores arranged on a line. In Figure 3 an example is given. Pores have an average distance of c and the distribution function of the distance has a standard deviation of Δc. If an external load is applied above a characteristic stress, cracks will propagate from all pores. The crack length will be described in the initial stage by equation (4). If two cracks come close to one another, they

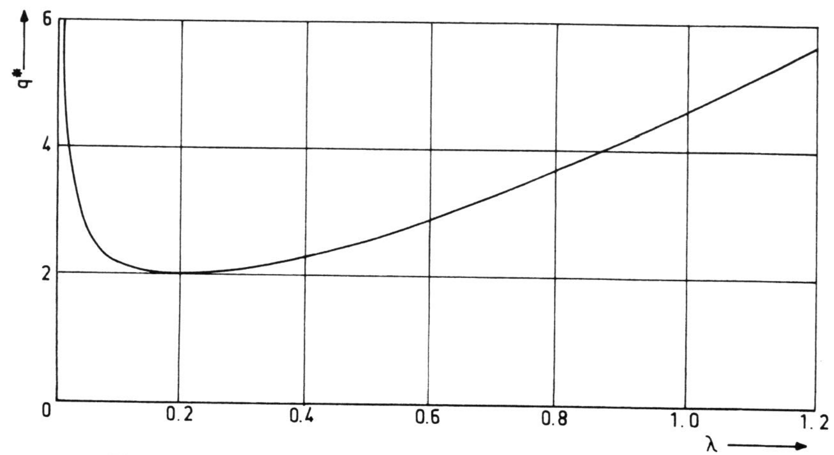

Fig. 2. Graphical representation of relation (4).

interact. It can be shown (Wittmann and Zaitsev, 1974) that they attract one another and finally they coalesce. This means a sudden discontinuous jump in crack length. In Figure 4 the calculated crack length of a computer simulation is shown. In fact the sum of all crack lengths

$$S = \sum_{i=1}^{n} (2l_i - 2r) \qquad (5)$$

is plotted versus the related load (r = 1; n = 100). Dashed lines represent the undisturbed situation according to equation (4). But now the crack length increases more rapidly and finally a critical state is reached. At a certain total crack length S_c further crack propagation is unstable. The corresponding load may be defined to be an ultimate load and thus a failure criterion is found. In the way just described here, porous structures may be studied in a systematic way. For a given porosity optimum pore distributions can be determined. Crack propagation and failure process of hardened cement paste and other porous ceramic materials may be realistically simulated in this way.

In concrete materials this is not of primary interest, however, because cracks are influenced by aggregates. Therefore crack propagation in a composite material shall be described in the following section.

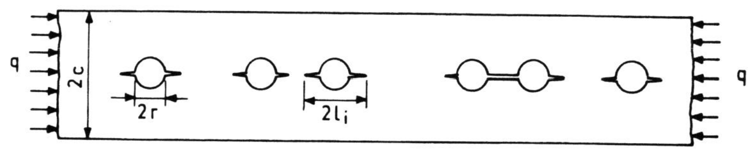

Fig. 3. Example for a simplified simulation of a porous structure with randomly distributed pores.

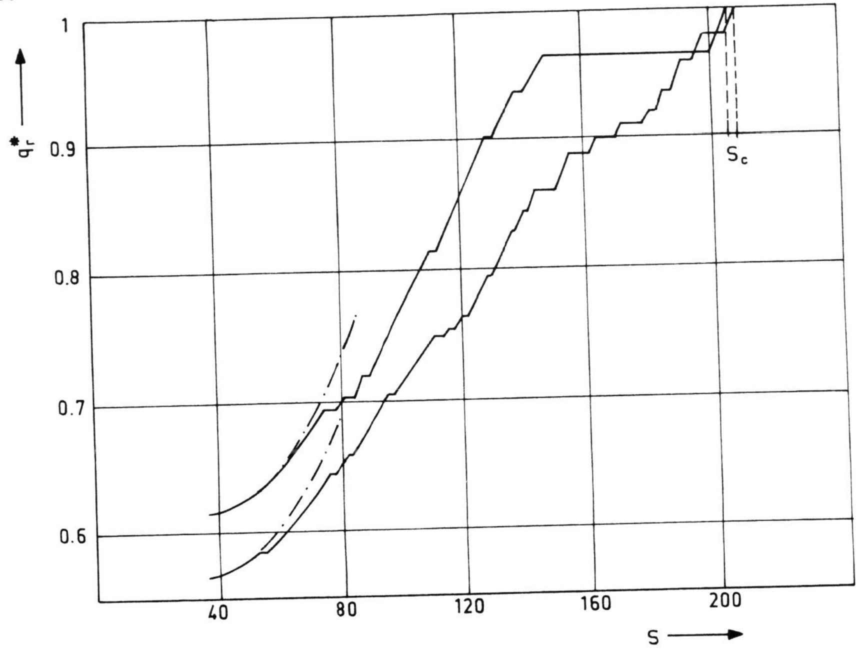

Fig. 4. Total crack length S (see eq. (6)) as function of load. Two runs of randomly distributed pores are arranged as shown in Figure 5. The mean distance of the pores has been chosen to be four times the radius (r = 1; c = 4) and a sample with 100 pores has been studied by this computer simulation.

CRACK PROPAGATION IN A PRE-CRACKED MATRIX

In a homogeneous porous material cracks can develop in an arbitrary way. The direction and the crack length are exclusively determined by the external state of stress. In mortar and concrete as in all other composite materials crack propagation is also influenced by the structure of the material. The high strength of normal aggregates causes crack arresting and crack deviation. As a result composite materials with high strength aggregates become more ductile than the plain matrix.

As mentioned above, in mortar and concrete there are usually a-priori cracks present. These cracks are caused by bleeding, capillary or drying shrinkage or by thermal stresses. Preloading of course may also cause crack formation.

If an external load is applied cracks may propagate. Therefore on the mezzo-level and on the macro-level extension of existing cracks has to be studied.

First of all we have to investigate branching cracks starting from an arbitrary inclined crack in a homogeneous plate. In Figure 5 a crack with length $2\ l_1$ and inclination α with respect to the direction of an external load is shown. The shear

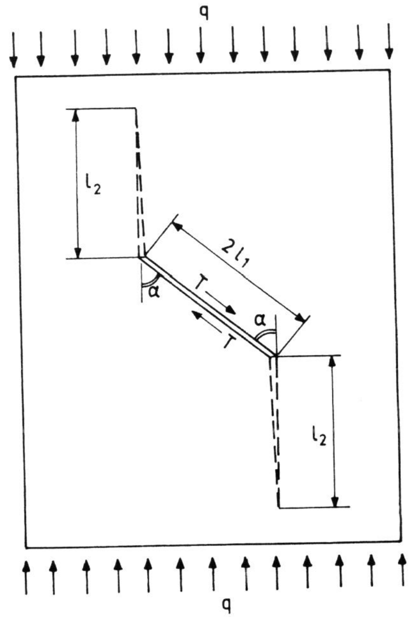

Fig. 5. Schematic representation of the development of branching cracks and definitions of symbols used in corresponding equations.

stress T acting along the inclined crack can be expressed in the following way :

$$T = 2\ l_1 \cdot q\ (\sin\alpha\cos\alpha - \rho\sin^2\alpha) \tag{6}$$

where ρ represents the coefficient of friction. The x component (perpendicular to the direction of stress) of the shear stress T at the ends of the crack is :

$$P = T \cdot \sin\alpha \tag{7}$$

and with equation (6) P becomes :

$$P = 2\ l_1\ q\ (\sin^2\alpha\cos\alpha - \rho\sin^3\alpha) \tag{8}$$

It is useful to introduce the following abbreviation :

$$A\ (\alpha,\rho) = \sin^2\alpha\cos\alpha - \rho\sin^3\alpha \tag{9}$$

With this expression the horizontal component P may be rewritten :

$$P = 2\ q\ l_1\ A\ (\alpha,\rho) \tag{10}$$

By introducing some simplifying assumptions it may be shown that the length of the two branching cracks l_2 can be expressed as function of P by the following implicit equation (Zaitsev and Wittmann, 1977) :

$$P = K_{IC}\sqrt{\pi\ l_2} \tag{11}$$

From equations (10) and (11) it follows :

$$q = \frac{K_{IC}}{A(\alpha,\rho)} \frac{1}{2\,l_1} \sqrt{\pi\, l_2} \qquad (12)$$

or in a slightly modified form :

$$q = \sqrt{\frac{l_2}{l_1}}\; \frac{K_{IC}}{2A(\alpha,\rho)} \sqrt{\frac{\pi}{l_1}} \qquad (13)$$

Equation (13) has been found to be in good agreement with experimental findings (Wittmann and Zaitsev, 1980). Crack propagation of this type may be characteristic for failure of a structure on the mezzo-level i.e. mortar with preexisting cracks. In concrete, however, the interface between matrix and aggregate has to be taken into consideration.

CRACK PROPAGATION IN A COMPOSITE MATERIAL

In Figure 6 a polygonial aggregate is supposed to be embedded in a homogeneous matrix. There exists a crack with length 2 l_1 on one side. As has been shown earlier (Wittmann and Zaitsev, 1971), in this case the crack will spread along the interface if a critical load q^{IF} is reached. This crack extension is unstable and follows mode II. The critical load essentially depends on stress intensity factor of the interface K_{IIC}^{IF} :

$$q^{IF} = \frac{K_{IIC}^{IF}}{B(\alpha,\rho)} \frac{1}{\sqrt{\pi\, l_1}} \qquad (14)$$

where $B(\alpha,\rho)$ is given by :

$$B\,(\alpha,\rho) = \sin\alpha \cos\alpha - \rho \sin^2\alpha \qquad (15)$$

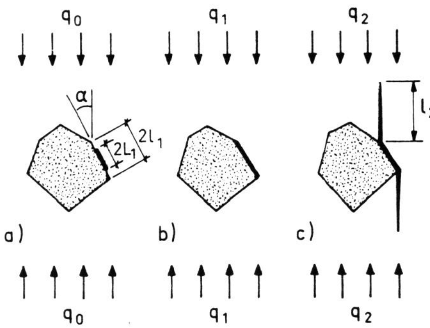

Fig. 6. Schematic representation of crack growth starting from an interface : (a-b) initial crack grows unstable along the interface, (b-c) stable branching cracks are developed.

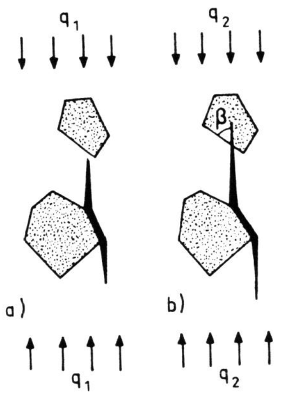

Fig. 7. A crack as created according to Figure 6 meets a second inclusion.

The now created crack with length 2 l_1 (for the meaning of symbols see Fig. 6) will further behave like an inclined crack in a matrix. Branching cracks will propagate into the matrix in a stable way if the load is increased. By using equation (12) this conditions may be written as follows :

$$q = \frac{K_{IC}}{A(\alpha,\rho)} \frac{1}{2l_1} \sqrt{\pi\, l_2} \qquad (16)$$

In concrete, cracks which penetrate into the matrix, may meet another aggregate along their path. In Figure 7 this situation is shown schematically. When the crack reaches the second inclusion further crack growth is dependent on both the inclinations of the first and second interface. The conditions for opening (I) and shear (II) for crack propagation can be given as follows :

$$q_I = \frac{2K_{IC}^{IF} \sqrt{\pi\, l_2}\,/\,l_1}{A(\alpha,\rho)\;3\cos\,{}^{\beta}/_2 + \cos\frac{3\beta}{2} - 3C(\alpha,\rho)\;\sin\,{}^{\beta}/_2 + \sin\frac{3\beta}{2}} \qquad (17)$$

and

$$q_{II} = \frac{2K_{IC}^{IF} \sqrt{\pi\, l_2}\,/\,l_1}{A(\alpha,\rho)\left[\sin\,{}^{\beta}/_2 + \sin\frac{3\beta}{2}\right] + C(\alpha,\rho)\left[\cos\,{}^{\beta}/_2 + 3\cos\frac{3\beta}{2}\right]} \qquad (18)$$

where $C(\alpha,\rho)$ has the following meaning :

$$C(\alpha,\rho) = B(\alpha,\rho) \cdot \cos\alpha \qquad (19)$$

It is important to note that further crack growth depends also on the sign of β because sin β appears in equations (17) and (18). In fact it turns out that crack propagation is favoured if the inclinations of α and β have the same sign.

But a crack, meeting an inclusion, has a third possibility, the crack may extend through the inclusion. Whether a crack penetrates an inclusion or whether it is deviated along the interface depends on equations (17) and (18) and the condition for straigth crack formation through the aggregate (see also equation 12) :

$$q_I^A = \frac{K_{IC}^A}{A(\alpha,\rho)} \frac{1}{2l_1} \sqrt{\pi\, l_2} \qquad (20)$$

With the formulae (17), (18) and (20) it is possible to calculate crack formation in a composite material. In this connection computer simulation methods proved to be successful. In a "computer experiment" different stages of loading can be studied. The validity of this approach can then be checked by comparison with real experiments.

CRACK PROPAGATION IN NORMAL AND HIGH STRENGTH CONCRETE

By means of a computer a random structure of concrete can be generated. The aggregates can be chosen to be distributed at random. Also the size and geometry of the polygonial aggregates can be generated by means of a stochastic process. In addition at each particle an interfacial crack at a randomly chosen side can be generated.

In the computer experiment the influence of an increasing compressive load on the crack formation can be studied. According to the formulae mentioned in the previous

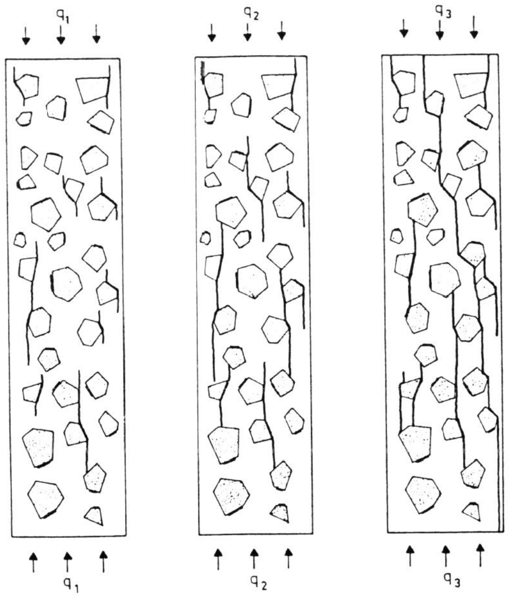

Fig. 8. Three stages of crack formation under increasing compressive load as simulated for normal concrete by means of a computer experiment.

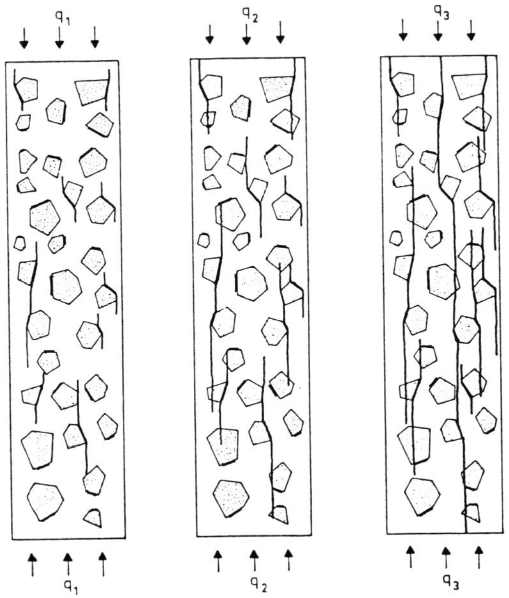

Fig. 9. Three stages of crack formation under increasing compressive load as calculated for high strength concrete.

sections some a-priori cracks start growing above a certain stress-level. Cracks extend into the matrix and eventually they merge. As the aggregates are stronger than the matrix all cracks pass along interfaces :

$$K_{IC} < K_{IC}^{A} \qquad (21)$$

Therefore the condition (20) never becomes critical.

Three stages of increasing load are shown graphically in Figure 8. It has to be noted that q_1 is already above 50% of the ultimate load. It is obvious that more and longer cracks are created as the load increases and the structure disintegrates gradually. Finally an inclined crack runs through the specimen. This situation is defined to be materials failure. The inclination of the fracture line is based on conditions (17) and (18) and has been experimentally verified very often.

In normal concrete it is supposed that the matrix is much weaker than the aggregates. Therefore cracks are deviated. In high strength concrete this is not the case. Here strength of matrix and strength of aggregate are of the same order of magnitude :

$$K_{IC} \approx K_{IC}^{A} \qquad (22)$$

Under these conditions a crack may penetrate a particle. Whether it actually penetrates or not is mainly dependent on the geometrical arrangement. In Figure 9 an example of crack formation in high strength concrete is shown. As can be seen, with increasing load an increasing number of aggregates is spilt by growing cracks. Failure is again defined from the crack pattern i.e. when the first crack is running through the whole specimen. In high strength concrete, cracks are more likely to extend along the axis of applied load. It may be mentioned here that a similar crack pattern is obtained for lightweight aggregate concrete. This is further outlined in a separate paper (Wittmann and Zaitsev, 1980).

High strength concrete reacts more brittle than normal concrete. In the computer experiment this is shown by the fact that once stage q_1 is reached, ultimate failure occurs after a small increase of load. In fact the total work absorbed by crack formation is largely reduced in high strength concrete.

DETERMINATION OF LIFETIME UNDER HIGH LOAD

In the previous section crack propagation in normal and high strength concrete has been investigated. As a second example for the application of micromechanics the determination of the lifetime of hardened cement paste under high load shall be used. It must be mentioned here that the same approach can be applied in order to predict the lifetime of loaded concrete (Wittmann and Zaitsev, 1974).

Until now we have neglected time dependence of crack growth. In Figure 2 a critical crack length S_c is shown. If the corresponding load is reached, cracks will spread in an unstable way without further increase of load. If the load is kept constant on a level slightly below the critical load, the overall crack length increases as function of time due to creep in the vicinity of crack tips. Other mechanisms, such as stress corrosion, may also contribute to crack extension. In this way after some time a critical crack length is reached although the load remains under the "short term" critical load. The lifetime under high load is thus dependent on the visco-elastic properties of the material and is determined by the level of applied load.

If the hardening process of concrete while under load may be neglected, that is to say if we deal with matured concrete, the following formula for stress $\sigma(t)$ which causes failure after a lifetime t, related to the short term strength σ_o can be derived :

$$\frac{\sigma(t)}{\sigma_o} = \sqrt{\frac{1}{1+\phi(t)}} \qquad (23)$$

In this equation $\phi(t)$ stands for the creep number. If the creep deformation is high a comparatively low lifetime under high load may be expected. A more rigorous treatment of this problem is described elsewhere (Wittmann and Zaitsev, 1974).

As mentioned above, the lifetime of hardened cement paste and concrete can be determined by means of this approach. As an example some experimental results are compared with theoretical prediction in Figure 10. The lifetime is plotted as function of the related stress $\sigma(t)/\sigma_o$. For the very young samples the calculated curve bends upwards after having reached a minimum. The lifetime which corresponds with this minimum value is shifted towards higher values as the material matures. At the minimum the strength decrease due to crack propagation is just counterbalanced by the hardening process due to further hydratation. Samples which survived this critical lifetime will never fail under constant sustained load.

DISCUSSION AND CONCLUSIONS

It has been shown that failure of concrete is caused by crack formation in the heterogeneous structure of the material. Crack theory is a powerful tool to investigate crack formation and final degradation of a porous composite material. The observed load bearing capacity is dependent on the pore size distribution within the matrix and the geometrical arrangement and the structure of the interfaces.

Micromechanical methods can be used in two different ways. First it is possible to investigate and simulate materials behaviour in a realistic way. This approach finally leads to a better description and understanding of experimental findings. Second micromechanical analysis can be used to optimize composite materials. This latter mentioned possibility may turn out to be most important for the development of new high strength materials.

The well-known relation between strength and water-cement ratio can be interpreted by means of pore size distribution. At very low water-cement ratios the expected high strength is not reached because an increasing number of compaction pores is created due to insufficient workability. Superplasticising agents avoid the development of compaction pores at comparatively low water-cement ratios and thus the extreme value distribution of pore size in a given sample is drastically changed. In this way, high strength and other superior properties are obtained.

Another method to reduce porosity and to avoid the presence of large pores is mechanical compaction. Compacts with strength of 250 N/mm^2 and above can be easily prepared.

The simulation of a cracked structure can be further developed. The orientation of a-priori-cracks, for instance, can be chosen according to their physical origin instead of random orientation. The effect of cracks initiated by preloading can also be studied in this manner.

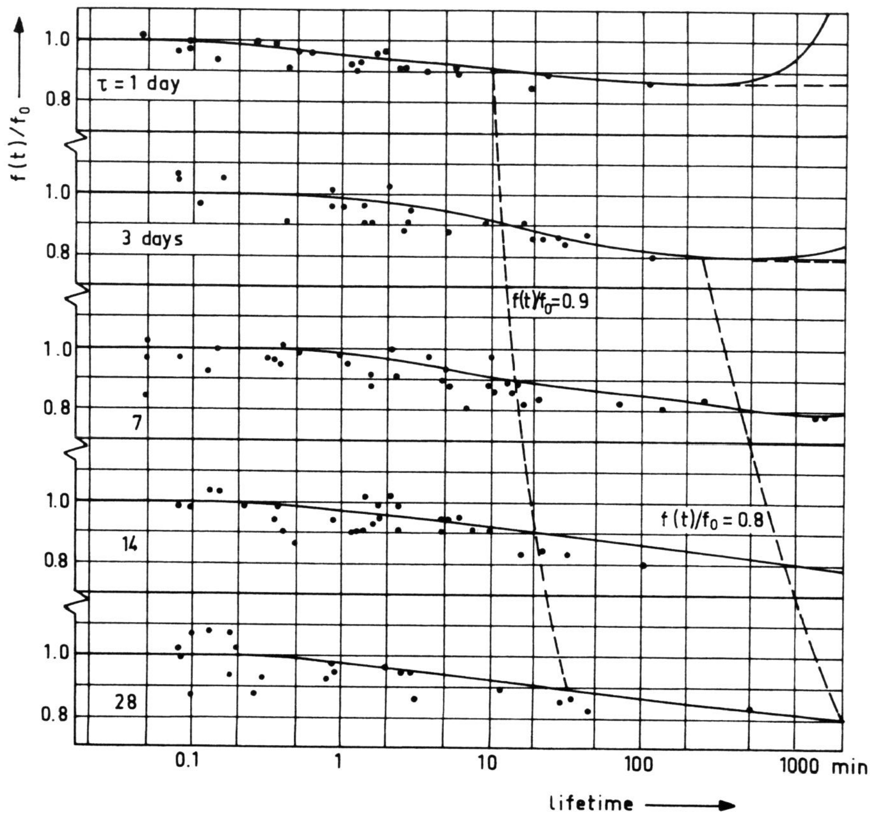

Fig. 10. Comparison of theoretical prediction of lifetime of hardened cement paste under high load with experimental results. Samples with an age ranging between 1 day and 28 days have been tested.

A further extension may be the replacement of a two-dimensional composite structure by a three-dimensional one.

Crack theory is one possibility to replace phenomenological studies. Although often claimed, purely phenemenological studies are by no means economic and their results do not meet requirements of modern computerized structural analysis.

Therefore more quantitative and more mathematical concepts have to be introduced in materials science of concrete (Wittmann, 1978). The validity of modern methos of investigation has to be checked carefully by comparing the predictions made with appropriate experimental data. Furthermore the usefulness of all concepts will be determined by the fact whether they are applicable to the solution of problems arising in advanced concrete structural engineering.

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