BRITTLE FRACTURE INITIATION CHARACTERISTICS UNDER BI-AXIAL TENSILE LOAD WITH LARGE SCALE AND GENERAL YIELDING

Y. Ueda*, K. Ikeda**, T. Yao*, M. Aoki***,
S. Shibasaki*, and T. Shirakura***
* Welding Research Institute, Osaka University, Suita, Osaka, Japan
** Mechanical Engineering Research Laboratory, Kobe Steel Ltd., Kobe, Hyogo, Japan
*** Structural Engineering Laboratory, Kobe Steel Ltd., Amagasaki, Hyogo, Japan

ABSTRACT

In the previous report, the brittle fracture initiation characteristics of plate subjected to bi-axial tensile load were investigated in the elastic state and small scale yielding state by using PMMA (Poly-methylmethacrylate) and SM41 (Mild steel), respectively.

In this report, the brittle fracture initiation characteristics with large scale yielding are investigated. A series of fracture tests is conducted on the cruciform specimens of SM41 under bi-axial tensile load, of which load ratios are 0/1, 1/2 and 1/1. Elastic-plastic stress analysis by the finite element method is also carried out assuming the plane stress condition.

The results obtained by the experiments and the finite element analysis lead to the following conclusions:

(1) The direction of fracture initiation is nearly normal to the vector of crack opening displacement (COD) near the tip of notch obtained by the finite element analysis.

(2) Brittle fracture stress under mixed mode conditions may be conservatively predicted by the critical COD for the opening mode fracture based on the application of the COD-concept.

KEYWORDS

Bi-axial load; Brittle fracture; Combined modes; Mild steel

INTRODUCTION

In the preceding paper (Ueda and colleagues, 1977) the initiation characteristics of brittle fracture under bi-axial stress were investigated in the elastic state and small scale yielding state. In the case of perfectly brittle fracture, the fracture stress and direction of fracture initiation are well predicted by the criteria based on $G_{\theta,max}$ (Anderson and colleagues, 1971). In the case of brittle fracture with small scale yielding, the direction of fracture initiation is well predicted by the criterion of $G_{\theta,max}$, but the predicted fracture stresses are somewhat lower than the test results.

In this report, brittle fracture under combined modes of MODE I (Opening) and MODE II (Sliding) is investigated in not only the large scale yielding atate, but also in the general yielding state.

EXPERIMENT

Material and Test Specimen

The mechanical properties and the chemical compositions of the mild steel, SM41, tested are represented in Table 1. The specimen is of a cruciform type, containing a center notch which is inclined at an angle, β, to the vertical direction, and is machined through the thickness of the plate. The details of the test specimen are shown in Fig. 1.

TABLE 1 Mechanical Properties and Chemical Compositions

Material	Mechanical properties			Chemical composition (%)				
	σ_s (kg/mm^2)	σ_B (kg/mm^2)	EL. (%)	C	Si	Mn	P	S
SM 41	41	50	26	0.14	0.25	0.86	0.028	0.014

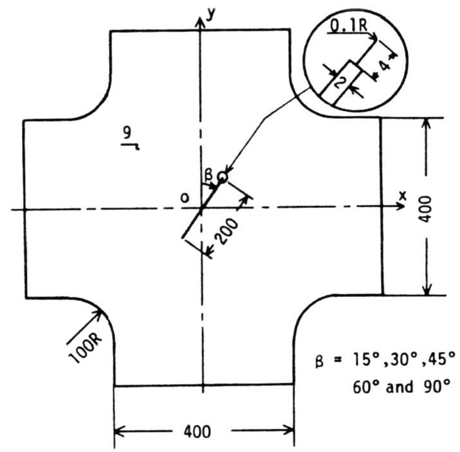

Fig. 1 Test specimen

Test Procedure

A series of fracture tests is conducted in the temperature range between -60 and -90°C. The specimens are fractured in a brittle manner, being subjected to large scale yielding or general yielding. A vertical testing machine of 300 tons is used, in combination with a specially designed horizontal testing machine of 150 tons which can move in accordance with the vertical displacement of the specimen. The loads are applied proportionally in two directions.

In the test, the direction of propagation of the initial crack and the fracture load are measured. The crack opening displacement, COD, is also measured by clip-gauges during loading. Two clip-gauges are attached to the specimen between A and B, and A and C in Fig. 2. Small circular holes are machined at points A, B and C, where the clip-gauges are attached. Point A is 30 mm from the tip of the notch. COD under a mixed mode condition can be separated into two components, V_I and V_{II}, which correspond to those for MODE I and MODE II, respectively. Assuming that the distance between B and C does not change, each COD is graphically obtained by measuring the distances $\overline{BA'}$ and $\overline{CA'}$ in Fig. 2.

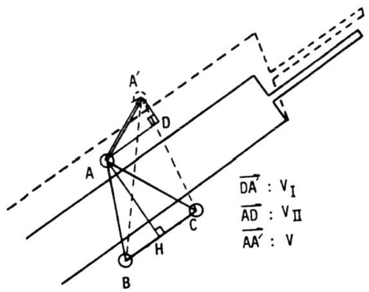

Fig. 2 Measurement of COD under mixed mode

Deep notch tests are also carried out in the temperature range between -60° and -90°C, and the fracture toughness (Critical crack opening displacement, Φ_c) is evaluated from these test results in addition to the previous ones (Ueda and colleagues, 1977).

ELASTO-PLASTIC STRESS ANALYSIS

Finite Element Analysis

A series of elasto-plastic stress analysis by the finite element method is carried out to obtain the crack opening displacement for the plane stress condition employing the constant strain element. Different finite element meshes are employed according to the notch inclination angle, β. However, mesh size near the tip of the notch is kept the same regardless of the β and bi-axial load ratio, P_x/P_y, and the meshes for analysis are shown in Fig. 3. The accuracy of the calculated COD depends on the mesh size especially near the tip of the notch. It is confirmed that the mesh size shown in Fig. 3 is fine enough to secure the accurate COD, since the results is scarcely affected by decreasing the mesh sizes more than those shown in Fig. 3. In the analysis, the substructure method is employed to save computer time and labor for generation of the input data. The analysis is performed on the test specimens for all possible combinations of the notch inclination angle (β = 15°, 30°, 45°, 60° and 90°) and the bi-axial load ratio (P_x/P_y = 0/1, 1/2 and 1/1). Bi-axial loads are applied proportionally in both directions. Young's modulus, E, strain hardening rate, H', and the yield stress, σ_s, are assumed to be 21,000 kg/mm^2, 210 kg/mm^2 and 30 kg/mm^2, respectively, and von Mises's yield condition is employed.

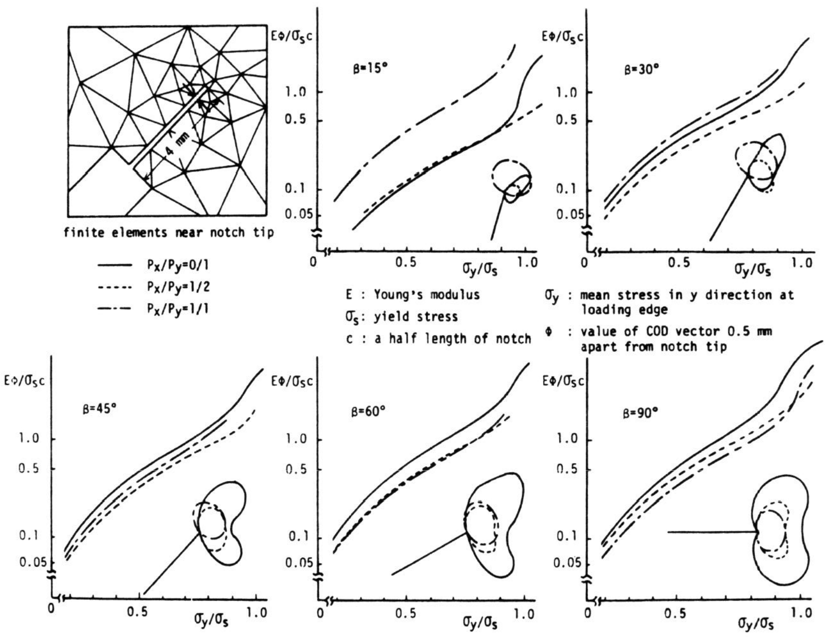

Fig. 3 Mean stress - COD curves and spread of plastic zone (σ_y/σ_s = 0.75)

Elastic-plastic Behavior of the Specimen

The calculated mean stress - COD curves and the plastic zone size for all cases are shown in Fig. 3. The mean stress, σ_y, is non-dimensionalized by the yield stress, σ_S, and COD, Φ, by σ_S, E, and half crack length, c (Ueda and colleagues, 1979). Crack opening displacement, Φ, under mixed mode condition is defined as the vector which is composed of the displacement in MODE I, Φ_I, and that in MODE II, Φ_{II}, at a distance of 0.5 mm from the tip of the notch. This compound COD vector inclines to the notch line by an angle of $\tan^{-1}(\Phi_I/\Phi_{II})$ and its magnitude is equal to $\sqrt{\Phi_I^2+\Phi_{II}^2}$. For all notch angles, spreads of plastic zone and $\Phi E/(\sigma_S c) - \sigma_y/\sigma_S$ curves are almost the same when $P_x/P_y = 1/1$. However, when $\beta \neq 90°$ or $P_x/P_y \neq 1/1$, the development of the plastic zone from the tip of the notch is influenced by β and P_x/P_y. Consequently, the crack opening behavior is quite different in the individual cases. Based on the results of FEM analysis, some specimens are fractured with large scale yielding, and the other after general yielding. Figure 4 shows the spreads of plastic zones at fracture when $\beta = 60°$.

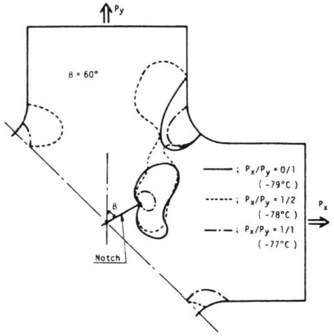

Fig. 4 Spread of plastic zone at fracture ($\beta = 60°$)

Mean Stresses around a Notch

The mean stresses σ_{xe} and σ_{ye} around the notch are in general different from σ_x and σ_y along the loading edges in the case of the cruciform specimen. Figure 5 shows the calculated stress distributions in the specimen when $\beta = 60°$ and $P_x/P_y = 1/2$. In this report, σ_{xe} and σ_{ye} are defined as the mean stresses between points f and f', and g and g', respectively, which can be obtained either by a theoretical analysis such as FEM or by measurement of elastic strains (refer to the written discussion on Ref., (Ueda and colliagues, 1979)). In the case of a notch being 160 mm long, as reported in the previous paper (Ueda and colleagues, 1977), the stress ratios, σ_{xe}/σ_x and σ_{ye}/σ_y, were the same regardless of the notch angle, β, when the specimen is in the elastic state. However, in the case of a notch being 200 mm long in this study, these stress ratios vary somewhat according to the notch angle, β, and the bi-axial load ratio, P_x/P_y. Figure 6 shows such variation, and it is observed that the stress ratios increase slightly with σ_y when $\sigma_y/\sigma_S > 0.5$. The stress ratio, σ_{xe}/σ_x, has the same tendency with σ_x, but it is confirmed that the stress ratio, σ_{xe}/σ_{ye}, does not change even when σ_x/σ_S or σ_y/σ_S is greater than 0.5. Consequently, it may be said that σ_{xe}/σ_{ye} is -0.09, 0.41 and 1.0 when P_x/P_y is 0/1, 1/2 and 1/1, respectively, which are the same values as those in the previous report (Ueda and colleagues, 1977).

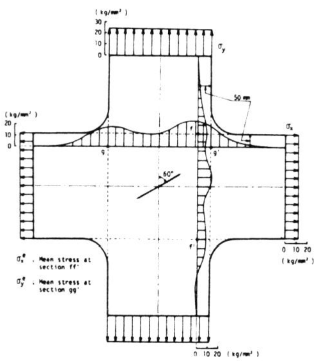

Fig. 5 Stress distribution in the cruciform specimen calculated by FEM ($\beta = 60°$ and $P_x/P_y = 1/2$)

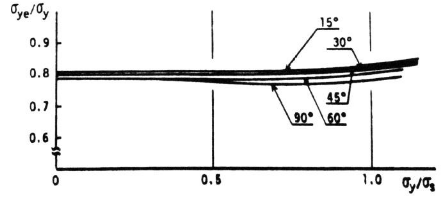

Fig. 6 Field stress versus applied stress (y-direction, $P_x/P_y = 0/1$)

TEST RESULTS AND DISCUSSION

Comparison between Observed and Theoretically Calculated CODs

COD was measured at a position 30 mm from the tip of the notch for several cases where the load ratio, P_x/P_y, and the notch angle, β, are changed. For the case of $\beta = 30°$ at $P_x/P_y = 0/1$, the observed and calculated CODs are shown against the applied load in Fig. 7. As the load exceeds about 100 tons, the experimental COD becomes greater than the calculated one. If there is short slow crack growth at the actual notch, this might increase the value of the COD. Taking this into account, the measured COD is considered to agree with the calculated one. Anyhow, it is difficult to measure COD under mixed mode loading in the test and it is complicated to convert the measured COD to that at the notch tip. In the following discussion, the calculated COD vector at the position 0.5 mm from the tip of the notch is regarded as that at the tip of the notch as shown in Fig. 3.

Direction of Crack Initiation

The relationship between the notch angle, β, and the direction of propagation of the crack with respect to the initial notch angle, $(-\theta)$ is shown in Figs. 8(a), (b) and (c) for load ratios, $P_x/P_y = 0/1$, 1/2 and 1/1, respectively. In the case of $\beta = 15°$, 30° and 45° at $P_x/P_y = 0/1$, the fracture initiates in a direction of $\beta + (-\theta) = 45°$, that is, direction of the maximum shear stress in a plate without a notch. The fracture is normal to the plate surface and shows shear appearance of elongated dimple. Hereafter, this type of fracture is referred to as shear fracture. The solid lines in Fig. 8 show the direction of propagation of the crack estimated from the assumption that the fracture initiates

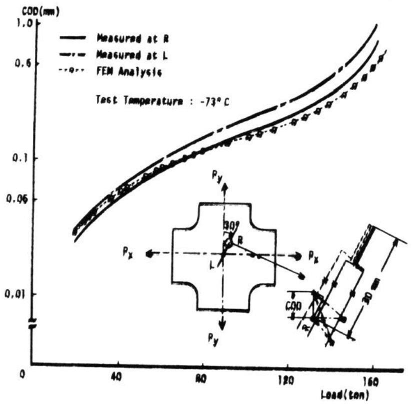

Fig. 7 Comparison of measured COD with calculated one

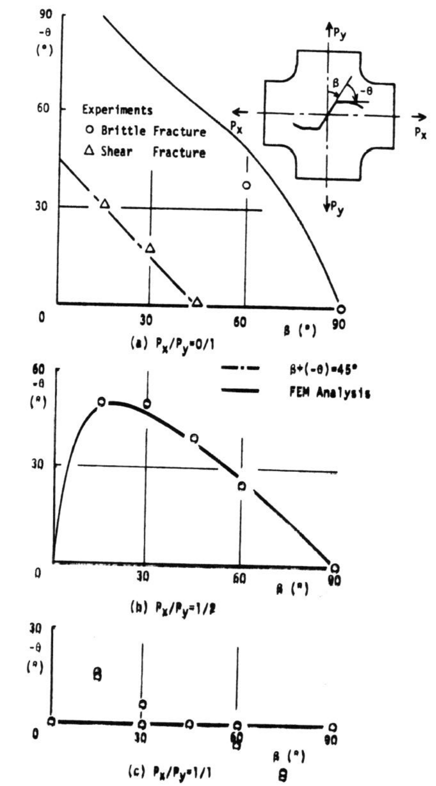

Fig. 8 Direction of crack propagation

in the direction perpendicular to the COD vector obtained from the elasto-plastic analysis by FEM. The estimated line for $P_x/P_y = 1/1$ coincides with the horizontal axis in Fig. 8.

The estimated directions show good agreement with the observed ones except in the case of the shear fracture. This agreement may be attributed to the fact that the COD vector at the tip of the notch is closely related to the vector of the stretched zone depth formed prior to the fracture, and that the fracture initiates in the direction perpendicular to the stretched zone (Ohtsuka and colleagues, 1974).

In contrast with this, the maximum energy release rate criterion, $G_{\theta,max}$, and the COD vector criterion can be applied for prediction of the direction of the fracture initiation from a notch with small scale yielding under mixed mode loading. The direction which is predicted by the former criterion, that is, $G_{\theta,max}$, shows better agreement with the observed direction than that by the latter COD criterion. In the above two cases, the extent and size of the yielded zone at the tip of the notch under mixed mode loading are different. Judging from the result of comparison, it is recognized that each criterion should be applied to an appropriate case. From this conclusion, the direction of the fracture initiation from the notch with small scale yielding should be predicted by the $G_{\theta,max}$ criterion, based on energy balance. On the other hand, in the case of being accompanied with large scale yielding, the direction can be predicted by the COD criterion, based on deformation at the tip of the notch.

Moreover, when the specimen is subjected to pure MODE II in the same temperature range at which brittle fracture of pure MODE I with large scale yielding or general yielding occurs, the COD vector becomes parallel to the notch. Therefore, the direction of fracture initiation can be estimated to be perpendicular to the notch. However, in fact, the shear crack initiates from the tip of the notch prior to complete fracture. Therefore, the behavior to fracture is anticipated to be more complicated.

Fracture Stress

Variation of the fracture stress for different notch angles is shown in Fig. 9. The vertical coordinate is the ratio of the effective fracture stress in the y-direction at the central part of the specimen, $\sigma_{ye,f}$, to that for $\beta = 90°$ and $P_x/P_y = 0/1$. As explained above, the fracture for $\beta = 15°$, $30°$ and $45°$ under $P_x/P_y = 0/1$ was the shear type, and the maximum stress is indicated, which does not correspond to the fracture load. (The test specimen did not fracture completely.) While the fracture stress for $P_x/P_y = 0/1$ and $1/2$ increases with a decrease in β, that for $P_x/P_y = 1/1$ is kept nearly constant irrespective of β. The lines in Fig. 9 denote the estimated fracture stresses based on the COD criterion which will be discussed in the following section. The experimental fracture stresses under pure MODE I, that is, for $P_x/P_y = 1/1$ of $\beta = 90°$, are plotted in the neighborhood of the estimated line. On the other hand, those under various mixed mode conditions, which are, for $\beta \neq 90°$, and $P_x/P_y = 0/1$ or $1/2$, scatter a little above the estimated lines.

Application of COD Criterion to Mixed Mode Conditions

The COD concept has been proposed as a fracture criterion especially in the field of the fracture with large scale yielding or general yielding under pure MODE I and pure MODE III conditions, and the applicability of that concept has been verified. In parallel with this, the strip yield model has been proposed as a elasto-plastic crack model for calculation of COD by Dugdale (1960) and Bilby and

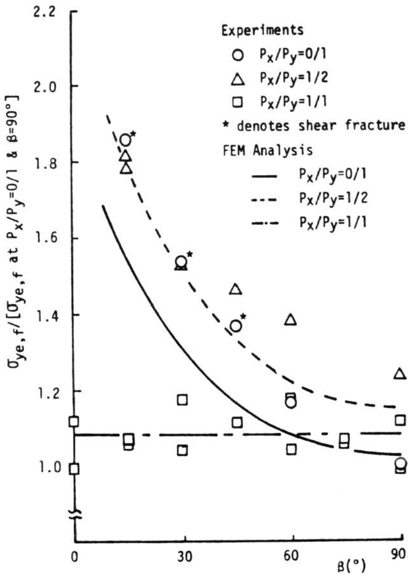

Fig. 9 Variation of fracture stress with notch angle

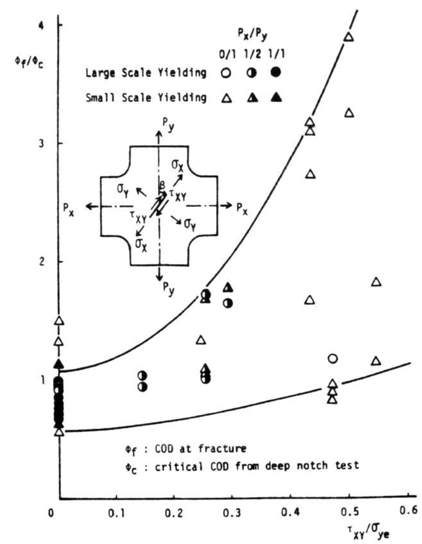

Fig. 10 Variation of COD at fracture with shear stress parallel to notch

colleagues (1963). Moreover, even the strip yield model under mixed mode loading is proposed by Sakai, Iino and Sakano (1976). In these strip yield models, COD can be determined from the cancellation of stress singularity at the tip of the plastic region on the prolonged line of the notch. Therefore, since stress singularity does not occur under uniform tension applied parallel to the notch, such as σ_x in the case of being $\beta = 90°$, the size of the plastic region is not dependent upon σ_x. However, as shown in Fig. 3, the actual size of the plastic zone is significantly influenced by the magnitude of σ_x, and so is COD. Thus, the size of the plastic zone in the strip yield model is not actual but conceptual, and this indicates a reasonable value only for the limiting case of being $\beta = 90°$ at $P_x/P_y = 0/1$. Essentially, the size and extending direction of the plastic zone should be related closely to COD. However, it is very difficult to take into account the respective components of stress in the dynamical model of COD proposed so far. To this point, COD under mixed mode conditions was analyzed by FEM, and the applicability of the COD concept was investigated. As reported previously, the test results of the brittle fracture with small scale yielding under the mixed mode conditions revealed that the critical fracture parameter, G_{cr}, tends to increase with an increase in K_{II}, which is the stress intensity factor corresponding to MODE II. From the results of experiment and analysis by FEM, the ratio of CODs, Φ_f/Φ_c, is plotted against the non-dimensionalized shear stress τ_{xy}/σ_{ye}, in Fig. 10. Here, Φ_f denotes the COD at fracture, which is determined from the relation shown in Fig. 3, with the introduction of fracture stress, and Φ_c denotes the critical COD at the corresponding test temperature from the deep notch test. In this figure, the result of fracture with small scall yielding by the previous test is also shown. This figure indicates that Φ_f increases with an increase of the shear stress in the same manner as G_{cr}. Therefore, as shown in Fig. 9, the actual fracture stresses under the mixed mode loading are higher than the estimated ones which are calculated based on the COD criterion. That is, the component of COD corresponding to MODE II is not so influential on brittle fracture of elastic-plastic material as that which corresponds to MODE I. This problem should be studied also from the microscopic aspect in the future. However, as the estimated fracture strength under mixed mode loading is conservative, such estimation by the COD concept is effective from the practial viewpoint.

CONCLUSIONS

The brittle fracture tests of the cruciform specimen of mild steel with an inclined notch were conducted, applying bi-axial tension at such temperatures as the double edge notched specimen or the deep notch test specimen fractures with large scale yielding. The results of the fracture with small scale yielding by the previous test as well as those by the present one were compared with the elastic-plastic behaviors of the specimens analyzed by FEM.

The following conclusions were drawn:

(1) The fracture with small scale yielding under mixed mode loading initiates in the direction in which the energy release rate, G_θ, becomes maximum, and that with large scale yielding or general yielding initiates perpendicularly to the COD vector analyzed by FEM.

(2) The COD at the fracture, Φ_f, calculated by FEM increases with an increase of the shear stress parallel to the notch in the same manner as G_θ at the fracture with small scale yielding, G_{cr}, apparently increases with an increase of the stress intensity factor corresponding to MODE II, K_{II}.

(3) The brittle fracture strength under mixed mode loading can be estimated conservatively on the basis of the COD criterion, in which the COD as a function of the load is analyzed by FEM and the critical COD is obtained from the deep notch test. Therefore, such estimation is considered to be effective from a viewpoint of practical design.

REFERENCES

Anderson, G.P., V.L. Ruggles, and G. Stibor (1971). Use of finite element computer programs in fracture mechanics, Int. J. Frac. Mech., Vol. 7, 63-76.

Bilby, B.A., A.H. Cottrell, and K.H. Swinden (1964). The spread of plastic yield from a notch, Proc. Roy. Soc., A279, 304-314.

Dugdale, D.S. (1960). Yielding of steel sheets containing slits, J. Mech. Phys. Solids, 8, 100-104.

Ohtsuka, A., and colleagues (1974). The stretched zone and COD criterion on the fracture, J. Soc. Naval Arch. of Japan, 136, 249-257.

Sakai, K., N. Iino, and K. Sakano (1977). In G.C. Sih and C.L. Chow (Ed.), Fracture Mechanics and Technology, Vol. II, Sijthoff and Noordhff, Netherlands, Sec. XII, 907-916.

Ueda, Y., and colleagues (1977). In D.M.R. Taplin (Ed.), Fracture 1977, Vol. II, Univ. of Waterloo, Canada, Part I, 173-181.

Ueda, Y., and colleagues (1979). Characteristics of brittle fracture under bi-axial tensile load (2nd report), Trans. JWRI, Vol. 8, 121-130.