ANALYSIS OF STEADY STATE CRACK GROWTH BY DISCRETE DISLOCATION THEORY

H. Homma*, T. Matsui**, and H. Nakazawa***

* School of Energy Engineering, Toyohashi Univ. of Technology, Toyohashi, Japan
** Mitsubishi Motors Co. LTD. Kawasaki, Japan
*** Dept. of Physical Engng., Tokyo Institute of Technology, Tokyo, Japan

ABSTRACT

A dislocation model is developed to analyse plastic behavior of a growing crack. The plastic zone size and the crack tip opening displacement for the steadily propagating cracks are computed. The plastic zone size for a steadily propagating crack is around 80 percents of that for a stationary crack loaded by the nominally equal stress intensity factor. The opening displacement for the steadily propagating crack is about one fourth as small as that for the stationary crack. Secondary yield zones formed near the newly-created crack surfaces appear. This model can be applied to crack propagation events under the other modes, cyclic loading, monotonically increasing loads and so on.

KEYWORDS

Dislocation model; steadily propagating crack; plastic zone size; crack tip opening displacement; secondary yield zone.

INTRODUCTION

It is essential for understanding crack propagation events to clarify elastic-plastic behavior of material near the crack tip. However, elastic-plastic analysis near a growing crack tip have hardly been made.

A crack in metallic materials grows in the plastic zone ahead of its tip. Material element which was at the crack tip before the crack grew is unloaded in elastic manner and material element which was ahead of the crack tip is deformed further, after the crack propagates. Therefore, size of plastic zone and crack tip opening displacement for a growing crack may be fairly different from those for a stationary crack under the equivalent load.

Chitaley and McClintock(1971) computed the size of plastic zone and the crack tip opening displacement for a steady state propagating crack subjected to mode III loading in an elastic-perfectly plastic material. They indicated that the size of plastic zone and the crack tip opening displacement for the propagating crack are respectively four ninth and one fourteenth in comparison with those for a stationary crack with the identical length.

Anderson(1974) calculated behavior of a steady-state propagating crack under mode I loading in various work-hardening materials by use of finite element method. For almost all fracture problems, mode I crack propagation is a major factor. In fatigue crack propagation and stable slow crack growth observed in plane stress fracture, behavior of these cracks can not be completely understood without the knowledge of plastic zone size and opening displacement of a growing crack in elastic-plastic materials.

In the present paper, a steadily growing crack under mode I loading is analysed by means of a model of linear discrete dislocation arrays. A dislocation model was successfully used to calculate plastic relaxation of a stationary crack subjected to anti-plane shear and tension by Bilby et al(1963,1964). In the model, both of plastic zone and a crack were represented by linear dislocation arrays collinear with the crack itself. On the other hand, Atkinson and Kanninen (1977), Vitek(1976) and Miyamoto and Kageyama(1977) analysed the stress distribution near an edge dislocation oriented in a given direction in a cracked infinite plate using complex stress function. They and also Riedel(1976) applied the solution to the problem of the plastic relaxation of a stationary crack with inclined slip lines ahead of its tip, a model first used by Bilby and Swinden(1965). In the present paper, the solution of stress for an edge dislocation near a crack given by Vitek(1976) is utilized.

DISLOCATION MODEL FOR STEADY CRACK PROPAGATION

Foundation of The Dislocation Model

It is postulated that when a crack is remotely loaded under plane strain conditions, a pair of the inclined slip lines on which density of edge dislocations continuously varies, represents plasticity of the crack tip as shown in Fig.1. When material is perfectly plastic, resolved resultant shear stress on the lines is equal to shear yield strength of the material. The resultant shear stress is the sum of shear stress due to the crack alone under the applied load and that due to interaction between the crack and the dislocations. We have the following equation for equilibrium.

$$\tau_{ys} = \tau^{c}(y) + \sum_{k=1}^{4} \int_{0}^{y_p} D(y_k)\tau^{d}(y,y_k)dy_k \qquad (1)$$

where τ_{ys} is the yield strength of the material, $\tau^{c}(y)$ is shear stress at y due to the crack, $D(y_k)$ is density of the edge dislocation at y_k, $\tau^{d}(y,y_k)$ is shear stress at y due to the dislocation with an unit Burger's vector at y_k. Vitek(1976) solved numerically the above equation by replacing the continuous distribution of the dislocation density by discrete dislocations with Burger's vector b_{jk} placed at equal intervals. The above equation is rewriten for equilibrium on the slip line l as follows:

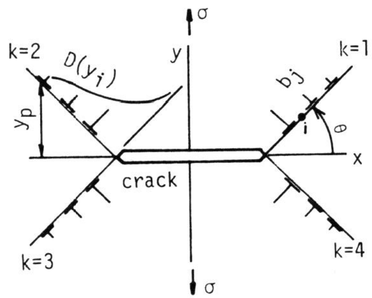


Fig.1. Dislocation model for stationary crack tip plasticity

$$\tau_{ys} = \tau^{c}(y_i) + \sum_{k=1}^{4} \sum_{j=1}^{N} b_{jk}\, \tau^{d}(y_i,y_{jk}), \quad i=1,\cdots,N \qquad (2)$$

where $\tau^{c}(y_i)$ is shear stress at point i due to crack, b_{jk} is magnitude of Burger's vector of a dislocation to be placed at a point j on the slip line k and $\tau^{d}(y_i,y_{jk})$ is shear stress at a point i due to a dislocation with an unit Burger's vector placed at a point j on the slip line k. For symmetry, magnitudes of Burger's vector at identical points on the every slip line are the same. So, we can drop

the subscript k from b_{jk}. These are linear simultaneous equations with unknown variables b_j. Point i, where the shear stress is evaluated, is placed midway between the distributed dislocations as used by Miyamoto and Kageyama(1977). Crack tip opening displacement can be calculated by

$$v = 2.0\, \sin\theta \sum_{j=1}^{N} b_j \qquad (3)$$

and plastic zone can be judged by the condition that

$$\tau_{ys} > \tau^{c}(y_{N+1}) + \sum_{k=1}^{4} \sum_{j=1}^{N} b_j\, \tau^{d}(y_{N+1},y_{jk}). \qquad (4)$$

Model for Steady crack propagation

When a crack grows under a constant load in an infinite plate, element of material at the crack tip is unloaded in an elastic manner. Stress-strain history of the element as the crack approaches it and passes through it is shown in Fig.2. It should be noted that plastic strain is kept invariable in the unloading process.

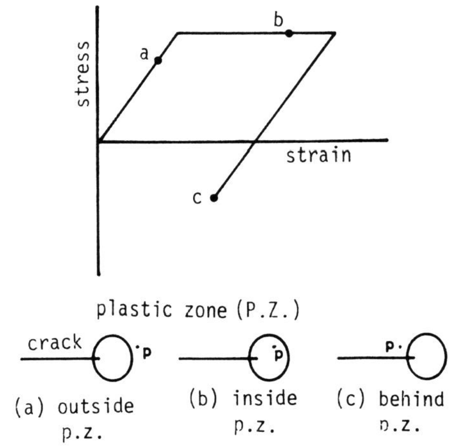


Fig.2. Stress-strain history experienced by an element near a propagating crack.

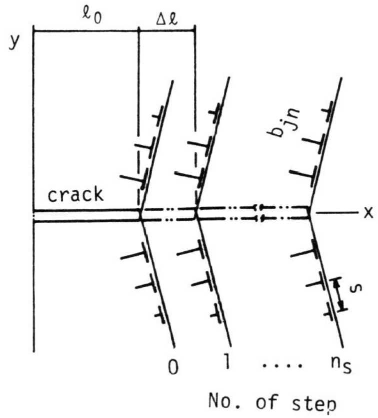


Fig.3. Model for plasticity of a propagating crack tip

Since Burger's vector represents plastic deformation, dislocations formerly distributed at slip lines should not be redistributed after the crack grows, providing that the resultant shear stress on the slip lines behind the current crack tip is between the yield strength on tension and that on compression. Governing equation for a propagating crack based on Eq(2) is,

$$\tau_{ys} = \tau^{c}(y_i,\ell_{ns}) + \sum_{n=0}^{ns} \sum_{k=1}^{4} \sum_{j=1}^{N} b_{jn}\, \tau^{d}(y_i,y_{jk},\ell_n), \quad i=1,\cdots,N. \qquad (5)$$

where $\tau^{c}(y_i,\ell_{ns})$ is the shear stress at a point i on the active slip line for the current crack of length ℓ_{ns} due to the crack itself, b_{jn} is Burger's vector distributed at a point j on the slip line at the tip of the crack of which length was ℓ_n, $\tau^{d}(y_i,y_{jk},\ell_n)$ is the shear stress at a point i on the active slip line for the current crack due to a dislocation with unit Burger's vector placed at a point j on the slip line k at the crack tip of its length ℓ_n. In the above equation, unkown variables are $b_{jns}(j=1,\cdots,N)$. This situation is shown in Fig.3.

COMPUTATION RESULTS

Computation Parameters

Since it is postulated that a crack propagates in high strength steel, the computation is made using the following parameters;

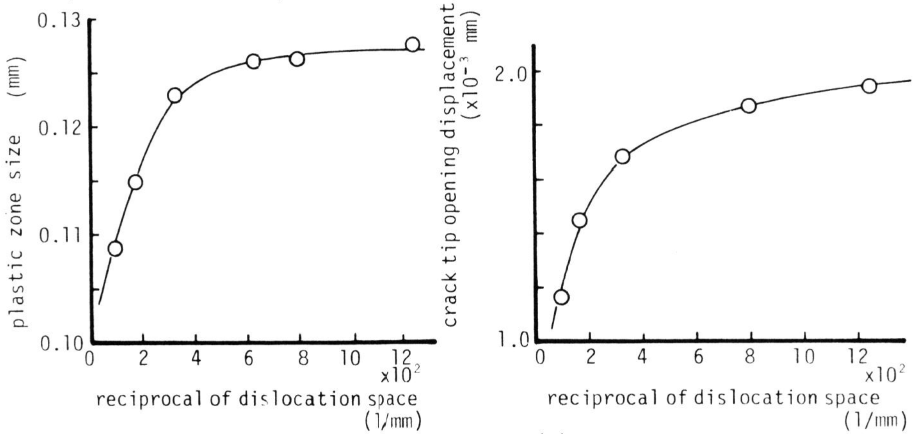
Shear modulus: 79380 MNm^{-2}; Poisson ratio: 0.3

Shear yield strength: 490 MNm^{-2}, Angle of slip line against x axis: 90°

The model used in this analysis has two arbitrary parameters. One of them is the space between the dislocations and another is the crack growth amount at each propagation step as shown in Fig.3. It can be, however, expected that behavior of crack tip plasticity would converge to that of a steadily and continuously propagating crack as their values approach zero.

Plasticity of Stationary Crack

First, plastic zone size and crack tip opening displacement of a stationary crack with length of 4 mm in an infinite plate loaded by uniform tension of 294 MNm^{-2} are computed for various spaces of dislocations. The results are shown in Fig.4.



(a) plastic zone size (b) crack tip opening displacement

Fig.4. Dependency of plastic zone and crack tip opening displacement on dislocation space.

Plastic zone size rapidly increases as the space of dislocations decreases and approaches a converged value of 1.27×10^{-1}mm for the space below 1.58×10^{-3}mm. The opening displacement also increases with decrease of the space of dislocations, but the rate is moderate. It still varies with the space below 1.58×10^{-3}mm to approach a converged value of 1.95×10^{-3}mm. It can be seen from the results in Fig.4 that the plastic zone size is insensitive to the dislocation space while the crack tip opening displacement is sensitive. Magnitudes of Burger's vectors of the dislocations arrayed at intervals of 3.37×10^{-3} mm on a slip line are shown in Fig.6.

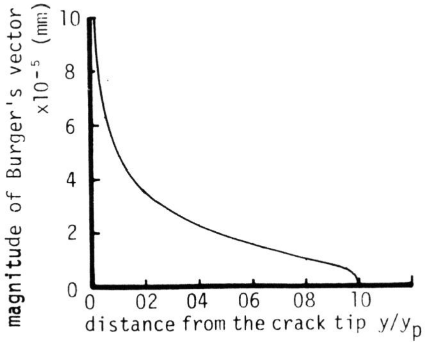


Fig.5. Distribution of magnitudes of Burger's vector possessed by dislocations.

A dislocation nearest the crack tip has the greatest Burger's vector. This fact corresponds to that the dislocation density at the crack tip is infinite in BCS model(Bilby et al, 1963,1964). Since edge dislocations have a function depressing the stress at points behind them regarding the direction of their Burger's vectors, a dislocation with the bigger Burger's vector must be placed at the point closer the crack tip as shown in Fig.5.

Plasticity of Steadily Propagating Crack

To examine the effect of a crack growth amount $\Delta\ell$ on the crack tip opening displacement, calculation is performed for various values of the crack growth amount from a half to two times of the crack tip opening displacement for the initial crack. Loading condition is the same as that for the stationary crack. In Fig.6, crack tip opening displacement values for a propagating crack are plotted against the growing steps for various dislocation spaces. The amounts crack grows at each step are chosen to have the following three values: 5.0×10^{-4}mm, 1.0×10^{-3}mm and 3.7×10^{-3}mm.

As seen in the figures, the crack tip opening displacement value drops considerably after a crack has grown at the first step and then, gradually increases with the following steps to converge to a constant value for each of the dislocation spaces. Drop of the crack opening displacement at the first step is bigger for the longer dislocation spacing and for the smaller crack growth step. Dependency of the drop on the crack growth step can be seen from the fact that the initial slip lines become closer to the slip lines in the second step as the crack growth amount becomes shorter, and therefore, dislocations on the first-step slip lines more significantly affect the distribution of the dislocation on the second-step slip lines because stress components around a dislocation are reduced as a function of the reciprocal of the distance from the dislocation. Also, the dependency of the drop in crack tip opening displacement value on the dislocation space can be explained from the fact that on the initial slip lines, dislocations having bigger Burger's vector are located near the crack tip for the longer dislocation spacing, and the

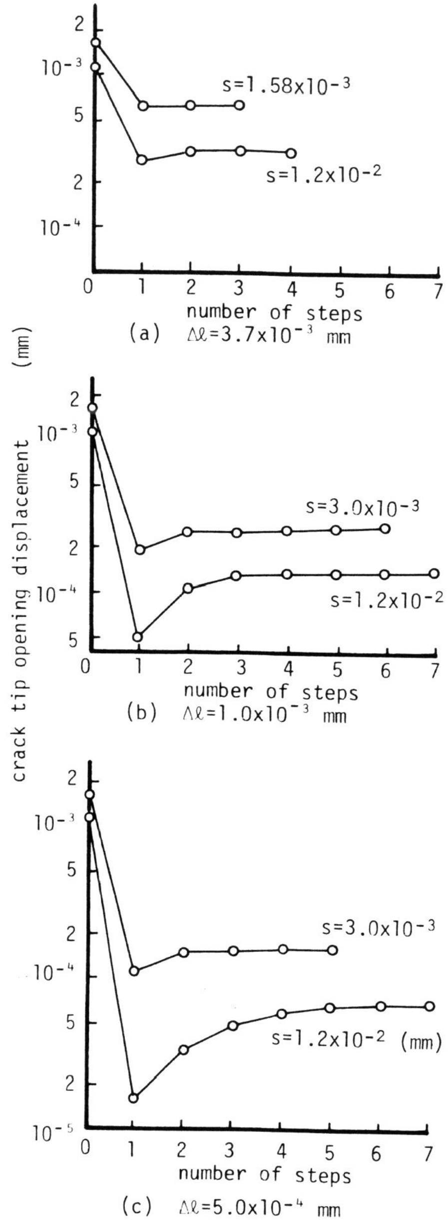


Fig.6. Dependency of crack tip opening dispacement value during crack propagation on dislocation space and crack growth amount.

dislocations reduce considerably the stress on the second-step slip lines near the crack tip.

It is noticed that a steady state value of crack tip opening displacement for a propagating crack takes place earlier for the longer crack growth amount and the shorter dislocation space and the crack tip opening displacement value in the steady state is dependent on the these parameters.

Their effects on the crack tip opening displacement are now considered in more detail. In Fig.7, the crack tip opening displacement values in the steady state for four crack growth amounts are plotted against the reciprocal of the number of dislocations placed on the slip line corresponding to the dislocation space. These gradually increase with reduction of the dislocation space. Except the result for the crack growth amount of 3.7×10^{-3}mm, the results for three crack growth amounts focus at 5.3×10^{-4} mm when the dislocation space approaches zero.

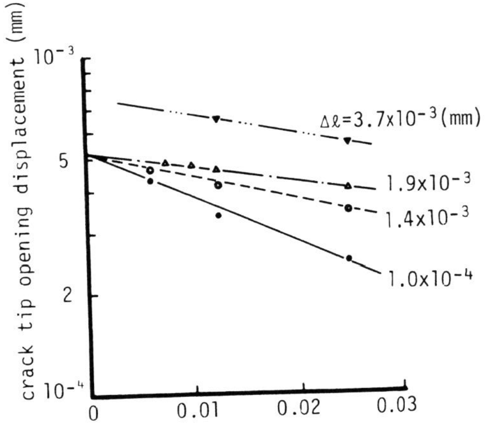


Fig.7. Crack tip opening displacement plotted agaist the number of arrayed dislocations

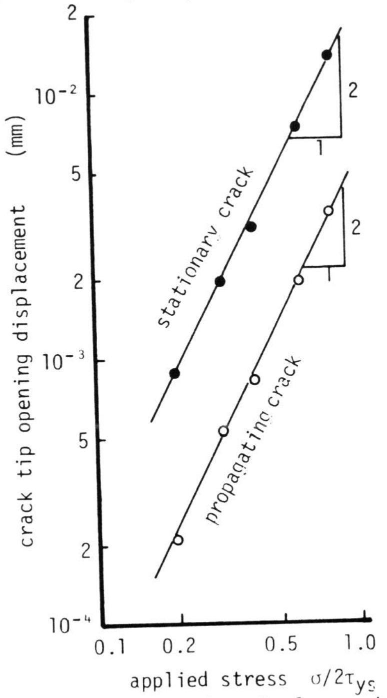


Fig.8. Crack tip opening displacement as a function of applied stress.

TABLE 1 Ratios of CTOD values and plastic zone size for the both cracks

$\sigma/2\tau_{ys}$	$(CTOD)_{pc}/(CTOD)_{sc}$	$(PZS)_{pc}/(PZS)_{sc}$
0.2	0.24	0.70
0.3	0.27	0.87
0.4	0.26	0.77
0.6	0.26	0.80
0.8	0.26	0.86

We can deduce from this result that the crack tip opening displacement for the steadily propagating crack under uniform tension of 294 MNm^{-2} may be 5.3×10^{-4}mm, which is 27 percent of that for a stationary crack loaded to a nominally equal stress intensity factor. In Fig.8, the steady crack tip opening displacement values for propagating cracks are shown as a function of applied stress normalised by twice of the shear yield strength. In the figure, the crack tip opening displacement values for the corresponding stationary crack are also indicated. Ratios of the crack tip opening displacement values for the both cracks are listed in Table 1. The ratios are around 0.26 for all applied stresses. Ratios of the plastic zone size for the both cracks are also given in the third column. They are about 0.83.

DISCUSSION

We will compare the results computed in this analysis with the results which Anderson(1974) obtained using a finite element method. He calculated the opening displacement of crack surface behind the crack tip by one mesh for the steadily propagating crack. That was about 60 percent of one for a stationary crack under nominally equal load intensity in contrast to 26 percent in this analysis. However, as mentioned by himself in his paper, it does not seem that his results fully converge regarding the mesh size. Additionally, it should be noticed that it is very difficut for the crack tip opening displacement to be expressed by a finite element analysis using constant strain elements.

On the other hand, plastic zone shapes in two dimensions around a growing crack tip were accurately obtained by the finite element analysis while in this analysis, they are represented by two lines symmetrically emanating from a crack tip. It is impossible directly to compare the plastic zone shapes analysed by the both methods. The maximum size of plastic zone perpendicular to the crack line will be considered because the slip lines are normal to the crack line in this analysis.

According to the finite element results, the plastic zone size for a steadily growing crack was around 80 percent of that for a stationary crack under the nomially equal load. This is very similar to those listed in Table 1. It should be mentioned that the relationship between the crack tip opening displacement and the plastic zone size for a stationary crack can not be directly applied to the case of a propagating crack.

As stated in the section on the model for steady crack propagation, dislocations placed on slip lines at the previous crack tip are not redistributed through the crack propagation process for the sake of reducing the computation time. In order to examine the propriety of the simplification in the computation, the shear stress distribution on the slip lines behind the current crack tip are calculated and are shown in Fig.9. The stress distributions are results on the slip line at the initial crack tip and the fourth-step slip line after the crack propagated by seven steps under tensile stress of 196 MNm^{-2}. The stress is beyond the shear strength in the range of y/y_p from 0.2 to 1.0. It is a reason for the result that the shear stress component near the crack tip loaded by uniform tension is larger at θ above 90° than at θ=90° when the distance from the crack tip is identical.

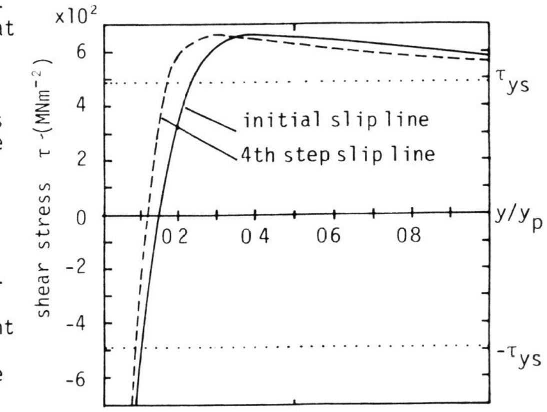


Fig.9. Stress distribution on the slip lines behind the crack tip after 7 steps.

The crack tip opening displacement obtained in this analysis may be conservative because redistribution of the dislocations is not performed.

It should be noticed from Fig.9 that a secondary compressive yield zone may be formed near the newly-created crack surfaces as in the results calculated by Anderson(1974) and by Chitaly and McClintock(1971). Although Anderson suggested that the existence of the secondary yield zone probably depends on the way in which the relaxation near the crack tip is performed, it seems from the present work that this may be inherent.

CONCLUSIONS

A dislocation model is developed to analyse plastic behavior of a propagating crack. Plasticity of the crack tip is represented by inclined slip lines at the crack tip and descrete dislocations on the lines. The plastic zone size and the crack tip opening displacement for the steadily propagating cracks in high strength steel plate are computed. Computed results depend on the dislocation spacing and the amount of crack growth at each step. When values of these parameters approach zero, the plastic zone size and the crack tip displacement converge to constant values. The following conclusions are deduced:

(1) The plastic zone size for a steadily propagating crack is around 80 percent of that for a stationary crack loaced by nominally equal stress intensity factors.
(2) The opening displacement for the steadily propagating crack is about 26 percent of that for the stationary crack.
(3) Secondary yield zones formed near the newly-created crack surfaces appear as in the results obtained by the other researchers.

The model developed here can be applied to calculate plasticity of a crack propagating in other modes, under cyclic loading, under monotonically increasing load and so on.

ACKNOWLEDGEMENT

The authors would like to express their gratitude to Kiichiroh Kurihara who performed a part of the computation.

REFERENCES

Anderson, H. (1974) J. Mech. & Phys. Solids, 22, 285-308
Atkinson, C. and M. F. Kanninen (1977) Int. J. Fract., 13-2, 151-163
Bilby, B. A., A. H. Cottrell and K. H. Swinden (1963) Proc. R. Soc., A272, 304
Bilby, B. A., A. H. Cottrell, E. Smith and K. H. Swinden (1964) Proc. R. Soc., A279, 1
Bilby, B. A. and K. H. Swinden (1965) Proc. R. Soc., A285, 22
Chitaley A. D. and F. A. McClintock (1971) J. Mech. & Phys. Solids, 19, 147
Miyamoto H. and K. Kageyama (1977) Preprint of JSME, 770-2, 75-77
Riedel, H. (1976) J. Mech. & Phys. Solids, 24, 277
Vitek, V. (1976) J. Mech. & Phys. Solids, 24, 263-275