AN EXPERIMENTAL STUDY OF THE DUGDALE MODEL

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ABSTRACT

An experimental study of the displacements at the tip of a fatigue crack was conducted to evaluate the Dugdale relation between displacement and applied load. Displacements were measured in two kinds of aluminum compact tension specimens of different thicknesses using a laser-interferometric technique with a very short gage length. The relation is found to be quite accurate for brittle fracture conditions. The geometrical relation between the crack mouth and crack tip displacement was determined; it is different for the two materials.

KEYWORDS

Crack tip displacement, Dugdale model, laser interferometry, aluminum, compact tension specimens.

INTRODUCTION

Once the plastic zone at the tip of a crack becomes significantly large, the linear elastic fracture mechanics equations do not describe the displacements of the crack surface. A popular method of predicting the displacements at the tip of the crack incorporates the Dugdale model (Dugdale, 1960) of plasticity. Other theoretical analyses (see discussion in Broek (1978)) predict identical or very similar displacements. A very important experimental work was conducted by Robinson and Tetelman (1974) who investigated that displacement relation for Charpy specimens of various materials subjected to slow-bend tests. They measured the interior crack-tip displacement at discrete loads with dental impression material. This work is similar to theirs except that the surface displacements at the crack tip were measured continuously using laser-based interferometry. Another major difference is that aluminum compact tension specimens of various thicknesses were tested.

A relation between the crack tip opening displacement (CTOD) and the displacement measured at the crack mouth with a clip gage (CMOD) has been proposed (again, see Broek (1978)). This relation has been experimentally determined for the Charpy specimen geometry and various materials (Robinson and Tetelman, 1974).

The laser interferometric strain/displacement gage (ISDG) measures in-plane displacement over a gage length on the order of 100 microns at a position 100 microns behind the crack tip. Furthermore, it permits recording of this displacement in real-time. The objectives of this experimental work are threefold:

1. to evaluate the Dugdale CTOD model
2. to empirically determine the CTOD/CMOD relation for a compact tension specimen
3. to discover if a measurement at the crack tip would give a more sensitive indicator of "pop-in" —— particularly in specimens too thin to produce valid K_{Ic} data.

The paper is organized into four major parts. First, the ISDG is briefly described and the experimental details presented. The Dugdale model is considered next with a brief review followed by a presentation and discussion of the results. A similar format is used for the CTOD/CMOD data. The paper closes with a discussion of the sensitivity to "pop-in" and some overall conclusions.

EXPERIMENTAL PROCEDURE

Basics of the Interferometric Strain/Displacement Gage (ISDG)

The fundamental concept behind the ISDG can be explained with the aid of Fig. 1. Two small reflecting indentations are formed in the specimen surface at A and B with a four-sided pyramidal-shaped diamond. The tool used to impress these indentations is a Vicker's micro-hardness tester. The indentations are nominally 25 microns on a side, 100 microns apart and located 100 microns behind the fatigue crack tip. Figure 2 is a photomicrograph taken after testing of a set of indentations astride a crack. The specimen surfaces were polished on cloth metallurgical polishing wheels with a 1.0 micron solution; however, sanding with 600 grit paper produces quite satisfactory surfaces. The indentations were applied after the fatigue cracks had been grown in the specimens.

The individual indentations are small enough that appreciable diffraction occurs when one is illuminated with visible radiation. When the radiation is coherent, such as supplied by a He-He laser, then the two diffracted patterns from A and B form interference fringes where they overlap. The phenomenon here is equivalent to Young's two-slit interference experiment (Jenkins and White, 1957) except that the interference pattern is reflected at an angle α_o with respect to the incident beam. The governing equation is:

$$d \sin \alpha = m\lambda \qquad (1)$$

where d is the spacing between indentations A and B, λ is the wavelength of light (0.6328 microns in this case), m is the fringe order, and α is defined in Fig. 1.

To measure relative displacement, one establishes an observation position α_o and measures the movement of the fringes, δm, in fractions of the fringe spacing. The displacement, δd, is then given by:

$$\delta d = \frac{\delta m \, \lambda}{\sin \alpha_o} \qquad (2)$$

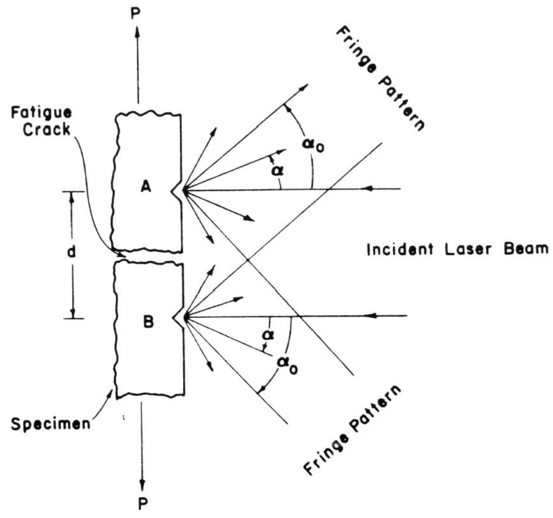

Fig. 1. Schematic of the ISDG.

Fig. 2. A set of indentations on a 7075-T651 specimen after testing. The indentations were originally 100 microns apart.

Fringe motion can be caused by rigid body motion of the specimen as well as by relative displacement of the indentations. When the specimen moves parallel to its surface and along a line between the indentations (i.e. vertically in Fig. 1), one fringe pattern moves toward the incident laser beam and one moves away. With fringe motion toward the laser beam defined as positive, averaging the two fringe motions eliminates this component of rigid body motion. This component is present whenever a specimen is loaded, so it is very important that it be eliminated. Other rigid body motions, such as one perpendicular to the specimen, must be eliminated by careful alignment of the testing machine. In practice, this turns out to be relatively easy. More information about the basics of the ISDG can be found in Sharpe (1971).

Specimen Geometries and Materials

Compact tension specimens of aluminum types 2024-T351 and 7075-T651 were used for this study —— the 2024 representing a tough aluminum, and the 7075 a brittle aluminum. The geometry of ASTM E399 with W = 50.8 mm was followed except that specimens were prepared with four nominal thicknesses —— 3 mm, 6 mm, 13 mm, and 25 mm. The specimens were oriented in the T-L configuration with respect to the plate.

Tensile tests on round 12.83 mm diameter specimens were conducted according to ASTM E8. The 2024 had a yield and ultimate strength respectively of 360 MPa and 487 MPa; the 7075 had 543 MPa and 620 MPa. Both materials had $E = 73 \times 10^3$ MPa.

Fatigue pre-cracks were grown to a nominal length of 25 mm. Again, the procedures of ASTM E399 were followed to assure sharpness of the crack tip. The approach was to prepare the specimens for valid K_{Ic} testing even though it was recognized that only the thickest specimen would likely provide a valid reading.

After the fatigue cracks were grown, indentations were placed on the polished specimen surface with a Vickers hardness tester. The indentations were placed approximately 100 microns behind the crack tip. It is very difficult to locate precisely the tip of a fatigue crack on an unloaded specimen. The distance of the indentations was measured more accurately after the specimen was tested. In all cases except one, the indents were 100 ± 14 microns behind the crack tip.

Displacement Recording Techniques

The compact tension specimens were loaded in a screw-driven Instron testing machine at the rate prescribed by ASTM E399. The crack-mouth displacement (CMOD) was measured with an E399 clip gage and the specimen was loaded until considerable plastic deformation occurred —— but not until complete fracture. In other words, the test was conducted in the prescribed manner for a fracture toughness test except that the specimens weren't broken in two.

A schematic of the measurement set-up is given in Fig. 3. A 5 milliwatt He-Ne laser illuminated the indentations. The two fringe patterns shown onto photomultiplier tubes —— each masked except for a narrow slit parallel to the fringes. As the fringes moved past the slits, the intensity of the signal varied in an approximately sinusoidal fashion. The two fringe pattern signals were fed into a small minicomputer which detected maxima and minima as the fringes moved. One bit (~ 5 millivolts) was output to the X-Y recorder each time either a max or a min was detected on either channel. This output was easily calibrated in terms of displacement through Eq. 2. Note that this arrangement doesn't detect the direction of the fringe motion, but this was known a priori.

As Fig. 3 shows, the CMOD was recorded simultaneously on the two-channel X-Y plotter. A typical record is shown in Fig. 4 —— note the different scales.

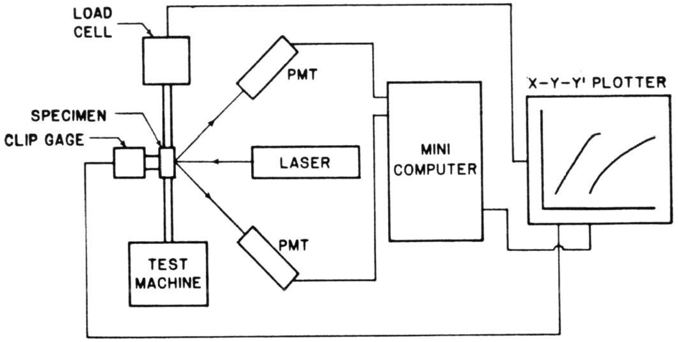

Fig. 3. Schematic of the data acquisition system

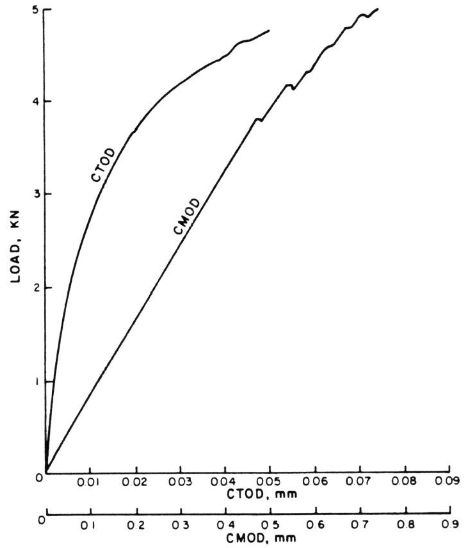

Fig. 4. Records of CTOD and CMOD for a 13 mm thick specimen of 7075-T651.

CRACK TIP OPENING DISPLACEMENT

Dugdale (1960) applied superposition principles to a remotely loaded central crack in an infinite sheet to determine the length of the plastic strip ahead of the crack tip. This length was computed to be

$$\rho = \frac{\pi K_I^2}{8\sigma_{ys}^2} \tag{3}$$

when the applied remote stress is much less than the material's yield stress σ_{ys}. K_I is the mode I stress intensity factor. Burdekin and Stone (1966) used this length to compute

$$\mathrm{CTOD} = \frac{K_I^2}{E\,\sigma_{ys}} \tag{4}$$

where E is the modulus of elasticity. This equation has become known as the Dugdale model for crack tip opening displacement. Other computations have been done — see Broek (1978) — but most of them end up with similar predictions. Equation 4 is developed for a plane stress situation and also applies only when the applied stress is much less than the yield stress — the usual situation.

The CTOD predicted by Eq. 4 should be measured right at the tip of the crack. As mentioned earlier, it is sometimes difficult to locate the tip precisely in an unloaded specimen. The indentations were placed nominally 100 microns behind the tip of the 25,000 micron long crack — one might argue that to be sufficiently close. The effect of the measured "COD" on the desired "CTOD" can be examined as follows.

The elastic solutions (Broek, 1978) for a centrally-cracked sheet give the displacement, COD, at any position x (see Fig. 5) as:

$$\mathrm{COD} = \frac{4\,K_I\sqrt{a}}{E\sqrt{\pi}}\sqrt{(1+\rho/a)^2 - (x/a)^2} \tag{5}$$

The length of the plastic zone, ρ, could be given by any model — not necessarily Eq. (3). If the indentations are located a distance, Δ, behind the crack tip in Fig. 5, then the CTOD can be computed from the COD by:

$$\mathrm{CTOD} = \frac{\mathrm{COD}}{\sqrt{1+\Delta/\rho}} \tag{6}$$

Here ρ and Δ are both assumed much less than the crack length. This can be a sizeable correction — especially for small loads when ρ is small.

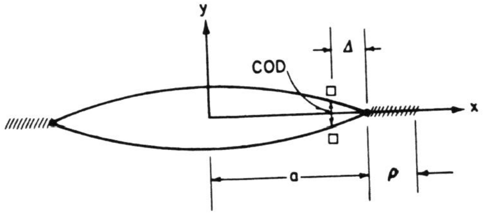

Fig. 5. Coordinate system.

Figure 6 illustrates the effect of this correction for the 7075-T651 material. Using a typical K_{IC} of 25 MPa $m^{1/2}$, from Eq. 3, $\rho_{max} \simeq 800$ microns. The curves in Fig. 6 were computed with Equations 6 and 3 and show that for Δ = 100 microns, only a 5 percent error is caused by ignoring the correction for loads greater than approximately half the maximum load. The correction would be smaller for the 2024 since the lower σ_{ys} produces a larger ρ.

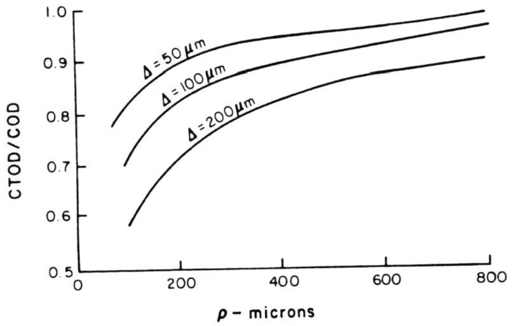

Fig. 6. Measurement location correction for 7075-T651.

All of the experimental CTOD data reported here were corrected via Eqs. 6 and 3. The K_I was computed from the ASTM E399 equation for a compact tension specimen. The data for the 2024 and 7075 aluminum are presented as plots of measured CTOD versus Eq. 4 in Fig. 7 and 8. The longer form of the Dugdale displacement equation is not used since Eq. (4) is accurate enough at these small loads.

Figure 7 shows excellent agreement with the Dugdale model only for the 25 mm thick specimens. The one 13 mm specimen was the only one with indentations out of place — they were found to be 195 microns back from the tip. The thinner specimens consistently produce smaller displacements than predicted. The CTOD corresponding to K_{IC} is 35 microns, so all the data shown are recorded before the critical stress intensity is reached. A linear regression of the data shows an intercept of -0.66 microns with a slope of 0.955 and a correlation coefficient of 0.94.

Figure 8 — for the more brittle 7075 — shows generally excellent agreement with the Dugdale model for all thicknesses. The deviations from agreement occur at values above the 15 microns corresponding to K_{IC}. This raises the interesting possibility that a suitable measure of K_{IC} for thin brittle materials could be obtained by plotting CTOD versus the load-squared and establishing the point of deviation from linearity. The linear regression yields a slope of 0.987 with an intercept of -0.07 microns and a correlation coefficient of 0.98.

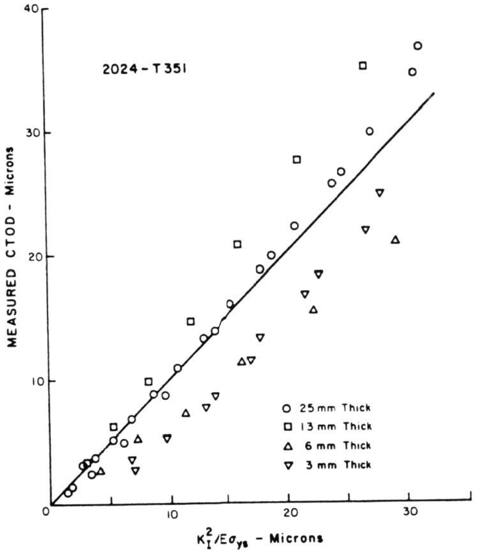

Fig. 7. CTOD measurements versus the Dugdale model for 2024-T351.

CRACK MOUTH OPENING DISPLACEMENT

As mentioned earlier, the crack mouth opening displacement (CMOD) was recorded for each experiment. It is then an easy task to plot the corrected CTOD versus the CMOD to determine if the CTOD can be accurately predicted from the easier clip-gage measurements for a range of materials and geometries.

Figures 9 and 10 present the data for 2024 and 7075 respectively. It is seen that a single curve suffices for all thicknesses in both materials, but the 7075 again shows less scatter. However, the curves are different for each material. Curves of the location of the center of rotation, r, versus the CTOD can be plotted (they are in the original work (Paleebut, 1978)), but a different curve will be obtained for each material. Robinson and Tetelman (1974) concluded a single curve of "r" versus CTOD was satisfactory for the several materials (including 2024 aluminum) they tested. Their specimens were small 3-point bend types which are somewhat different from the compact tension geometry used here.

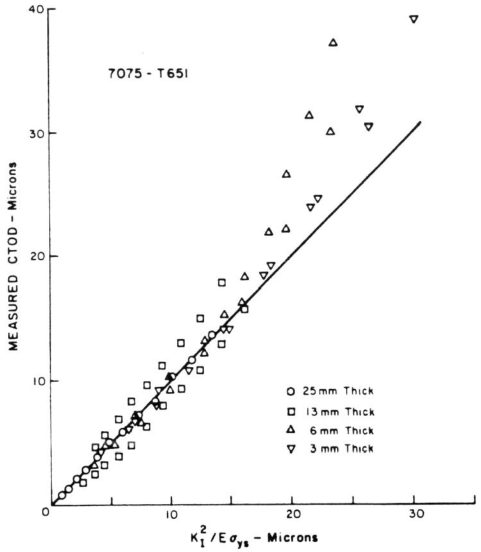

Fig. 8. CTOD measurements versus the Dugdale model for 7075-T651.

It seems unlikely that materials with different "brittleness" should exhibit the same CTOD-CMOD relation. 2024 is tougher, therefore has a larger plastic zone than 7075. So, at a given value ot the CTOD, the center of rotation should be further advanced ahead of the crack tip — producing a smaller CMOD. This is observed in comparing Figs. 9 and 10.

For the compact tension geometry with a crack length of 25 mm, the CTOD-CMOD relation is easily represented by a simple power law. These are shown on Figs. 9 and 10; note that the errant 13 mm specimen of 2024 is not well-represented by the power law. The "r" curve concept has the advantage of being adaptable to initial cracks of different lengths; an "r" curve could easily be generated using the measured CTOD-CMOD relation for compact tension specimens of these two aluminums.

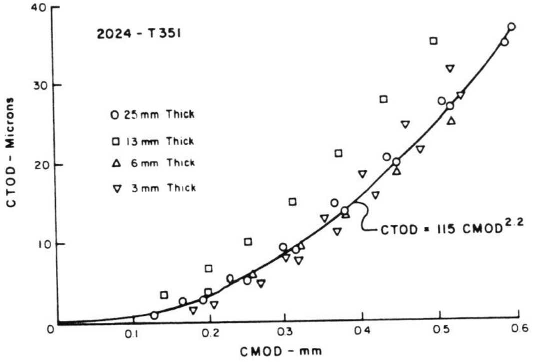

Fig. 9. CTOD-CMOD measurements for 2024-T351.

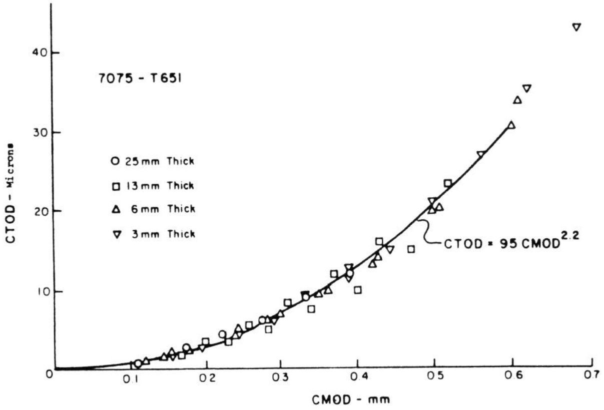

Fig. 10. CTOD-CMOD measurements for 7075-T651.

CONCLUDING REMARKS

The ISDG is demonstrated here to be a very easy-to-use means of measuring the displacement near the tip of a crack. A minicomputer is not really required; one can simply record the fringe intensities on a two-channel strip-chart recorder and compute the displacement later. In fact, strip-chart records were taken for all these experiments to provide redundancy in the measuring system. An advantage of the ISDG is that it can be used at very high temperatures (~ 730°C) with certain non-oxidizing superalloys (Sharpe and Martin, 1978).

The Dugdale model was developed for a centrally cracked specimen geometry. All the CTOD measurements here were close enough to the tip of the crack ($\Delta/a \sim 1/250$) that the stress intensity factor approach should apply. Note that these measurements were on the specimen surface —— as opposed to the interior measurements of Robinson and Tetelman (1974).

The measured and corrected CTOD agree quite well with the Dugdale model for brittle behavior, i.e. for the 7075-T651 aluminum and the thickest 2024-T351 aluminum. This is consistent with the findings of Robinson and Tetelman (1974) for a completely different experimental approach. Locating the indentations 100 microns behind the crack tip seems to work quite well, though moving them up to 50 microns would probably be better —— the correction procedure could be dropped. It would be unwise to try to put indentations right at the tip because of the uncertainty of locating the tip on the surface and the question of what the crack looks like just below the surface.

Crack tip opening displacement is found to be related to crack mouth opening displacement by a simple power law that is different for each material. This is in contrast to previous findings. The CTOD-CMOD relation is independent of specimen thickness, but further work would be needed to evaluate a universal relation for the compact tension specimen with various crack lengths.

One physical phenomenon watched very closely during these experiments was the "pop-in" behavior when the specimen initially cracked. It was thought that a displacement measurement near the crack tip might detect pop-in in the thinner specimens more readily than the coarser crack-mouth measurement. However, this was not observed in any of the tests. The pop-in that appeared on the CMOD record coincided exactly with that shown by the CTOD measurement. The only seemingly possible way of using CTOD measurements to determine K_{IC} in thin brittle materials would be to plot CTOD versus K^2. When the plot deviates from linearity, the data of Fig. 7 indicate a reasonable estimate of a critical CTOD corresponding to K_{IC} could be made.

These experiments support the use of the Dugdale model under the conditions for which it was developed —— namely small-scale plastic yielding. As so often happens in solid mechanics, the early uncomplicated work turns out to be the most useful.

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