AN ANALYSIS OF FAST FRACTURE AND ARREST IN DCB SPECIMENS USING CRACK TIP ELEMENTS

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ABSTRACT

A finite element model is presented of fast fracture and crack arrest in double cantilever beams under fixed grip loading conditions. Eight-node isoparametric elements and a special crack tip element incorporating terms of the Williams' eigenfunction expansion for the stress field at a crack tip are used. The model is very efficient at the static level for the determination of stress intensity factors throughout the whole crack history of the DCB specimen.

An efficient dynamic algorithm is employed maintaining long term stability even when subjected to rapid changes in boundary conditions. The phenomenological analysis, which uses an experimentally determined speed versus dynamic stress intensity factor characteristic, aims to reproduce existing experimental results. Results, in terms of the crack propagation history and the termination of the fracture event, make favourable comparison to those of earlier work and to experiment.

KEYWORDS

Finite elements; crack propagation; crack arrest; double-cantilever-beam specimen; fractures (materials).

INTRODUCTION

The dynamic crack propagation problem has come under increasing attention in recent years at both the theoretical and experimental levels. In particular, Kalthoff, Beinert and Winkler (1977) and Hahn and others (1973, 1976a, 1976b) have produced a substantial amount of experimental details and results of tests on double cantilever beam (DCB) geometries, under fixed grip loading conditions. This type of test model is highly suitable for a theoretical analysis due to the high degree of symmetry involved and the known crack path. A pioneering numerical model has been developed by Keegstra and colleagues (1977, 1978) and has achieved notable successes in regard to modelling crack propagation velocities and crack arrest lengths. The model is limited in effectiveness, however, due to the use of a highly refined mesh of numerically inefficient constant strain triangles.

The work, which is described here, was undertaken to improve the computational efficiency of earlier work and incorporates a base mesh of higher order elements and a novel crack tip element. This type of approach has the immediate advantage of providing improved completeness generally and is much better equipped in particular to model the complex stress field occurring in the critical region surrounding the tip of the propagating crack. The specification permits the use of relatively coarse meshes whilst still retaining a high degree of accuracy. Thus, whilst the generation of the element matrices becomes more involved and lengthy, the overall computational effort is reduced because of the smaller number of equations.

The element configuration operates within a dynamic algorithm, again chosen on the basis of computational efficiency, and attempts have been made to follow phenomenologically the experiments mentioned earlier. Keegstra's work has already shown that this is possible, but it is hoped that this analysis will offer certain worthwhile improvements.

CRACK TIP ELEMENTS

The fundamental problem confronting mathematical models of the fracture process, is to describe accurately the complex nature of the stress field in the immediate area of the crack tip. The $r^{-\frac{1}{2}}$ stress singularity at the tip implies that purely polynomial based elements will have a poor rate of convergence and hence will require extremely fine meshes for adequate tip representation. This difficulty may be overcome by incorporating the stress singularity in the formulation of a special crack tip element. Fawkes (1976) has investigated the effectiveness of these various modelling techniques and concluded that an element of the embedded singularity type (Fawkes, 1978) holds certain advantages over the rest.

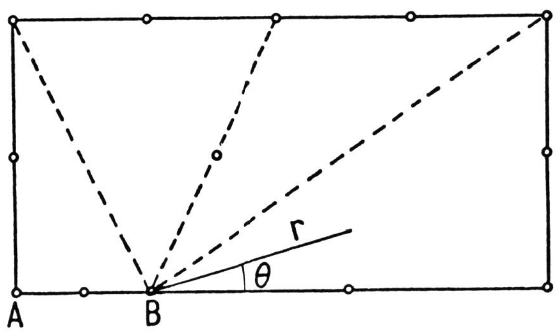

Fig. 1. Crack tip element

An element of this type is used in the investigation reported here. The full description has been detailed in earlier work by the authors (1979, 1980), but briefly it incorporates terms of the Williams' eigenfunction expansion for the stress field at a crack tip. It has been observed that elements with differing numbers of terms of the expansion are all admissible at the static level, the full formulation being completed by a mixture of polynomials. The element has 13 nodes, see Fig. 1, and is topologically equivalent to two adjacent 8-node

rectangular elements. This enables easy interchange of the element within a mesh of standard parabolic isoparametric elements. The crack runs from a corner node A to a point B on the base line. The position of the tip, and hence the crack length, may be varied by moving point B further along the base line into the element. In all, nine variants of the element are available with the extreme crack lengths being equivalent to the corresponding 8-node element mid-side node positions. By a subtle combination of changing variant and altering the position of the crack tip element within the overall mesh, it is possible to model tip incrementation on a much finer scale than from one 8-node element boundary to another. It will be noted that for certain crack lengths there is the possibility of more than one crack tip element configuration.

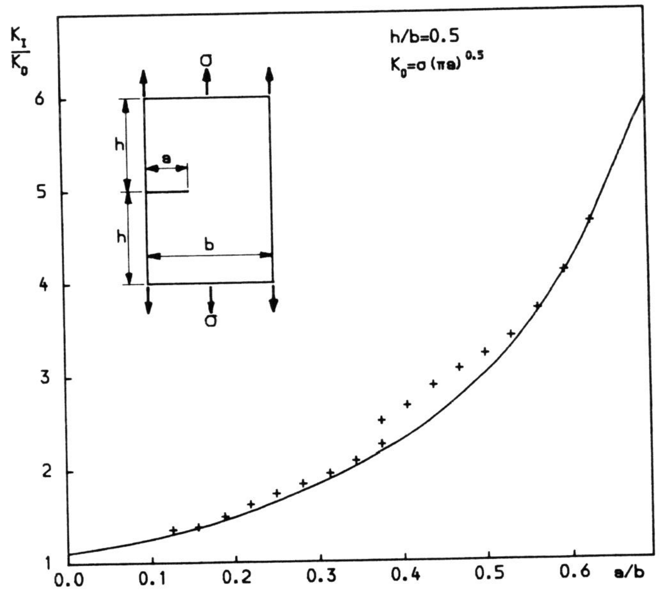

Fig. 2. Edge crack, uniform tensile stress

The elements have been tested on simple unimode test problems for which solutions are known (Rooke and Cartwright, 1976). The following results relate to an element specification combining the singular term only of the eigenfunction expansion with a complete polynomial, quartic in the crack-extension direction and quadratic in the $\theta = \pi/2$ direction. The problem depicted in Fig. 2 refers to an edge crack in a finite width sheet under loading conditions of uniaxial tensile stress. A mesh of 4 x 2 8-node elements, in which two elements had been replaced by the singularity element, produced results for a series of a/b ratios which are shown graphed against the theoretical curve. A discontinuity producing an error of the order of 10% occurs on element interchange for an a/b ratio of 0.375. Further refinement of the mesh results in this deviation being greatly reduced. Other simple problems concerning centre cracked specimens have been reported in an earlier paper by the authors (1980), together with the effect of mesh refinement.

In the course of the analysis, several versions of the crack tip element were tested. Variants were investigated in which the displacement incompatibility between the crack tip element and the surrounding 8-node elements was limited by the introduction of a form factor, restricting the range of the singular term involvement. No significant difference to the displacement field was found, in accordance with the argument of Patterson (1973), that the quality of convergence is not necessarily reduced when not using fully conforming elements.

As a further test of the reasonableness of the mathematical model at the static level, the opening mode stress intensity factor, K_I, was determined for the whole crack history of the DCB specimen and is shown in full in Fig. 3.

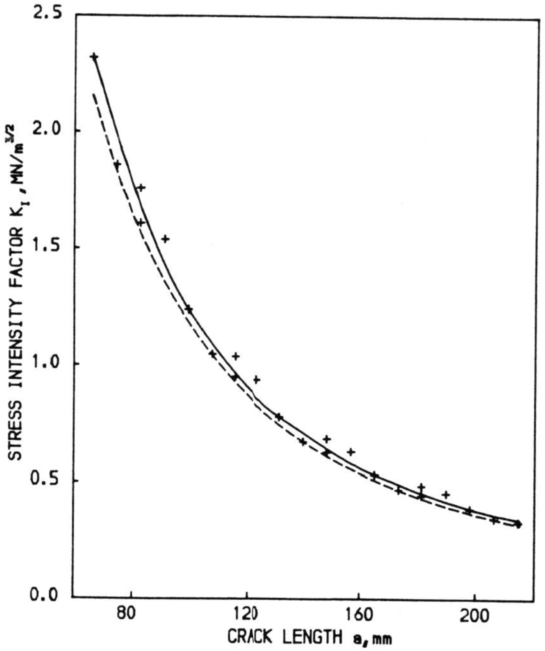

Fig. 3. Static stress intensity factor in DCB specimen

A theoretical curve, derived by Kanninen (1973), is represented by the dashed line in the same figure. A coarse 10 x 2 mesh was used and the results are highly acceptable, within 5% of Kanninen's solution. As would be expected, the maximum deviation from the curve occurs on element interchange where significant changes are made to the system matrices. Tip incrementation appears to have little or no malefaction to the equations.

DYNAMIC ANALYSIS

The discretised form of the equations of motion of a continuum can be written in matrix form as

$$[K]\,\underline{u} + [C]\,\dot{\underline{u}} + [M]\,\ddot{\underline{u}} = \underline{F} \qquad (1)$$

in which [K], [C] and [M] are the system stiffness, damping and mass matrices respectively, $\underline{F}$ is the generalised force vector and $\underline{u}$ is the displacement vector. The dot denotes differentiation with respect to time. Two basic methods exist for the solution of these equations but, due to the time dependent boundary conditions, a normal mode analysis must be discounted. We are left with the step by step integration technique and in terms of computational efficiency, the approach of Hitchings and Dance (1974) is most desirable since it preserves symmetric matrices and does not require the inversion of large matrices. The accelerations at sequential time points are related by a Lagrangian polynomial and the derivatives are used in Taylor expansions to determine estimates of the displacement and velocity at the next time point. Equation (1) may then be used to obtain the accelerations. The procedure is repeated iteratively until the process converges at each time step.

The dynamic problem is only conditionally convergent, requiring the time interval to be less than the period of the highest eigenvalue in the system. However, this imposes no increased restriction on the analysis since the time step must be small to be capable of following the rapid changes in geometry.

The algorithm has been tested in the context of a simple bar problem. A compressive load acts initially on the bar, it is released and the speed of the resultant stress wave is measured. Results to within 5% of the theoretical wave speed have been obtained using a coarse mesh and a convergence requirement within the iterative loop of 8 figure accuracy. This convergence criterion requires $\sim$ 12 iterations per time step, which makes the solution process much more efficient than other methods involving inversion of large matrices.

A further test of the validity of the dynamic analysis was performed on a model of the DCB specimen. A certain over-straining was applied to initiate the program and the boundary conditions were held at a fixed crack length over a large number of time intervals. The complex vibrations in both the horizontal and vertical directions were observed with no apparent decay. The two tests lead to the conclusion that the crack tip element and background mesh of quadrilaterals operate efficiently within the dynamic algorithm.

DYNAMIC CRACK ADVANCE

The program was to be validated by attempting to follow the results obtained experimentally by Kalthoff and colleagues (1977). They used wedge loaded DCB specimens of Araldite corresponding to the data given in Table 1. The crack propagation phase was followed by a high speed camera and dynamic stress intensity factors were derived using the method of caustics (Theocaris, 1972).

The finite element model consisted of two rows of ten identical elements, to permit interchangeability of the crack tip and isoparametric elements. The crack tip speed versus dynamic critical stress intensity factor, K_{ID}, characteristic was inferred from Kalthoff's observations and was used to control the tip advance mechanism. After a tip advance the stress intensity factor was allowed to build up until it attained the dynamic toughness, at which point the tip was advanced by replacing the current element mass and stiffness matrices by the advanced versions within the overall system mass and stiffness matrices, and by interpolation of the displacements, velocities and accelerations.

Table 1 DCB Specimen Data

Elastic modulus	3.66×10^{9} Nm^{-2}
Poisson's ratio	0.392
Fracture toughness	0.79×10^{6} Nm$^{-3/2}$
Initial crack length	66 mm
DCB half-depth	63.55 mm
DCB length	321 mm
DCB thickness	10 mm
Density	1.223×10^{3} kg m^{-3}
Pin opening displacement	3.607×10^{-4} m

To maintain the necessary energy balance resulting from the creation of new crack surfaces, energy was required to be dissipated in the region of the crack tip. This was modelled by the provision of 'holding back forces' to the opening edge immediately behind the crack tip. The rate of dissipation of energy, G_D, is given by

$$G_D = \frac{K_{ID}^2}{E} \qquad (2)$$

for the dynamic regime, where E is the elastic modulus. The forces are derived from the restraining surface stress,

$$\sigma_H = \lambda K_{ID} \left(1 - \frac{\bar{\delta}(x)}{\bar{\delta}(l_H)} \right) \qquad (3)$$

in which $\bar{\delta}$ is a time averaged displacement. The forces are tuned initially to agree with eqn. (2) by adjusting the constant λ and the length over which they act, l_H. With this model of energy dissipation, there should be no need for subsequent adjustment of the holding back forces in order to remove energy at the required rate.

The numerical model chosen, corresponded to the most seriously overstrained of Kalthoff's experiments with a crack initiation stress intensity factor, K_{Iq}, of 2.32 MNm$^{-3/2}$. The crack was allowed to advance in the manner previously described and the full crack history is shown in Fig. 4, together with the experimental results of Kalthoff shown by the dashed curves. The velocity curve is obtained by calculating the gradient to a fitted curve of crack length against time for a succession of crack lengths, a dynamic stress intensity factor, K_I^{dyn}, is then inferred.

This approach, whilst demonstrating the average condition, does not describe the complex stress wave variations which appear in the instantaneous velocity curve, Fig. 5. Here, two notable events can be observed; at a crack length of 145 mm the fast dilatational wave impinges on the crack tip after reflection from the far end of the specimen. Secondly, a combination of the joint coincidence of the slow rotational wave and the fast wave after a second reflection at the crack front is marked by a severe retardation of the tip at a length of ∿ 175 mm. Figure 5. amply demonstrates Kannien's view (1980) that the DCB specimen is the most dynamic of all possible structural configuations.

Results from the numerical model show that after initial tuning, G_D operates within a tolerance band of 10% of the required rate. K_{ID} builds up after tip advance to initiate further propagation, indicating that the dynamic algorithm can adequately withstand the disruption caused by tip advance.

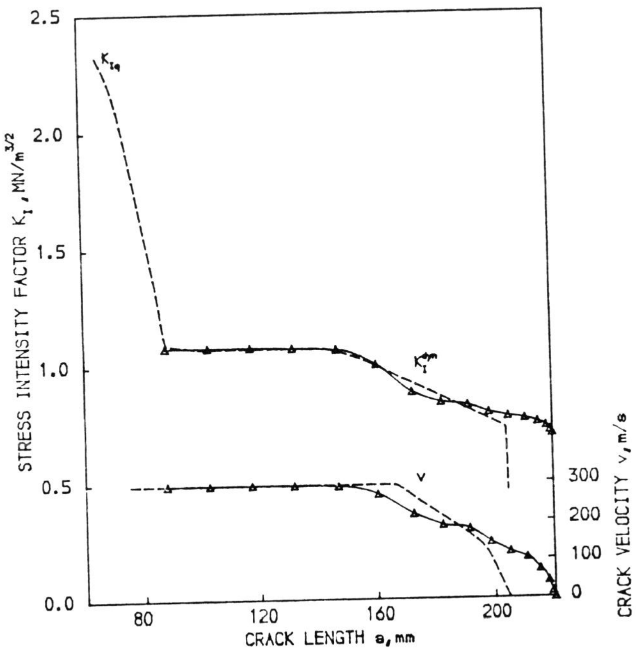

Fig. 4. Stress intensity factor for crack propagation

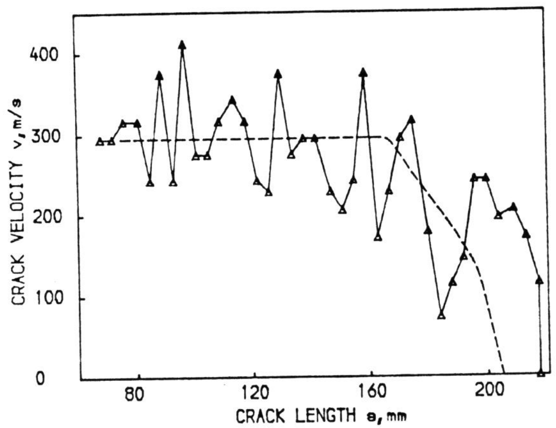

Fig. 5. Instantaneous crack speed chart

DISCUSSION AND CONCLUSIONS

A crack tip element has been designed which accurately represents the complex stress field at the crack tip. Although the element is relatively complicated, it can be used in very coarse meshes whilst still retaining a high degree of accuracy. The element can be used in conjunction with a mesh of 8-node quadrilateral elements to model the crack propagation process, using a computationally efficient dynamic algorithm. Since commencement of the investigation, other work has been reported in which quadrilateral meshes have been used with and without crack tip elements, Owen and Shantaram (1977), Nishioka and colleagues (1980) and Mall and Luz (1980). However, not one of these analyses has followed the Keegstra method in its attempt to model the dynamical situation.

Conditional stability of the algorithm presents no problem because the time stepping increment must be small to follow the rapid changes in boundary conditions. However, the conditional stability of the iterative process restricts the choice of eigenfunction terms to the low order ones only. Energy dissipation in the model is adequately represented by holding back forces which, after initial tuning, dissipate at the correct rate for the whole crack history.

Validation of the program was made by comparing results to available experimental data. The results, in terms of the crack propagation history and the termination of the fracture event, make favourable comparison to experiment and to those of Keegstra. But, whilst his analysis required a mesh containing 253 nodes and only permitted a crack increment of 6 mm, the present analysis employs only 85 nodes and a tip advance of 4 mm is possible.

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