A VIBRATION TECHNIQUE FOR THE DETERMINATION OF STRESS INTENSITY FACTORS

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ABSTRACT

A new method of determining stress intensity factors is described. The technique uses the compliance method; the compliance changes being determined from the changes in natural frequencies of the specimen as crack size increases. It is applied to two different crack geometries and offers the advantages of speed and minimal hardware requirements. Results suggest that the method is most accurate for small crack sizes or geometries where the crack is of a symmetric nature. The application of compliance measurement to a vibration non-destructive test technique is then considered.

KEYWORDS

Stress intensity factor; vibration; non-destructive test; compliance; single edge notch; axisymmetric notch.

INTRODUCTION

The method of non-destructive testing developed by Adams and others (1978) is based upon the measurement of the natural frequencies of a structure. It was shown that the changes in these natural frequencies arising from the presence of damage could be used to locate and to some extent quantify the damage, providing it was of a local nature.

The authors represented the damage by a spring and then by postulating a series of positions for the spring the value of spring stiffness that would give the measured changes in natural frequency could be calculated for each position. By assuming that this spring had a stiffness which was the same for all modes the damage could be located, and the actual spring stiffness determined. The authors found that this stiffness decreased as damage severity increased, being infinity for no damage and zero for total failure of the structure. Thus the test was able to locate and evaluate the damage.

Measurement of compliance, which is the reciprocal of stiffness, is often used as a means of determining stress intensity factors (Srawley, Jones and Gross, 1964; Cartwright and Ratcliffe, 1972; Underwood and others, 1972). The technique was originally established by Irwin and Kies (1954) as an experimental technique but Dixon and Pook (1969) later suggested the use of the compliance technique together with the finite element method of analysis as a general means of determining stress intensity factors.

The work described here introduces a new experimental method for determining stress intensity factors in mode I fracture and applies the results obtained to the non-destructive test described above.

NOTATION

- A Cross sectional area
- A_c Area of crack
- C Compliance
- G Strain energy release rate
- K_N Stress intensity factor for mode, N
- ℓ Length of bar
- P Applied load
- x Length of a section of bar
- β Receptance of system B
- β' Receptance of system B'
- δ Receptance of system D
- λ $\omega\sqrt{\rho/E}$
- ν Poisson's ratio
- ρ Density
- ω Frequency (rad.sec^{-1})

BACKGROUND

Irwin and Kies showed that the energy release rate of a cracked structure could be expressed in terms of the load and the rate of change of compliance with crack area as

$$G = \frac{P^2}{2}\frac{\partial C}{\partial A_c} \qquad (1)$$

The energy release rate can also be expressed in terms of stress intensity factor as

$$G = \frac{K_I^2}{E}(1 - \nu^2) \quad \text{for plane strain} \qquad (2a)$$

and
$$G = \frac{K_I^2}{E} \quad \text{for plane stress} \qquad (2b)$$

both of these expressions being for mode I fracture. Combining (1) and (2) yields

$$K_I = P\left[\frac{E}{2(1-\nu^2)}\frac{\partial C}{\partial A_c}\right] \quad \text{and} \quad K_I = P\left[\frac{E}{2}\frac{\partial C}{\partial A_c}\right] \quad \text{respectively.}$$

Since equation (1) expresses stress intensity factor in terms of the derivative of compliance it is not possible to determine K_I directly from a single compliance measurement, it is necessary to evaluate the derivative either analytically or by obtaining values of the compliance corresponding to a range of crack sizes.

Some crack geometries are considered in published literature (Srawley, Jones and Gross, 1964) and these can be used where applicable; a more general technique is the use of a suitable stress analysis technique such as the finite element method (Dixon and Pook, 1969; Hellen, 1975).

Srawley, Jones and Gross (1964) carried out an experimental investigation into the compliance changes in a single edge notch specimen. Crack sizes ranging from zero to one half of the specimen width were used and the authors claim an accuracy of 0·5% within the range of interest. Their results were in good agreement with concurrent results obtained by boundary collocation of a stress function (Gross, Srawley and Brown, 1964).

The original method of using the finite element method proposed by Dixon and Pook (1969), was to evaluate the specimen compliance for a range of crack sizes and then to take the derivative with respect to crack area. The technique has the disadvantage of requiring a complete run of the finite element analysis for each crack size before any stress intensity factors can be determined. The method has been refined by Hellen (1975) who considers energy differences between two slightly different meshes. This method has inherent error cancellation but the main advantage is a reduction in the computational effort required to determine the energy release rate. However some modification to a normal finite element package is required.

The proposed method of establishing the relationship between compliance and crack size is to use a vibration technique similar to that used by Adams and others (1978) but without the damage location facility. Such a technique would offer the advantage of speed as natural frequencies can be determined accurately in a short time using simple electronic instrumentation, the main requirement being a stable oscillator covering the appropriate frequency range and a digital frequency meter. This new technique is referred to as the dynamic measurement technique.

THEORY

Consider a bar with a crack as shown in Fig. 1. The crack is located at position x along the bar and is represented by the insertion of a spring of compliance C. The crack will also affect the damping of the bar but the effect that this has on the natural frequencies is small and may therefore be ignored.

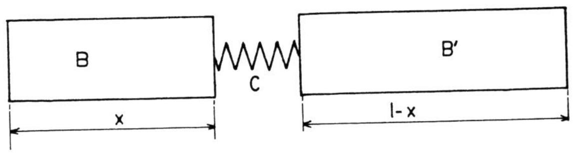

Fig. 1 Bar with spring to represent damage.

If the sections of bar either side of the damage are defined as B and B', having receptances β_{xx} and β'_{xx} respectively, then the natural frequencies of the bar are such that the following equation is satisfied (Bishop and Johnson, 1960)

$$\beta_{xx} + \beta'_{xx} + C = 0 \qquad (3)$$

In the case of a bar of constant cross section the receptances are given by

$$\beta_{xx} = \frac{-\cos\lambda x}{AE\lambda \sin\lambda x} \quad ; \quad \beta'_{xx} = \frac{-\cos\{\lambda(\ell - x)\}}{AE\lambda \sin\{\lambda(\ell - x)\}}$$

where all symbols have the meaning given in the notation section (Bishop and Johnson, 1960).

Substituting these values in equation (3) yields

$$C = \frac{\cos \lambda x}{AE\lambda \sin \lambda x} + \frac{\cos\{\lambda(\ell - x)\}}{AE\lambda \sin\{\lambda(\ell - x)\}}$$

or $$C = \frac{1}{EA\lambda}\left(\cot \lambda x + \cot\{\lambda(\ell - x)\}\right) \quad (4)$$

The evaluation of the right hand side of equation (4) requires the knowledge of the dimensions of the bar together with the velocity of sound within the bar. Since quoted values are not accurate enough other means of determining the material properties must be found. The easiest method is to use the undamaged natural frequencies to determine these properties.

If there is no damage within the bar then sections B and B' become one continuous section, D, of length ℓ with a receptance, $\delta_{\ell\ell}$, which is given by

$$\delta_{\ell\ell} = \frac{\cos \lambda\ell}{AE\lambda \sin \lambda\ell}$$

The natural frequencies of the undamaged bar are therefore given by

$$\sin \lambda\ell = 0.$$

This is satisfied by $\lambda\ell = n\pi$ where n is integer and refers to the mode of vibration being considered.

If the natural frequencies for the undamaged bar are measured then a value of λ can be determined for each mode. As λ tends to be slightly different for each mode greatest accuracy is obtained if the value found for a particular mode is only used for that mode. If the natural frequencies are then measured after a crack has been introduced into the bar then the compliance of the crack can be calculated using equation (4).

EXPERIMENTAL VERIFICATION

To verify the technique described above tests were carried out using two different crack geometries. The first was the single edge notch specimen and the second a cylindrical bar with an axisymmetric notch. In both cases aluminium bar was taken from stock, a section of about ·4m in length was cut off and the ends of this section machined square.

The technique used to simulate a crack in the single edge notch specimen was the same as that used by Srawley, Jones and Gross (1964). A small hole is drilled in the specimen and then a fine slot is made joining the hole to the edge of the specimen. The actual crack tip is then considered to be a quarter of the diameter of the hole from the base of the resultant slot. For the cylindrical specimen the notch was produced in a lathe using a pointed tool.

The crack was placed mid-way along the bar as this position gives no change in the natural frequencies of the even modes. Any changes that were measured for these modes were attributed to temperature variations and used to correct the frequencies of the other modes to a constant temperature.

RESULTS

Single edge notch. The compliance values obtained using the first and third modes are shown in Fig. 2 together with results obtained by other authors. For crack sizes corresponding to values of a/W greater than 0.4 it was found that there were two resonances near the frequency expected for the 3rd mode and so

definition of the 3rd natural frequency became difficult. This effect was also observed by Adams and co-workers (1978) who attributed it to coupling between axial and flexural vibration.

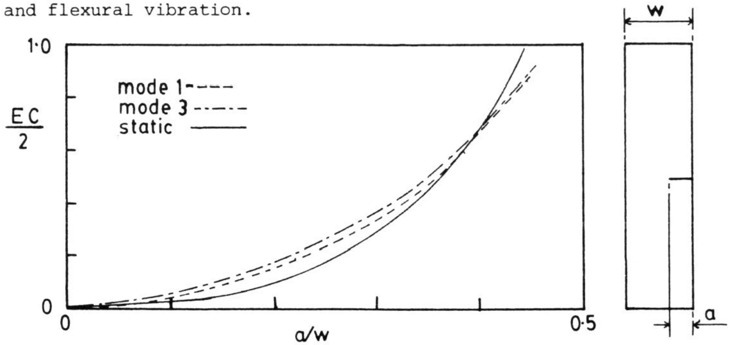

Fig. 2. Compliance of single edge notch specimen.

It can be seen from Fig. 2 that the compliances measured dynamically are greater than those measured statically for values of a/W less than 0.38. The discrepancies between these two sets of results, and between results obtained using different modes are likely to arise from two sources.

The first of these is the assumption made in the vibration analysis that the compliance change occurred over an infinitessimal length of the bar. In practice the compliance change would occur over a section of bar which became longer as the crack depth increased. When compliances are measured statically this section is subjected to a uniform load along its length but in the case of resonant vibration the load at any instant will vary along this section. This non-uniformity will generally be greater in the third mode than in the first and thus lead to a greater error in this mode.

The second source of error is the axial-flexural coupling already referred to above. This can be removed by using a specimen with a symmetrical type of damage such as the one discussed in the next section.

Cylindrical Specimen. Figure 3 shows the variation of compliance with the diameter ratio d/D for the cylindrical specimen used. As no comparable data was available none is presented.

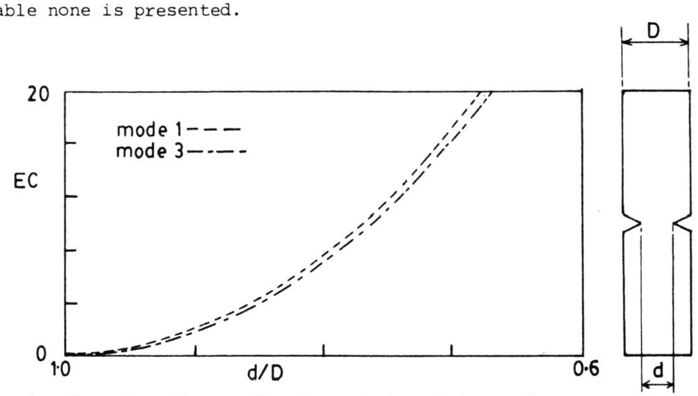

Fig. 3. Compliance of axisymmetric notch specimen.

The agreement between the first and third modes is better than for the single edge notch specimen but in this case the third mode gives lower compliance values than the first mode. As stated above the cylindrical specimen with an axisymmetric notch does not exhibit any coupling between axial and flexural vibration and these results suggest that this may be the major source of error for the single edge notch specimen.

DETERMINATION OF STRESS INTENSITY FACTORS

Method. To determine stress intensity factor from compliance data it is necessary to obtain the derivative of compliance with respect to crack area. The method used was to approximate the compliance/crack area relationship by a polynomial. A minimax curve fit was used which selects the polynomial of the required degree with the minimum value of the maximum error for all data points. This error is reduced as higher degree polynomials are used but if the order is too high the polynomial can oscillate between data points. It was found in practice that a polynomial of degree 4 or 5 gave a good compromise.

Having calculated the appropriate coefficients for the polynomial the derivative can easily be obtained and the resulting polynomial used to calculate corresponding stress intensity factors.

Results.

Single edge notch specimen. The results obtained for stress intensity factor are shown in Fig. 4 together with results from static experiment and boundary collocation.

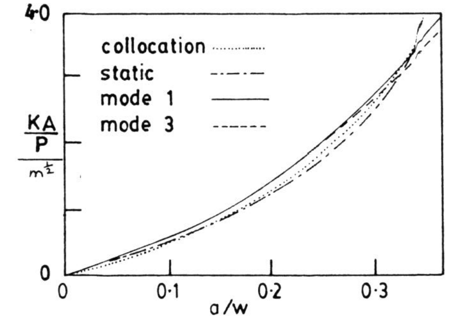

Fig. 4. Single edge notch

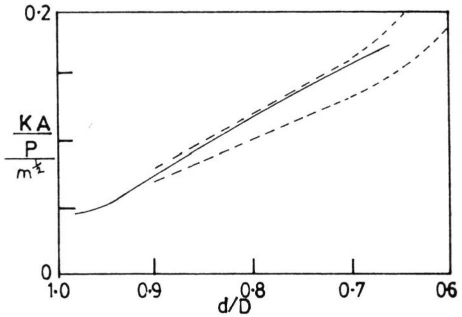

Fig. 5. Axisymmetric notch

For values of a/W of less than 0.3 the results obtained from the new method are slightly higher than those obtained by the other methods, which are in good agreement. However, for values of a/W greater than 0.3 the stress intensity factor as determined by dynamic measurement is lower than that determined by the other methods, this discrepancy becoming serious for values of a/W greater than 0.35; for example, with a/W equal to 0.4 the stress intensity factor is underestimated by 18% with vibration mode 1 and 25% with mode 3.

Cylindrical specimen. Hellen (1975) presents various results from several authors for this crack geometry. The upper and lower limits of these are shown in Fig. 5 together with the results obtained in this work. Bueckner (1965) obtained results which lie close to the upper bound and have a claimed accuracy of 1%, a claim which Hellen concludes could well be valid. Over the range of crack sizes for which

Hellen presents results the values obtained by the natural frequency measurement technique are between the upper and lower limits, tending towards the upper limit for larger cracks (that is lower values of d/D).

As is to be expected from the compliance results both modes are in close agreement, the largest discrepancies occur at the extremes of crack size considered but are always less than 5%.

Discussion. The above two cases show that the technique described can be used to obtain stress intensity factors for various crack geometries. The greater accuracy of the results for the cylindrical specimen suggest that the main source of error is the coupling between axial and flexural vibration which is introduced when the crack is of an asymetric nature. In this case the technique can still be applied to smaller crack sizes. Errors arising from the assumption that all the compliance change occurs at one point could be overcome by adding a large mass to each end of the bar and then considering it to be a two mass/spring system, as shown in Fig. 6. This would apply a load on the specimen that was uniform along the length of the specimen at any instant during the vibration cycle. This would reproduce the situation which occurs during static compliance measurement.

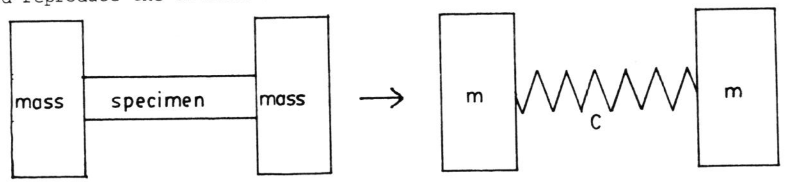

Fig. 6. Two mass/spring approximation of speciment with masses.

The determination of the crack size/compliance relationship for one geometry using the dynamic measurement technique could usually be completed in less than one hour if six or seven experimental points were used. This is likely to be less time than is required for static measurement where the specimen must be mounted in the testing machine and extensometers attached for every measurement. The technique will therefore offer the possibility of carrying out a compliance/crack size calibration in a relatively short time.

APPLICATION TO NON-DESTRUCTIVE TESTING

Having established the relationship between crack size, compliance and stress intensity factor it should be possible to use the compliance values obtained from the non-destructive test described by Adams and co-workers (1978) to determine the other two parameters. To establish the accuracy of this procedure an aluminium bar, similar to that used in the single edge-notch calibration, was tested over a range of crack sizes. The crack was located at a randomly chosen position, the only criteria being that the positions of the anti-nodes for the first three modes be avoided.

For each crack size the crack location and compliance were calculated from the changes in the natural frequencies of the specimen. The crack size was then determined using the polynomial relationships previously established. Table 1 shows the crack sizes obtained using polynomials determined by three methods; dynamic measurement using modes 1 and 3 and static measurement.

The values listed in Table 1 were obtained by taking the average of the values calculated using each of the three possible combinations of two modes out of the three considered. It can be seen that the crack is very accurately located over the range of sizes considered, the error being about 0·3% of the specimen length.

TABLE 1. Calculated crack size and position

Actual Crack Position = 0.118 m   Specimen Length = 0.387 m

a/W	Posn m	EC/2	a/W (Calculated values) Dynamic Mode 1	Dynamic Mode 3	Static	Frequency Hz Mode 1	Frequency Hz Mode 3
0.0	-	0.0	0.0	0.0	0.0	6566	19673
0.122	0.119	0.052	0.124	0.125	0.137	6547	19671
0.173	0.119	0.094	0.169	0.170	0.186	6525	19668
0.264	0.119	0.222	0.254	0.254	0.275	6464	19645
0.346	0.119	0.422	0.334	0.333	0.349	6361	19590

The crack sizes calculated using the crack size/compliance relationship determined from dynamic measurements are the most accurate and are very similar for both modes. This is to be expected as both the non-destructive test and the calibration use the same analysis and crack representation. The errors obtained from using a statically determined crack size/compliance calibration reflect the differences in the two calibration curves. These errors are smallest for the largest crack size used, which has a value of a/W near that at which the calibration curves cross (Fig. 2). For the smaller crack sizes the errors are approximately 10%.

Having obtained a value for a/W, it is then possible to determine the stress intensity factor using the derivative of one of the polynomials determined during the compliance calibration. Table 2 shows the values obtained using relationships determined by the vibration method with values of crack sizes from the non-destructive test, together with the values recommended by Srawley, Jones and Cross (1964) for the *actual* crack size.

TABLE 2. Stress intensity factors determined from ndt

a/W	KA/P ($m^{1/2}$) from ndt Mode 1	Mode 3	Recommended KA/P ($m^{1/2}$)
0.122	0.79	0.75	0.66
0.173	1.12	1.12	0.97
0.264	1.91	1.96	1.93
0.346	3.45	3.34	3.66

The values obtained are higher than the recommended values for small cracks, and lower for the largest crack used. For a/W ≈ 0.26, the values obtained from the non-destructive test straddle the recommended value. The results indicate that the stress intensity factors can be evaluated to within 20% using the non-destructive test described which on many engineering situations will be sufficiently accurate particularly as, for small, less critical cracks, the method will over-estimate the stress intensity factor.

CONCLUSIONS

A new experimental method for determining compliance changes, and hence stress intensity factors, for different crack geometries has been described. The method is not as accurate as static compliance measurement but is quicker and will often provide answers of sufficient precision for engineering applications. When this technique is combined with a previously described non-destructive test it is possible to obtain accurate values of crack location and size in a damaged structure. From this information stress intensity factors can then be determined.

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