A FINITE ELEMENT ANALYSIS OF TRANSIENT CRACK PROBLEMS WITH A PATH-INDEPENDENT INTEGRAL

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ABSTRACT

The use of a path-independent dynamic J-integral to compute the time-dependent stress intensity factor in the finite element analysis of elastodynamic crack problems is presented. The potential application of this dynamic J-integral, in the case of instrumented impact tests, is also demonstrated.

KEYWORDS

Dynamic J-integral; dynamic finite element analysis; instrumented impact tests; fracture dynamics; crack tip singular element.

INTRODUCTION

The notched beam impact tests have been traditionally used to determine a material's resistance to fracture based on energy consideration. For the past several years, these tests have been modified to obtain the fundamental material property, the dynamic fracture initiation toughness, K_{Id}; and these are commonly referred to as instrumented impact tests. Initially, the dynamic fracture initiation toughness, K_{Id}, was computed on the assumption that the crack growth initiation occurs at the maximum impact load and that the static relations can be applied. Several investigators (Aberson and colleagues, 1977a; Kalthoff and colleagues, 1977; Mall and colleagues, 1978, 1980a) have studied critically the various aspects of instrumented impact tests. These investigations have shown that computation of K_{Id} based on static relation and the peak value of impact load can lead to erroneous results; and therefore, for proper evaluation of K_{Id}, fully dynamic analyses must be performed that take inertia effects into account.

Dynamic analyses of impact tests have utilized both finite difference and finite element methods. The special singular crack tip elements (Aberson and colleagues, 1977b) as well as conventional elements (Mall and colleagues, 1978) have been employed in the finite element method. An alternate singular element, obtained by placing the mid-side node at the quarter position near the crack tip in 8-node isoparametric element, has been also utilized to study the transient response of impacted specimens (Ayres, 1976; Mall, 1980a). This distortion in configuration of 8-node isoparametric element produces the elastic inverse square root singularity at the crack tip. This simple singular element is especially attractive to users since 8-node isoparametric elements are now available in most computer programs and thus saves the extra effort of developing a special singular element. However, the special singular element has the distinct characteristics which permit the direct computation of stress intensity factor. On the other hand, a scheme should be devised to extract the dynamic stress intensity factor with the quarter-point singular element. Mall (1980a) employed an indirect procedure which consisted of the comparison between the dynamic crack opening displacement of the node close to the crack tip to its static counterpart obtained from the same finite element mesh. This ratio of the dynamic and static crack opening displacement is the ratio of the dynamic stress intensity factor to its static value. Ayres (1976) computed the transient stress intensity factor from the conventional J-integral in the dynamic analysis of the Charpy specimen with the quarter-point singular element. Since the conventional J-integral is not a path-independent in the dynamic case, Ayres (1976) determined J-integral on a contour passing through the node points nearest to the crack.

The objective of this paper is to present an alternate procedure to compute the time-dependent stress intensity factor with the help of a path independent J-integral for elastodynamic crack problems, as well as to demonstrate the potential application of this path-independent dynamic J-integral in instrumented impact tests.

THE J-INTEGRAL FOR ELASTODYNAMIC CRACK PROBLEMS

The conventional J-integral has been extended by Bui (1977) for elastodynamic crack problems as follows:

$$J_{Id} = \int_{\Gamma} (Wn_1 - \sigma_{jk}n_k u_{j,1} - \frac{1}{2}\rho\dot{u}_j\dot{u}_j n_1)ds + \frac{d}{dt}\int_{A} \rho\dot{u}_j u_{j,1}\, dA - \int_{\Gamma} \rho\dot{u}_j u_{j,1} V n_1 ds \qquad (1)$$

where W is the elastic energy density, σ_{ij} and u_i are stress and displacement components respectively, $\dot{u}_i = \partial u_i/\partial t$ and ρ is mass density. The n_i is the unit outward normal to the contour Γ joining two points on opposite sides of the crack's surface while going around the tip, and moving with the velocity V, and A is area enclosed within the contour Γ. This J-integral for dynamic crack problems is not a line integral, due to the second term. However, it is a path-independent as shown by Bui (1977). The dynamic J-integral as given by Eq. (1) can also be expressed as follows:

$$J_{Id} = \int_{\Gamma} (Wn_1 - \sigma_{jk}n_k u_{j,1})ds + \int_{A} \rho\ddot{u}_j u_{j,1} dA \qquad (2)$$

where the first term is the line integral and is the static J-integral of crack problems. The second term in Eq. (2) is the area integral over the area A enclosed by an arbitrary contour Γ; and it is the modification of the static J-integral due to the presence of inertia force in case of dynamic crack problems. This has been shown by Mall (1980b). The J_{Id} as expressed in Eq. (2) is preferred since it is simple to incorporate in the finite element program by adding the area integral term in the existing subroutine of the static J-integral. The dynamic stress intensity factor, in case of elastodynamic stationary crack problems, can be thus computed as follows:

$$K_{Id} = \sqrt{\kappa E\, J_{Id}}$$

where $\kappa = 1/(1-\upsilon^2)$ and 1 for plane strain and stress conditions, respectively. The E and υ are Young's modulus and Poisson's ratio, respectively.

A STRIP WITH A CENTRAL CRACK

This is a theoretical problem, and was analyzed to compare the present results with previously available results (Aberson and colleagues, 1977b; Chen, 1975). This problem involves the dynamic analysis of a strip (20 x 40 mm) with a central crack (2a = 4.8 mm) subjected to a suddenly applied and maintained tension, σ. Figure 1 shows the finite element mesh of one quadrant of this strip which consists of 90 quadratic elements. Two elements at the crack tip are the singular elements obtained by placing the mid-node at the quarter-point position near the crack tip. The material of the strip is a linear elastic having Young's modulus = 200 GPa, Poisson's ratio = 0.3 and density = 5 g/cm^3. The finite element NONSAP code (Bathe and colleagues, 1974) was employed with Newmark β integration scheme (with $\beta = 1/4$) and lumped mass formulation. Three time steps corresponding to the transit time of longitudinal wave across the two side nodes, side and mid-nodes, and side and quarter nodes of the singular element were used. These time steps yielded the dynamic stress intensity factor within 1 percent of each other. The time step based on the overall dimension of the element can be, thus, employed with the quarter-point singular element.

The relations between time and normalized dynamic stress intensity factor ($K_{Id}/\sigma\sqrt{a}\sqrt{\pi}$) are shown in Fig. 2. The solid line in Fig. 2 shows the normalized dynamic stress intensity factor obtained by comparing the dynamic and static crack opening displacement of the node at quarter position of the singular element. The data points, in Fig. 2, show the normalized dynamic stress intensity factors obtained from J_{Id} for three contours, shown in Fig. 1. These results clearly demonstrate the path-independence of the dynamic J-integral. The dynamic stress intensity factors obtained from J_{Id} are in excellent agreement with the indirect displacement procedure which has been shown as a simple and accurate technique to compute the time-dependent stress intensity factor for elastodynamic stationary crack problems (Mall, 1980a). Further, the results shown in Fig. 2 are in excellent agreement with those obtained by previous investigators (Aberson and colleagues, 1977b; Chen 1975) and are not compared here for the sake of brevity.

A THREE-POINT BEND SPECIMEN

A series of drop weight impact tests of 3-point bend specimens of A533 B steel at different temperatures were performed by Loss (1976) where the tup load as well as dynamic strain at a location near the crack tip were measured. Here one specimen tested at 12°C will be analyzed. The specimen having length, height and thickness of 228.6, 50.8 and 25.4 mm, respectively, was fatigue precracked with a nominal crack length of 25.4 mm. Figure 3 shows the finite element breakdown of half of the specimen. The two elements near the crack tip are quarter-point singular elements and the rest are 8-node isoparametric elements. The specimen was instrumented with a 3.2 x 3.2 mm strain gage at the location shown in Fig. 3. Prior to impact, the specimen was calibrated statically to establish a relationship between the strain of this gage and the applied load at mid-span. This calibration with the measured dynamic strain of the same gage during actual impact test provided the equivalent specimen load in three-point bend for the equivalent static condition. This procedure of assessing the equivalent specimen load is based on the assumption that the static calibration between the strain (at a location near the crack tip) and load at mid-span are independent of strain rate. This assumption is valid as will be shown from the present dynamic analysis.

The dynamic analysis was conducted under the plane stress condition where the smoothed impact load-time relation, shown in Fig. 4, was input at the node

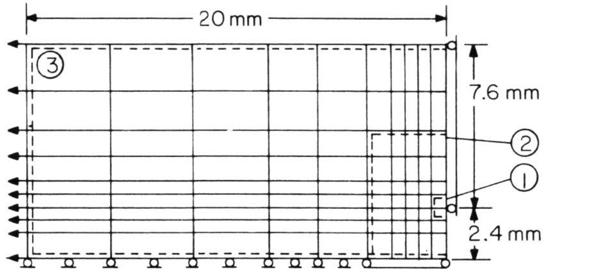

Fig. 1. Finite element mesh of a centrally cracked strip.

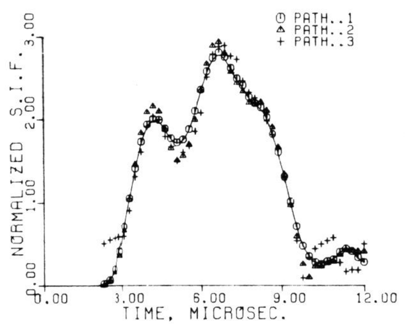

Fig. 2. Dynamic stress intensity factors of a centrally cracked strip.

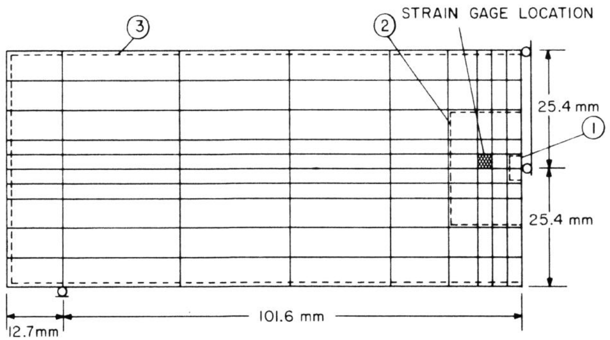

Fig. 3. Finite element mesh of a bend specimen.

corresponding to the impact point. This node was also given the experimental impact velocity of 2.4 m/s as the initial condition in the analysis. The computed dynamic strain at the gage location (shown in Fig. 3) is compared with its experimental counterpart in Fig. 5 and is in good agreement. The time of crack initiation is shown by the sudden drop in the experimental strain in Fig. 5. Figure 6 compares the dynamic stress intensity factor obtained from the indirect displacement procedure (solid line) with those obtained from the dynamic J-integral for three contours (data points), shown in Fig. 3. The dynamic stress intensity factor obtained from J_{Id} is, thus, independent of integration path; and also in agreement with its counterpart obtained from the displacement method.

The equivalent specimen load during impact, obtained from the computed dynamic strain at the gage location, is expressed as the function of displacement of node corresponding to impact in Fig. 7. This relation between the equivalent load and concurrent displacement is practically linear (see the dotted line in Fig. 7); and it can be employed to determine the critical dynamic J-integral from the following:

$$J_{Id} = \frac{2}{B} \int P d\delta \tag{3}$$

where P is the applied load, δ is the displacement at loaded point and B is the uncracked ligament area. The term $\int P d\delta$ is the area under the load-displacement curve of Fig. 7 up to the instant of crack initiation, 0.43 millisecond. The critical dynamic J-integral is computed to be 86.4 kJ/m^2 from Eq. (3). This value of critical J_{Id} is in excellent agreement with the J_{Id} obtained from Eq. (2) for three contours whose average value is equal to 86.5 kJ/m^2 at the instant of crack initiation. This agreement, further, verifies the validity of the present analysis.

The calculation of equivalent specimen load is based on the assumption that static calibration of the strain gage near the crack tip is independent of strain rate. This can be shown in the following manner. The dynamic stress intensity factor can also be computed from this equivalent specimen load from the following static stress intensity factor, K - expression for the bend specimen (Srawley, 1976)

$$K = \frac{PS}{BW^{3/2}} f(a/w) \tag{4}$$

where P is equivalent specimen load, S is span, B is width and W is height of specimen. Figure 8 shows the dynamic stress intensity factor obtained from the equivalent specimen load with Eq. (4) as well as its counterpart obtained from the dynamic analysis. The dynamic stress intensity factor obtained from the equivalent specimen load in conjunction with static analysis is in excellent agreement with its counterpart obtained from dynamic analysis. It can be, thus, concluded that the equivalent specimen load is independent of strain rate. Further, the equivalent load can be utilized to obtain the dynamic fracture initiation toughness or critical dynamic J-integral from the usual static procedure in instrumented impact tests. However, the displacement during the impact will be required for J_{Id} analysis; and this measurement will be an extra task in instrumented impact tests. The present analysis can, therefore, serve a useful role in instrumented impact tests.

CONCLUSIONS

1. The dynamic version of J-integral, proposed by Bui (1977) and modified by the author, can be employed to compute the time-dependent stress intensity factor in the finite element analysis of elastodynamic crack problems. This dynamic J-integral is the path-independent and simple to incorporate in the existing conventional J-integral subroutine by introducing an area integral term of Eq. (2).

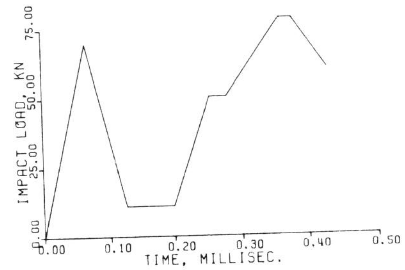

Fig. 4. Smoothed impact load of a bend specimen.

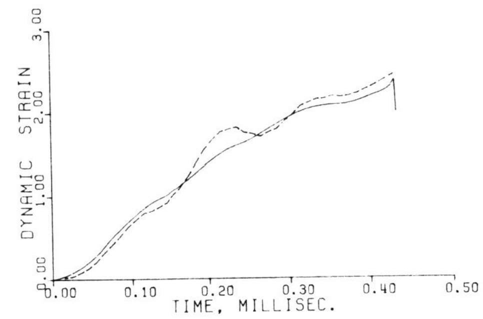

Fig. 5. Comparison of experimental (solid line) and computed (dashed line) dynamic strain (x 10^{-3}) of a bend specimen.

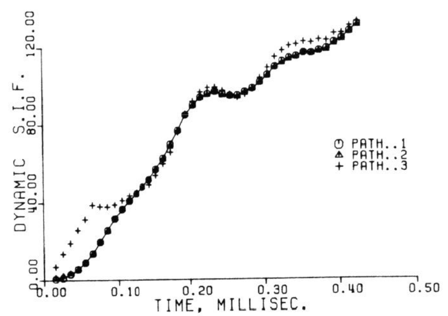

Fig. 6. Dynamic stress intensity factors (kPa$\sqrt{m}$) of a bend specimen.

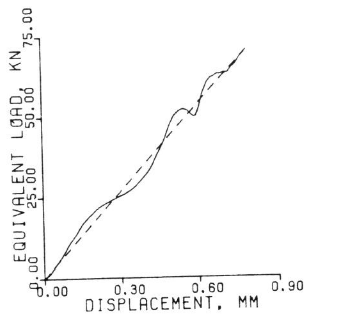

Fig. 7. Equivalent speciman load - displacement relation of a bend specimen.

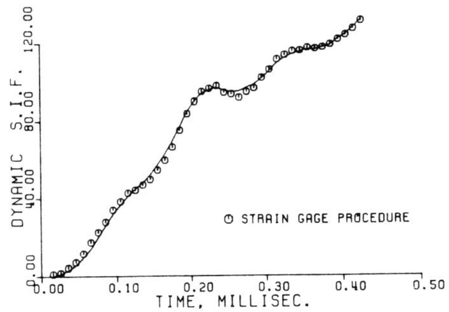

Fig. 8. Comparison of dynamic stress intensity factor (kPa$\sqrt{m}$) obtained from the indirect displacement procedure (solid line) and from equivalent specimen load (data point).

2. The present study also demonstrates that there is a potential application of this J-integral in instrumented impact tests to obtain the fundamental material property, the critical dynamic J-integral (J_{Id}) or dynamic fracture initiation toughness (K_{Id}).

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