STRAIN DEPENDENCE OF THE J-CONTOUR INTEGRAL IN TENSILE PANELS*

D. T. Read and H. I. McHenry
Fracture and Deformation Division
National Bureau of Standards
Boulder, Colorado 80303

ABSTRACT

The J contour integral has been experimentally measured as a function of applied strain for single edge notch tensile panels under elastic-plastic loading conditions. The results have been compared to analytical predictions based on finite element analysis and to theoretical estimates based on models representing the two behavioral extremes, uniform strain and perfect plasticity. The experimental, analytical and theoretical results have the expected form: the J-integral initially increases as the square of the applied strain, and at strains above yield the J-integral is a linear function of strain. The experimental and analytical results are in reasonable agreement, while the uniform strain and perfect plasticity models under- and over-estimate J respectively. An extension of the perfect plasticity model is proposed to treat behavior between these two limiting cases.

KEYWORDS

Crack driving force; elastic-plastic conditions; J-integral; fracture mechanics; tensile panels; finite-element-analysis; theoretical models

INTRODUCTION

The J-integral has gained widespread acceptance as a measure of the driving force for fracture under elastic-plastic conditions, that is, when notch tip plasticity is extensive. This report describes experimental measurements of applied J-integral values as a function of strain in simple configurations relevant to structural components. The experimental results are compared with results obtained from finite element analysis and theoretical models.

EXPERIMENTAL PROGRAM

Experimental measurements of the applied J value as a function of strain for several notch lengths have been performed. Three different experimental approaches were applied to allow verification of results: direct measurement of the contour integral; measurement of the pseudo-potential-energy change with crack extension (compliance technique); and measurement of crack tip opening displacement (CTOD). Each approach applies a known relationship between J and some set of experimentally measurable quantities, as described in detail by Read and McHenry (1980).

Experimental Procedures

The material chosen for the present study was a normalized C-Mn steel with a yield strength of 340 MPa. Single-edge-notched tensile panels with gage section 100 mm wide, 348 mm long, and 12 mm thick were tested with notch lengths of 0, 2, 10, 30 and 46 mm. Tension-tension fatigue precracking was used to sharpen the crack tips (except for the 2 mm notch which was tested with the notch tip as cut with a jeweler's saw).

The instrumentation required for direct measurement of the J contour integral consisted of twenty electrical resistance strain gages and three linear variable displacement transducers (LVDT's) mounted on the specimen (Fig. 1) and a minicomputer for acquisition and storage of the strain and displacement values. The terms of the integrand were derived from the measured strain and displacement data and the integration was performed numerically using the trapezoidal rule.

A qualitative measure of the shape of the strain fields around the crack tips was obtained by coating the specimen gage sections with brittle lacquer before straining and then observing darkened regions in the lacquer which corresponded to regions of high specimen strain. Photographs were made of the strain patterns at regular intervals during the test.

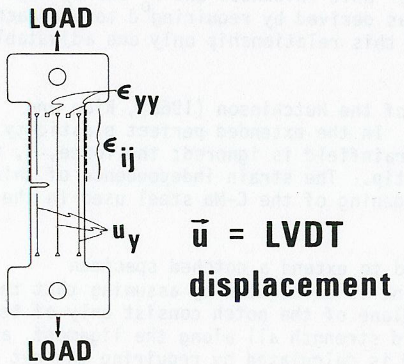

Fig. 1. Experimental arrangement for direct measurement of the J-contour-integral.

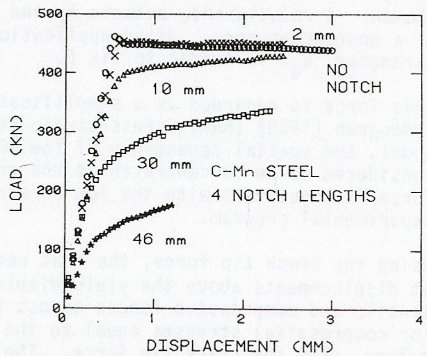

Fig. 2. Load plotted against displacement for all specimens.

RESULTS

The load-displacement records are shown in Fig. 2. J-integral values were determined as a function of strain for the four notched tests. Strain was defined as the average displacement measured by the three LVDT's over a gage length of 348 mm. Data points were obtained at strain intervals of about 0.00015. J-values were determined by the direct evaluation of the J-contour-integral, Fig. 3, and by the compliance method, Fig. 4. The CTOD at maximum strain was measured using the replication technique; the results are shown in Table 1. The general form of the J vs. ε curves is a parabolic dependence of J on strain at low strains and a linear dependence at strains above yield. The parabolic-then-linear form of the J-ε curves consistent with previous experimental studies employing the compliance techique by Bucci, Paris,

Landes and Rice (1972), with theoretical studies by Begley, Landes and Wilson (1974), and with theoretical and experimental studies in support of British COD design curve (Burdekin and Stone, 1966). Significant deviations from the parabolic-then-linear dependence of the J on strain were observed in certain of the tests of the present study. In the 2 mm notch, the rapid rise in J at the yield strain occured as narrow shear bands emanated from the crack tip, Fig. 5. At higher strains (>0.0025), the shear bands spread due to strain hardening, and effectively masked the 2 mm notch so that J no longer increased with strain. In the 10 mm notch test, the shape of the J-ε curve was influenced by plastic deformation near the holes machined through the specimen to attach the LVDT's. Note that these holes did not cause plastic deformation (detectable by cracking of the brittle lacquer) in the deeper-notch tests; holes were not used in the 2 mm notch specimen. The results for the 46 mm notch test were influenced by the loading history which included complete unloadings at strain values of 0.0012 and 0.0019. In addition, shear strains, which are greater for the deeply notched case, were not accounted for in the direct measurement of J.

Table 1 Crack tip opening displacements (CTOD) and resulting J values, calculated using m = 2.2.

Notch length (mm)	Strain	CTOD (mm)	J(kN/m)
10	0.011	1.57	1152
30	0.0077	1.90	1394
46	0.0034	0.56	409

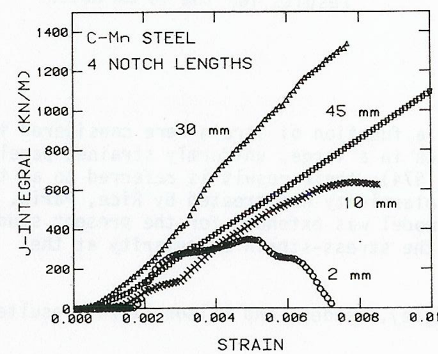

Fig. 3. Experimental results for the J-integral obtained by direct measurement of the contour integral.

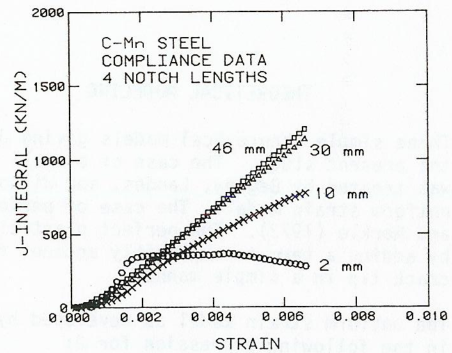

Fig. 4. Experimental results for the J-integral determined by the compliance technique.

Analytical Results

The two-dimensional elastic-plastic finite element analysis computer code of Gifford (1975, 1978) and Hilton and Gifford (1979) was used to calculate J-integral vs. strain. This program incorporated special nonlinear crack tip elements and conventional 12-node quadrilaterial isoparametric elements. Finite element analysis calulations of J vs strain were carried out for 30 and 46 mm single-edge-notches to model the experimental situation. Limitations of the finite element analysis program in the treatment of large strains prevented calculation of results for the 2 mm notch and prematurely terminated the results for the 10 mm notch. The general form of the finite element results, shown in Fig. 6, was parabolic-then-linear as expected.

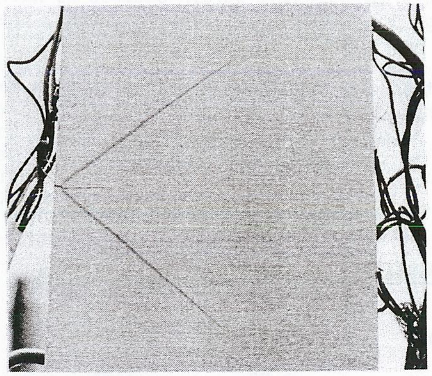

Fig. 5. Photograph of strain pattern revealed by brittle lacquer, in C-Mn steel specimen with 2.25 mm notch.

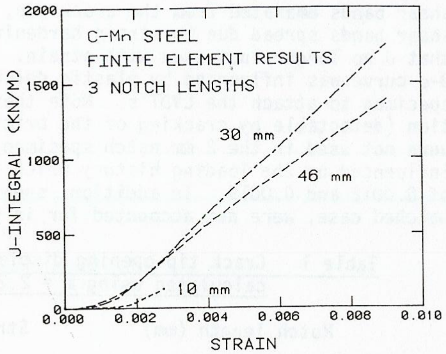

Fig. 6. Finite element results simulating the behavior of two specimens of the present study in the 46 mm notch and the 30 mm notch, and partial results for the 10 mm notch.

THEORETICAL MODELING

Three simple theoretical models giving J as a function of strain were considered in the present study. The case of a small notch in a large, uniformly strained panel was treated by Begley, Landes, and Wilson (1974); their result is referred to as the uniform strain model. The case of perfect plasticity was treated by Rice, Paris, and and Merkle (1973). The perfect plasticity model was extended for the present study by adding a term to explicitly account for the stress-strain singularity at the crack tip in a simple manner.

The uniform strain model as developed by Begley, Landes, and Wilson (1974) resulted in the following expression for J:

$$J = \frac{\sigma_y^2}{E} \pi a \left(\frac{\varepsilon}{\varepsilon_y}\right)^2 \quad \text{for } \varepsilon/\varepsilon_y \leq 1 \qquad (1a)$$

$$J = \frac{\sigma_y^2}{E} \pi a \,(2\varepsilon/\varepsilon_y - 1) \quad \text{for } \varepsilon/\varepsilon_y \geq 1 \qquad (1b)$$

These formulas were derived for the case of a notch that is small enough that plastic strains are distributed throughout the panel rather than concentrated at the notch section. This situation mininizes J, and therefore, the uniform strain model is a lower bound solution for J vs. ε.

The elastic-perfectly plastic model for J as a function of strain follows the work of Paris, Tada, Zahoor, and Ernst (1979). Limit load per crack tip, P, is governed by yielding of the ligaments (W - a):

$$P = \sigma_y \,(W - a)\, T \qquad (2)$$

and plastic displacements, Q_p, are concentrated at the crack tips, that is,

$$J_p = Q_p \sigma_y \qquad (3)$$

where J_p is the plastic component of J. Adding the elastic component, J_e, yields:

$$J = J_e + J_p = \sigma^2 \pi a/E + Q_p \sigma_y \qquad (4)$$

The strain distribution assumed in this model maximizes J, and therefore the perfect plasticity model usually provides an upper bound solution for J vs. ε.

Equation 2 neglects the existence of a stress-strain singularity at the crack tip. A simple way to approximately include the singularity is to postulate a line tensile force which operates along the crack tip perpendicular to the crack plane. This force was assumed to be proportional to a for $a/W \leq 0.1$ and to approach a constant value for longer cracks. A convenient functional form for such a force is the exponential relationship:

$$F = F_o \,(1 - e^{-a/a_o}). \qquad (5)$$

Here, F is the force across the crack tip per unit thickness and F_o and a_o need to be chosen. A relationship between F_o and a_o was derived by requiring J to approach zero as a approaches zero. After application of this relationship only one adjustable parameter, a_o, is needed to fix F.

This force is regarded as a simplification of the Hutchinson (1968), Rice and Rosengren (1968) (HRR) Stress-strain field. In the extended perfect plasticity model, the spatial dependency of the HRR strainfield is ignored; the force, F, is considered to be concentrated at the crack tip. The strain independence of this force is consistent with the low strain hardening of the C-Mn steel used in the experimental program.

Using the crack tip force, the load required to extend a notched specimen at displacements above the yield displacement is calculated by assuming that the tensile and compressive forces across the plane of the notch consist only of tensile (or compressive) stresses equal to the yield strength all along the ligament, as before, plus the crack tip force. The load is calculated by requiring the net load and net moment applied to each half of the specimen to be nil. Once the load has been calculated, Eq. 8 is used to calculate J. The result for J_p, is:

$$J_p = Q_p \,(\sigma_y - \partial F/\partial a) \qquad (6)$$

The result for the single-edge-notched specimen is:

$$J_p = \sigma_y Q_p \left[1 - f' + \frac{[2f + (1-2\alpha)(1 - f')]}{\sqrt{1 + 2f(1-2\alpha)} - 2\alpha(1-\alpha)} \right] \qquad (7)$$

where $f = F/\sigma_y TW$, $\alpha = a/w$, and $f' = \partial f/\partial \alpha$. The full expressions for the applied J value were formed by adding the linear elastic part to the plastic part.

Comparison of Results

The experimental load-displacement data for the specimen with the 30 mm notch are compared to calculated values from finite element analysis and the extended perfect plasticity theory in Fig. 7. The differences between experimental and calculated values are attributed to strain-hardening effects not accurately accounted for in the calculations. Figure 8, in which load at a strain of 2 times yield is plotted against crack length, shows that the crack length dependences of both the experimental and the analytical results are well represented by the extended perfect plasticity theory.

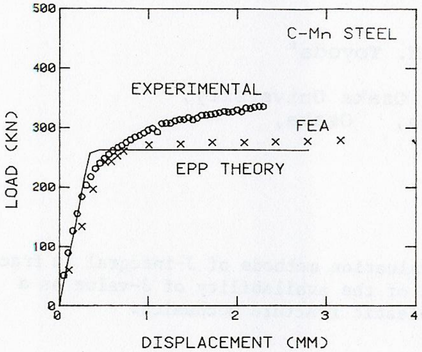

Fig. 7. Experimental, analytical, and theoretical results for the single-edge-notched specimen with the 30 mm notch; included are experimental, finite element analysis (FEA), and extended perfect plasticity (EPP) theory results.

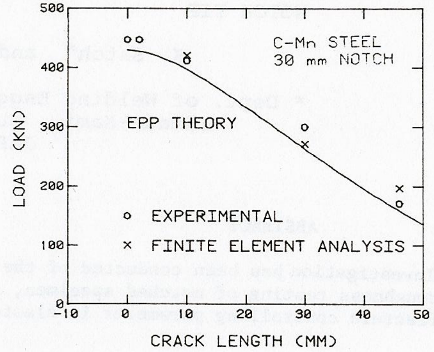

Fig. 8. Load at a displacement of 2 times yield as a function of crack length; experimental, finite element analysis, and extended perfect plasticity (EPP) theory results are shown.

Figure 9 displays the contour integral, compliance, CTOD, finite element analysis and the extended perfect plasticity model results for the 30 mm notch plotted as functions of strain. The agreement among these different methods of determining the J-integral was not as good for the other notch lengths, because of the experimental problems noted above.

Practically all the measured and calculated J integral results had the same type of strain dependence. But the results of the theoretical models differ significantly in their dependence on crack length. This is shown in Fig. 10, which displays J as a function of crack length at a strain of 4 times the yield strain. Experimental results by the compliance and contour integral techniques, finite element analysis results, and the three theoretical models are plotted. This figure shows that the experimental results disagree with both the perfect plasticity and the uniform strain theories. The extended perfect plasticity theory predicts J values which lie between those of the uniform strain and perfect plasticity models, and best represents the experimental and analytical results. Similar conclusions were drawn from Fig. 11, in which the uniform strain, perfect plasticity, and extended perfect plasticity models are compared with one another and with finite element results for a center-notched-panel.

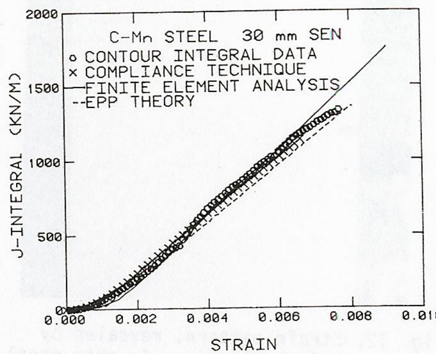

Fig. 9. Experimental, analytical, and theoretical results for single-edge-notched specimen with 30 mm notch; included are direct contour integral (CI), compliance technique (CT), finite element analysis (FEA), and extended perfect plasticity theory (EPP) results.

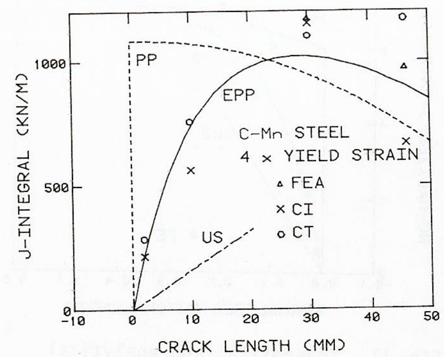

Fig. 10. Experimental, analytical, and theoretical results for the single-edge-notched specimens; included are direct contour integral (CI), compliance technique (CT), finite element analysis (FEA), uniform strain theory (US), perfect plasticity theory (PP) and extended perfect plasticity theory (EPP) results.

DISCUSSION

When this study was begun, it was hypothesized that the uniform strain model could be verified over a significant range of crack lengths. However, the experimental and analytical studies consistently produced J values several times those predicted by the uniform strain model; they were often in the neighborhood of the perfect plasticity result. But the perfect plasticity model was clearly unsatisfactory for short notch lengths. A physical interpretation of this result has been developed over the course of this study. This interpretation is that the uniform strain model holds only when plasticity conditions are such that plastic strains are spread over the whole length of the strained panel and are prevented from concentrating at the crack tip. A dramatic case of strain concentration at a crack tip is illustrated in Fig. 5. Slip bands at ± 45° and 90° to the tensile axis emanated from the crack tip. Each increment of applied displacement contributed to the strain in the slip bands. Because the slip bands terminated at the crack tip, all the strain contributed to the opening of the crack. The photograph in Fig. 12 shows that the strains were not concentrated at the tip of the 10 mm notch as much as for the 2 mm notch, and the J values shown in Fig. 3 for strains between 0.002 and 0.003 are lower for the 10 mm notch.

It is concluded that the limiting cases for the behavior of J as a function of strain in tensile panels are provided by the uniform strain and perfect plasticity models. In the uniform strain model only limited strain concentration at the crack tip is allowed; in the perfect plasticity model all the plastic strain is concentrated at the crack tip. The extended perfect plasticity theory is intermediate between these two extremes.

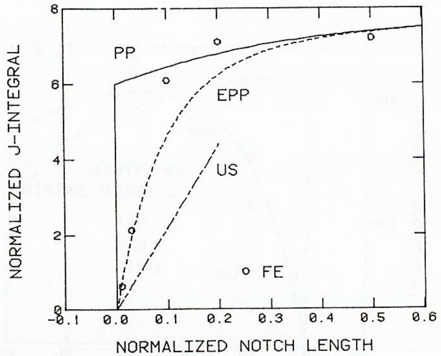

Fig. 11. Theoretical and analytical results for a center-notched-panel.

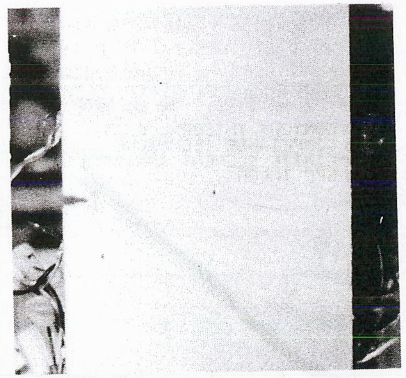

Fig. 12. Strain pattern, revealed by brittle lacquer, in ship steel specimen with 10 mm notch.

ACKNOWLEDGMENTS

The financial support of the U.S. Naval Sea Systems Command through J. P. Gudas, Project Monitor, and the technical assistance of J. W. Elmer, V. Hauptman and J. D. McColskey are gratefully acknowledged.

REFERENCES

Begley, J. A., J. D. Landes, and W. K. Wilson (1974). An Estimation Model for the Application of the J-Integral. Fracture Analysis, ASTM STP 560, American Society for Testing and Materials, 155-169.

Bucci, R. J., P. C. Paris, J. D. Landes, and J. R. Rice (1972). J Integral Estimation Procedures. Fracture Toughness, ASTM STP 514, American Society for Testing and Materials, 40-69.

Burdekin, F. M., and D. E. W. Stone (1966). The Crack Opening Displacement Approach to Fracture Mechanics in Yielding Materials. J. Strain Analysis, 1, 145-153.

Gifford, L. N. (1975). APES-Second Generation Two-Dimensional Fracture Mechanics and Stress Analysis by Finite Elements. David Taylor Naval Ship Research and Development Center, Bethesda, MD 20084, Report 4799.

Gifford, L. N. (1978). J-Integral Analysis of a Compact J_{Ic} Specimen. David Taylor Naval Ship Research and Development Center, Bethesda, MD 20084, Report M-4.

Hilton, P. D., and L. N. Gifford (1979). Evaluation of Some Crack Tip Finite Elements for Elastoplastic Fracture Analysis. David Taylor Naval Ship Research and Development Center, Bethesda, MD 20084, Structures Department Research and Development Report DTNSRDC-79/052.

Hutchinson, J. W. (1968). Singular Behavior at the End of a Tensile Crack in a Hardening Material. J. Mech. Phys. Solids, 16, 13-31.

Paris, P. C., H. Tada, A. Zahoor, and H. Ernst (1979). The Theory of the Tearing Mode of Elastic-Plastic Crack Growth. Elastic Plastic Fracture, ASTM STP 668, American Society for Testing and Materials, 5-36.

Read, D. T. and H. I. McHenry (1980) Detailed Report on Strain Dependence of the J-Contour Integral. To be published.

Rice, J. R., P. C. Paris, and J. G. Merkle (1973). Some Further Results of J-Integral Analysis and Estimates. Progress in Flaw Growth and Fracture Toughness Testing, ASTM STP 536, American Society for Testing and Materials, 231-245.

Rice, J. R., and G. F. Rosengren (1968). Plane Strain Deformation Near a Crack Tip in a Power-Law Hardening Material. J. Mech. Phys. Solids, 16, 1-12.