# SIMPLIFIED DETERMINATION OF J-INTEGRAL AND ITS AVAILABILITY AS A STRAIN INTENSITY PARAMETER AT NOTCH TIP

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## ABSTRACT

Investigation has been conducted of the evaluation methods of J-integral in fracture toughness testing of notched specimen, and of the availability of J-value as a fracture controlling parameter in elasto-plastic fracture mechanics.

## KEYWORDS

J-integral; fracture toughness; strain intensity parameter; elasto-plastic fracture mechanics; fracture criterion.

## INTRODUCTION

It has become common to apply the J failure concept (Rice, 1968) for obtaining fracture toughness of non-linear characteristic materials. For fracture toughness testing, some experimental evaluation methods of $J_C$-value have been proposed(Landes, 1974 and 1976). Although J-integral has a sufficient physical meaning for linear and/or non-linear elastic material (Rice, 1968), the availability of J-integral as a plastic singularity parameter at the tip of notch is not always made clear. The applicable range of formulas of J-calculation proposed previously is not also clarified, though methods of estimation of J-integral in toughness testing specimen with a deep notch were proposed(Rice, 1973).

In the present paper, attention is paid to the following two fundamental considerations; 1) the simplified evaluation methods of J-integral in fracture toughness testing of notched specimen, and 2) the availability of J-integral evaluated by the proper methods as a strain intensity parameter at crack tip.

Consideration, at first, has been carried out of simplified formulas for estimating J-integral, in particular including in specimens with a shallow notch. The accuracy and applicable range of the previous conventional formulas and newly established formulas are investigated using FEM numerical analysis based on non-linear elastic and/or elasto-plastic characteristic.

The plastic strain distributions in the vicinity of a notch tip are measured using hardness distribution measurement techniques. According to these studies, it is investigated whether or not J-integral can be the controlling parameter of strain singularity in the vicinity of the blunting tip of notch .

## J-ESTIMATING FORMULAS FOR NON-LINEAR ELASTIC MATERIAL

A number of compuitional methods which are in varied stages of development and acceptance are available for calculating the J-integral in fracture toughness specimen( Landes, 1974 and 1976; Oji, 1978; Kanazawa, 1975; Sumpter, 1976; Hagiwara, 1979). Although they are simplified method by using load-displacement curve of notched specimen, they are applicable to specimens with a deep notch because it is assumed in these studies that deformation occures intensively at the net section of notched specimen. For example, in deeply notched tensile-specimen, Rice (1973) proposed the area approximation procedure, as shown in Fig. 1, under the following assumption in relation to load line displacement $U_{crack}$ (that is, the contribution due to the presence of the crack).

$$U_{crack} = b\ f(P/bt) \qquad (1)$$

These J-estimating formulas have been termed $J_d$ because these are for deeply notched specimen.

In the present study, the same area approximation procedures for estimating J-value in shallowly notched specimen are newly established. It is postulated that the load line displacement U separates into the following two components

$$U=U_{no\ crack} + U_{crack} \qquad (2)$$

$U_{crack}$ in this case can be estimated under the following supposition.

TABLE 1. Summary of J-estimating Formulas of Notched Specimens

( Specimen geometry : Fig. 2.)

| | | Assumption | J estimating formulas |
|---|---|---|---|
| Tensile notched specimen | Deep notch | $U_{crack}=b\,f(P/bt)$ | $J_d=\frac{1}{bt}[2\int_0^{U_{crack}} P dU_{crack} - PU_{crack}]$ (Rice) |
| | Shallow notch | $U_{crack}=c\frac{c}{W}f^*(P/Wt)$ | $J_s=\frac{1}{ct}\int_0^{P} U_{crack}\, dP$ (Authors) |
| Bending notched specimen | Deep notch | $\theta_{crack}=g(M/b^2t)$ | $J_d=\frac{2}{bt}\int_0^{\theta_{crack}} M d\theta_{crack}$ (Rice)<br>$(J_d=\frac{2}{bt}\int_0^{U_{crack}} P dU_{crack})$ |
| | Shallow notch | $\theta_{crack}=(\frac{c}{W})^2 g^*(M/W^2t)$ | $J_s=\frac{2}{ct}\int_0^{M} \theta_{crack}\, dM$<br>$(J_s=\frac{2}{ct}\int_0^{P} U_{crack}\, dP)$ (Authors) |
| Round bar with circumferential notch | Deep notch | $U_{crack}=r\,h(P/r^2)$ | $J_d=\frac{1}{2\pi r^2}[3\int_0^{U_{crack}} P dU_{crack} - PU_{crack}]$ (Rice) |
| | Shallow notch | $U_{crack}=c\frac{c}{2R}h^*(P/R^2r)$ | $J_s=\frac{1}{2\pi rc}\int_0^{P} U_{crack}\, dP$ (Authors) |

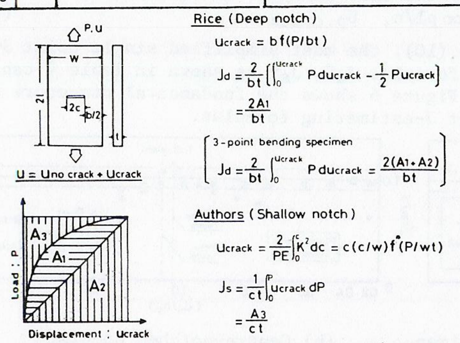



Fig. 1. Area approximation procedures for J-estimating in non-linear elastic material.

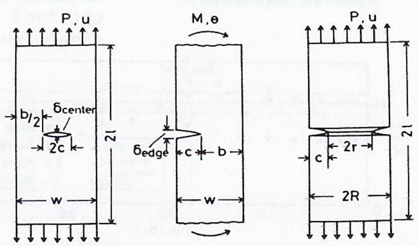



Fig. 2.



TABLE 2. Single Point J-estimating Formulas for Non-linear Elastic Material

| | | The single point J estimating formulas | |
|---|---|---|---|
| Tensile notched specimen | Deep notch | $J_d=\frac{1-n}{1+n}\sigma_N U_{crack}$ | $J_d=\frac{1-n}{1+n}\sigma_N \delta_{center}$ (Ōji) |
| | Shallow notch | $J_s=\frac{n}{1+n}\frac{b}{c}\sigma_N U_{crack}$ | (Authors) |
| Bending notched specimen | Deep notch | $J_d=\frac{2}{1+n}\frac{M}{bt}\theta_{crack}$ | $J_d=\frac{2}{1+n}\frac{P}{bt}U_{crack}$ (Ōji) |
| | Shallow notch | $J_s=\frac{n}{1+n}\frac{M}{ct}\theta_{crack}$ | $J_s=\frac{n}{1+n}\frac{P}{ct}U_{crack}$ (Authors) |

In LEFM,

$$U_{crack}=\frac{2t}{PE}\int_0^{c} K^2\, dc$$
$$= c(c/w)\cdot k\cdot P/wt$$
$$(k= \text{constant})$$

A similar(but non-linear) form is assumed for non-linear elastic materials :

$$U_{crack} = c(c/w)\cdot f^*(P/wt) \qquad (3).$$

The J-estimating formula is deduced in a way similar to Rice's(et al., 1973). Figure 1, Table 1 show the J-estimating formulas $J_s$ for shallowly notched specimens established newly.

In the above discussion of J-estimating formula, applicable ranges of $J_d$ and $J_s$ are not made clear. Consideration, here, has been carried out of the applicable ranges of $J_d$ and $J_s$ by applying deformation theory to material of which equivalent stress $\bar{\sigma}$-equivalent strain $\bar{\varepsilon}$ relation is given by

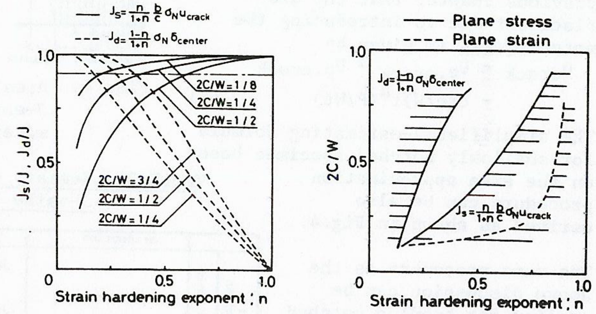
$$\bar{\sigma} = \sigma_o(\bar{\varepsilon}/\varepsilon_o)^n \qquad (4),$$

where $\sigma_o$, $\varepsilon_o$ are material constants and n is strain hardening exponent.

For deeply notched specimen of non-linear material according to pure power hardening law, Oji(1978) derived single point J-estimating formulas as shown in Table 2 by using results in Table 1 under the following assumptions.

$$U_{crack} \propto (P/bt)^{1/n} \qquad (5)$$
$$U_{crack} \propto \varepsilon_o$$

For shallowly notched specimen, single point J-estimating formulas can be derived under the same procedure. Here, we calculated J-value of tensile centre-notched specimen by FEM( Goldman, 1975; Shih, 1976) based on deformation theory. Figure 3(a) shows the relationship between the ratio of $J_d$ and $J_s$ determined by the above simplified formulas to J-value obtained by FEM analysis and strain hardening exponent n of material chosen. If 10 % error comparing with J-value obtained by FEM is allowed for estimating J-value of fracture toughness testing,the applicable range of $J_d$-, $J_s$-formulas is shown in Fig.3(b). For example, when n=0.2, that is for ordinary low C-Mn steel, $J_d$-formula is applicable in $2c/W > 0.5$, on the other hand $J_s$-formula in $2c/W < 0.25$.




(a) Accuracy (b) Applicable range

Fig. 3. Accuracy of simplified formulas $J_s$,$J_d$ and applicable range of each formula in non-linear material.

## J-ESTIMATING FORMULAS FOR ELASTO-PLASTIC MATERIAL

In general, structural metallic materials under loading show elasto-plastic behaviour. For incremental plasticity, which is the behavior more appropriate to real materials, path independence has not been proved theoretically. Elasto-plastic finite-element-analysis based on incremental strain theory, however, have shown J to be virtually path independent except for contours very close to the crack tip (JSME, PSC-21, 1979). In the present paragraph, regardless of existence of problems in relation to definition and meaning of J-integral, J-value calculated from the original path-integral-equation according to plastic analysis based on incremental strain theory is adopted for investigating the simplified J-estimating formulas.

According to Rices' analysis (1973) in elasto-plastic materials, the displacement between the load points is regarded as the sum of the elastic displacement $U_e$ plus the displacement due to plasticity $U_p$. In this analysis, it is presumed that the total displacement can be divided into the following four components, or

$$U = U_{e.crack} + U_{p.crack} + U_{e.no\ crack} + U_{p.no\ crack} \qquad (6)$$

For tensile centre-notched specimen, Rice has set up the J-estimating formula, as shown in Fig.4, based on the following assumption:

$$U_{p.crack} = b\ f(P/bt) \qquad (7)$$

The above assumption may be resonable for deeply notched specimen. For shallowly centre-notched specimen, it is postulated as same as in the previous chapter that the displacement due to introducing the notch $U_{crack}$ is given by

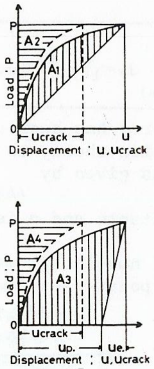
$$U_{crack} = U_{e.crack} + U_{p.crack} = c(c/W)f^{**}(P/Wt) \qquad (8)$$

The simplified J-estimating formula for shallowly notched specimen based on the area approximation procedure can be also derived as shown in Fig.4.

The same procedure as the above discussion can be applied for bending notched specimen and round bar with circumferential notch. Table 3 shows summary of simplified formulas for determining J in elasto-plastic materials with a notch.

In order to confirm the applicability of the above simplified formulas, elasto-plastic FEM analyses based on incremental strain theory have been carried out for determining J-value of tensile centre-notched and bending specimens of mild

$U = U_{e.crack} + U_{p.crack} + U_{e.no\ crack} + U_{p.no\ crack}$




**Tensile specimen**

Rice (Deep notch)

$U_{p.crack} = b f(P/bt) \rightarrow J_d = J_{(elastic)} + J_{(plastic)}$

$J_{(elastic)} = G$

$J_{(plastic)} = \frac{2}{bt}\left[\int_0^u P du - \frac{1}{2}Pu\right] = \frac{2A_1}{bt}$

Authors (Shallow notch)

$U_{crack} = c(c/w)f^{**}(P/wt)$

$\downarrow$

$J_s = \frac{1}{ct}\int_0^P U_{crack}\, dP = \frac{A_2}{ct}$

**3-point bending specimen**

Authors

(Deep notch)

$J_d = G + \frac{2}{bt}\left[\int_0^u P du - \frac{1}{2}P u_e\right] = G + \frac{2A_3}{bt}$

(Shallow notch)

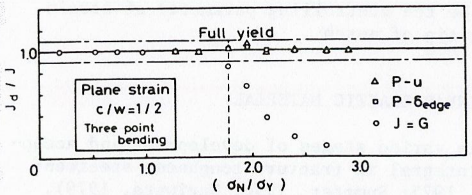
$J_s = \frac{2}{ct}\int_0^P U_{crack}\, dP = \frac{2A_4}{ct}$

Fig. 4. Area approximation procedures for J-estimating in elasto-plastic material.

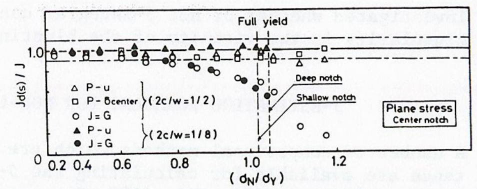
TABLE 3. Summary of Simplified Formulas of J-value for Elasto-Plastic Material

| | | Assumption | J estimating formulas |
|---|---|---|---|
| Tensile notched specimen | Deep notch | $U_{p.crack} = b\ f(P/bt)$ | $J_d = G + \frac{2}{bt}[\int_0^u P du - \frac{1}{2}Pu]$ (Rice) P-u<br>$J_d = G + \frac{2}{bt}[\int_0^{\delta center} P d\delta_{center} - \frac{1}{2}P\delta_{center}]$ P-δcenter |
| Tensile notched specimen | Shallow notch | $U_{crack} = c\frac{c}{W}f^{**}(P/Wt)$ | (General yielding) $J_s = \frac{1}{ct}\int_0^P U_{crack}\, dP$ (Authors)<br>(Full yielding) $J_s = \frac{1}{ct}[\int_0^P u dP - \frac{1}{2}P u_{e.no\ crack}]$ P-u |
| Bending notched specimen | Deep notch | $\theta_{p.crack} = g(M/b^2t)$ | $J_d = G + \frac{2}{bt}[\int_0^\theta M d\theta - \frac{1}{2}M\theta_e]$<br>$(J_d = G + \frac{2}{bt}[\int_0^u P du - \frac{1}{2}P u_e])$ P-u<br>$(J_d = G + \frac{2}{bt}[\frac{W}{c+rb}\int_0^{\delta edge} P d\delta_{edge} - \frac{1}{2}P\delta_{e.edge}])$ (Authors) P-δedge |
| Bending notched specimen | Shallow notch | $\theta_{crack} = (\frac{c}{W})^2 g^{**}(M/W^2t)$ | (General yielding) $J_s = \frac{2}{ct}\int_0^M \theta_{crack}\, dM$<br>$(J_s = \frac{2}{ct}\int_0^P U_{crack}\, dP)$ (Authors)<br>(Full yielding) $J_s = \frac{2}{ct}[\int_0^M \theta dM - \frac{1}{2}M\theta_{e.no\ crack}]$<br>$(J_s = \frac{2}{ct}[\int_0^P u dP - \frac{1}{2}P u_{e.no\ crack}])$ P-u |



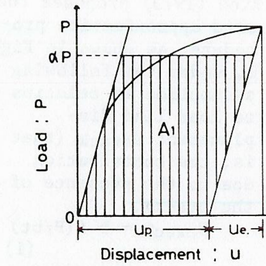



(a) Bending specimen




(b) Tensile notched plate

Fig. 5. Accuracy of various simplified formulas of J-value in elasto-plastic material.




Authors

(Deep notch)

$J_d^c = G + 2\alpha\frac{P}{bt}U_p = G + \frac{2A_1}{bt}$

$\alpha = \begin{cases} \frac{1-n}{2(1+n)} & \text{: Tensile} \\ \frac{1}{1+n} & \text{: 3-point bending} \end{cases}$

Sumpter

$\alpha = \frac{P_L}{P}$

($P_L$ : Limit load at full plastic)

Hagiwara

$\alpha = \frac{1}{3}\cdot\frac{2P+P_Y}{P}$

($P_Y$ : Load at initial yield)

Fig. 6. Fundamental idea of modified area approximation procedure for J-estimating in elasto-plastic material.

class steel (yield stress: 231 N/mm², strain hardening exponent $n$: 0.21). Figure 5 shows the accuracy of various simplified formulas shown in Table 3 by comparing with the J-integral calculated by elasto-plastic FEM analysis. In these figures, elastic contribution to J ($J=G$) is also plotted as a reference approach. If we use the proper simplified equation, the J-value calculated according to path-integral defined by Rice can be estimated even for incremental plasticity.

In the preceeding discussion, it was shown that in some crack configurations analysis of a single load-displacement record is sufficient to evaluate J-value. However, the single load-displacement record analyses require measurements of areas on those records. It is, therefore, appropriate to investigate whether an estimate of J can be made from a single point on a load-displacement record, such as the critical point of crack extension. Sumpter(1976) and Hagiwara(1979) proposed some procedures for estimating J-value of 3-point bending notched specimen from the above standpoint.

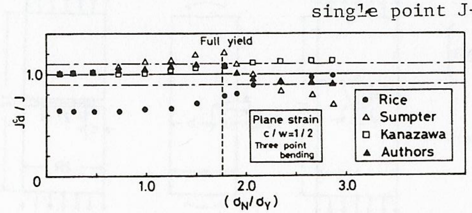
Here, we investigate single point J-estimating formulas for elasto-plastic materials under the same assumptions as eq.(5). Let consider a material of which equivalent stress-equivalent strain relation as follows;

$$\bar{\sigma} = E\bar{\varepsilon} \quad ; \bar{\sigma} \le \sigma_0$$
$$\bar{\sigma} = \sigma_0(\bar{\varepsilon}/\varepsilon_0)^n, \quad \sigma_0 = E\varepsilon_0 ; \bar{\sigma} > \sigma_0 \qquad (9)$$

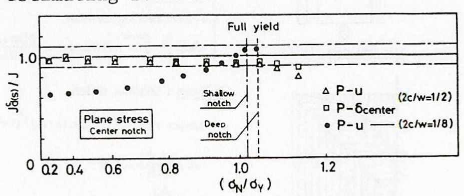
For ligament yielding or general (gross) yielding, it can be derived by the same analysis as eq.(5) that

$$U_{p.crack} \propto pl/n, \quad U_{p.crack} \propto \varepsilon_0 \qquad (10)$$

By using eq.(10), the most simplified single point J-estimating formulas $J_d^c$, $J_s^c$ as shown in Table 4 can be obtained. Figure 6 shows the fundamental procedure of single point J-estimating formulas.




(a) Three-point bending specimen




(b) Centre-notched specimen

Fig. 7. Accuracy of various most simplified formulas $J_d^c$, $J_s^c$ (cf. TABLE 4) in elasto-plastic material.

TABLE 4. Summary of the Most Simplified Single Point J-estimating Formulas in Elasto-Plastic Material

| | Assumption | J estimating formulas |
|---|---|---|
| Authors | $U_{p.crack} \propto P^{\frac{1}{n}}$ <br> $U_{p.crack} \propto \varepsilon_o$ | (Tensile notched specimen) <br> $J_d^c = G + \frac{1-n}{1+n}\sigma_N(U - U_e)$ [P-u] , $J_d^c = G + \frac{1-n}{1+n}\sigma_N(\delta_{center} - \delta_{e.center})$ [P-δcenter] <br> (Bending notched specimen) <br> $J_d^c = G + \frac{2}{1+n}\frac{P}{bt}(U - U_e)$ , $J_d^c = G + \frac{2}{1+n}\frac{P}{bt}\frac{W}{C+\gamma b}(\delta_{edge} - \delta_{e.edge})$ <br> (Round notched bar) <br> $J_d^c = G + \frac{2-n}{2+2n}\frac{P}{\pi r^2}(U - U_e)$ |
| | $U_{crack} \propto P^{\frac{1}{n}}$ <br> $U_{crack} \propto \varepsilon_o$ | (Tensile notched specimen) $J_s^c = \frac{n}{1+n}\frac{b}{C}\sigma_N U_{crack} F(C/W)$ , (Bending notched specimen) $J_s^c = \frac{n}{1+n}\frac{P}{Ct} U_{crack} G'(C/W)$ |
| Sumpter | | $J_d^c = G + \frac{2P_L}{bt}[\frac{W}{C+\gamma b}](\delta_{edge} - \delta_{e.edge})$ (Bending notched specimen) <br> $P_L$ ; Net yield load |
| Hagiwara Mimura | | $J_d^c = G + \frac{2(P+P_Y)}{3bt}[\frac{W}{C+\gamma b}](\delta_{edge} - \delta_{e.edge})$ (Bendig notched specimen) <br> $P_Y$ : Yield load |

Figure 7 shows accuracy of the various most simplified formulas in comparison with values of J-integral calculated by elasto-plastic FEM analysis. Except for at low loading condition of shallowly notched specimen, these most simplified formulas established newly in the present paper can be applicable in the same manner as the previous formulas.

## J-VALUE AS A FRACTURE CONTROLLING PARAMETER

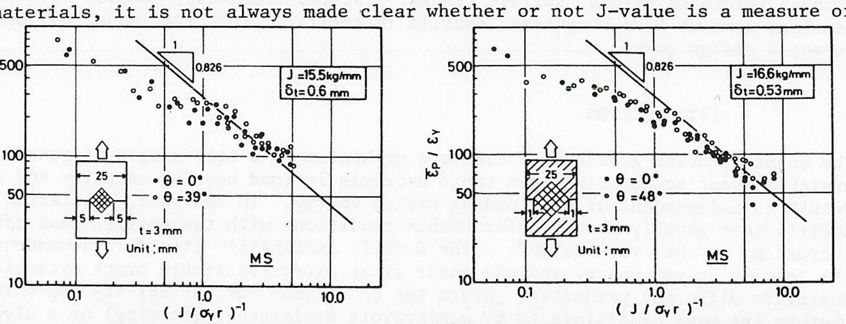
According to the deformation theory of plasticity, stresses $\sigma_{ij}$ and strains $\varepsilon_{ij}$ in the vicinity of notch tip under plastic deformation range are given by (Hutchinson, 1968)

$$\sigma_{ij} = \sigma_o \{ J/(I_n \varepsilon_o \sigma_o r) \}^{n/(1+n)} \tilde{\sigma}_{ij}(\theta)$$
$$\varepsilon_{ij} = \varepsilon_o \{ J/(I_n \varepsilon_o \sigma_o r) \}^{1/(1+n)} \tilde{\varepsilon}_{ij}(\theta) \quad (11),$$

where equivalent stress-strain relation of material is given by eq.(4), $I_n$ is a function of strain hardening exponet n and the co-ordinates (r,θ) show polar co-ordinates system of which the origin coincides the tip of notch. Moreover, J-value calculated for two-dimensional notched materials obeying laws of the deformation theory coincides with the rate of change of potential energy per unit thickness with crack length.

Nevertheless, for incremental plasticity, which is the behavior more appropriate to real materials, it is not always made clear whether or not J-value is a measure of the potential energy release rate, although the path independency of J-integral has been shown by some elasto-plastic FEM analyses.

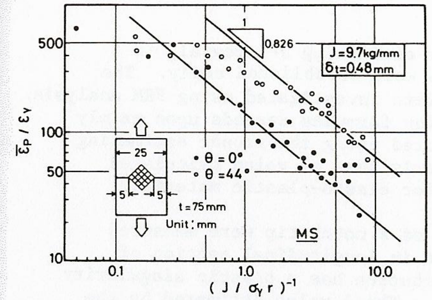
Here, experimental consideration of availability of J-integral as a strain intensity factor as shown in eq.(11) have been carried out. The distributions of equivalent plastic strain in the vicinity of the notch tip of notched specimens subjected to tensile loading were measured by using measurement techniques of hardness distribution. The materials used is mild steel (yield stress $\sigma_Y$=231 N/mm$^2$, ultimate tensile strength $\sigma_u$=414 N/mm$^2$). Equivalent plastic strain $\bar{\varepsilon}_p$ was obtained from Vickers hardness $H_V$ by using $\bar{\varepsilon}_p$-$H_V$ relationship prepared in advance of the mild steel used.




(a) Thin plate with deep notch (b) Thin plate with shallow notch

Fig. 8. Equivalent plastic strain distibutions in the vicinity of notch tip and plastic singularity of elasto-plastic notched specimen.






(c) Thick plate with deep notch

Fig. 8(c).

Figure 8 gives equivalent plastic strain distributions in the vicinity of notch tip for various specimen geometry and various loading levels as a function of the parameter $(J/\sigma_Y r)^{-1}$ considering the analytical results shown in eq.(11). The J-values in Fig. 8 were calculated by applying the proper J-estimating formulas in Table 3 or 4. It is known from the results of Fig. 8 that the singularity of distribution of equivalent strain agrees almostly with one derived analytically by Hutchinson(1968), except for concerning with strains in the very vicinity region of notch tip. That singularity is hardly dependent on notch size, loading level and plate thickness. Moreover, in the case of which the degree of constraint of plastic deformation in the vicinity of notch tipis not widely different, J-value control also quantitatively plastic strain level in the vicinity of notch tip.

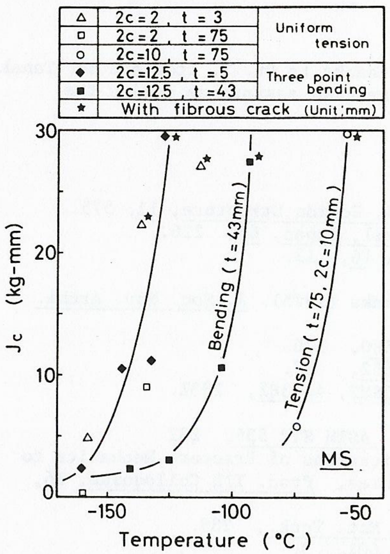



Fig. 9. Effect of specimen geometry on relation of $J_c$ vs. temperature.

Figure 9 shows the $J_c$-temperature relations obtained by fracture tests for various specimen geometry of mild steel. $J_c$-values were calculated by using J-estimating formulas shown in Table 4 (Table 3 only for shallowly notched specimen). For the notched thin-plate, $J_c$-temperature curves are almost independent of specimen geometry and loading condition, because the degree of plastic constraint does not considerably change. However, as plate thickness increases , $J_c$-temperature curve moves to higher temperature side. That is to say, as same as critical COD $\delta_c$-value shown in our previous report (Satoh, 1979), $J_c$-value in notched thick-plate at a certain temperature may depend upon even notch geometry because notch length affects plastic constraint.

It is, therefore, postulated that $J_c$-value becomes a available fracture controlling parameter if the $J_c$-value is obtained under the same level of plastic constraint in the vicinity of notch tip as practical crack problems. Moreover, two-parameters ($J_c$-$\delta_c$) criterion proposed by Toyoda(1979) on behalf of one-parameter criterion may be appreciate as a more proper measure which dominates the stress·strain characteristics in the vicinity of notch tip.

## CONCLUDING REMARKS

In the present study, the simplified formulas for estimating J-integral, in particular including shallowly notched specimens were established newly. The accuracy and applicable range of these formulas were investigated using FEM analysis. The applicable range of the simplified J-estimating formulas depends upon mainly strain hardening exponent n. The J-value calculated using the proper estimating formula proposed in the present paper becomes nearly same as value calculated according to path-integral defined by Rice even for elasto-plastic material.

The plastic strain distributions in the vicinity of a notch tip were measured using hardness distribution measurement technique in longitudinal section of deformed-notched specimen. Plastic strain distribution has a plastic singularity shown by Hutchinson for power-hardening material. The J-value estimated by the proper procedures dominates the singularity and the amount of the equivalent plastic strain in the vicinity of the blunting tip of a notch.

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