PROPAGATION OF CRACK BANDS IN HETEROGENEOUS MATERIALS

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ABSTRACT

Because of the material heterogeneity, the front of a crack in materials such as concrete as well as rocks is best treated as a large diffuse zone of microcracks the size of which is in a fixed relation to the size of the aggregate. At the same time, this treatment is the most effective one for the finite element modeling of propagating cracks, especially when the direction of the cracks is unknown and the cracks propagate skewly through the mesh. Recent work on this approach to fracture is summarized and reviewed. First it is demonstrated by computer results that the current approach in which the propagation of an element-wide crack band is determined by a strength criterion is unobjective, and does not converge as the element size is reduced. An energy criterion governing the stability of a crack band and its propagation is then formulated and consistent results obtained in computer calculations for this criterion with meshes of different sizes are presented. A further refinement is required in the case of a reinforced material; to achieve objectivity and proper convergence, it is necessary to account for the bond-slip between steel bars and concrete; and a method utilizing the concept of an equivalent free bond slip lengths is indicated; and computer results for crack band propagation in reinforced concrete are given. A consistent and properly convergent finite element scheme for propagation of crack bands of arbitrary direction now appears to be available.

Key Words: Fracture Mechanics, Crack Propagation, Crack Bands, Cracking, Reinforced Solids, Concrete, Reinforced Concrete Structures, Numerical Methods, Finite Element Analysis, Fracture Criterion, Energy Methods, Bond of Reinforcement.

INTRODUCTION

Although sharp inter-element cracks have been also applied to concrete, in finite element modeling, the prevalent approach is, in contrast to that for other materials, an element-wide smeared crack band. This approach seems to correspond to observations in that the cracks in concrete tend to be diffuse and spread over large zones, especially at the front of propagation. At the same time, the concept of an element-wide blunt smeared crack band is much more convenient and effective than inter-element sharp cracks, particularly when the direction of propagation is unknown and arbitrary, proceeding in a skew direction through the mesh. Compared to the use of sharp inter-element cracks, it is not necessary to split each node in two when the crack advances, thus avoiding the need for node renumbering and changes in topological connectivity of the mesh with the necessary recalculations of the structural stiffness matrix. Moreover, when the crack direction is unknown it is not necessary to vary the direction of the interface between two finite elements and move the location of the node into which the crack is about to extend.

It merely suffices to modify the stiffness of the matrix of the finite elements that undergo cracking, setting the normal stiffness in the direction across the cracks to zero.

The propagation of element-wide smeared crack bands has so far been determined on the basis of the stress compared to the strength limit. In a previous work it has, however, been demonstrated that this approach can give widely different results depending on the choice of the finite element mesh, and is therefore, unobjective. As the size of the finite elements in the region of the crack front is reduced to zero, the crack band tends to localize into a single element strip, and since the stress in the element in front of the crack band tends to infinity, the load which causes further extension of the crack band is always found to approach zero when the mesh refinement is considered. This fact, in itself, could perhaps be overlooked if the results for various reasonable finite element meshes agreed with each other. It has been, however, demonstrated that this is in reality far from true.

The purpose of this brief paper is to summarize and review recent works in which a consistent and physically justified method of determining the propagation of blunt smeared single element crack bands has been developed. We will outline the approach, both for plain and reinforced concrete and document the formulation by selected finite element examples.

ENERGY-CRITERION FOR CRACK BAND PROPAGATION

A propagation criterion that is independent of the mesh size is obviously the rate of the release of energy unit length (or in three-dimensions, unit area) of the crack band, which is the same concept as in fracture mechanics of sharp cracks. In a finite element scheme, the energy release rate, G, may be approximated as $\Delta U/\Delta a$, where ΔU is the energy release of the structure as the crack band advances the length Δa of a single finite element. If the value of $G = -\Delta U/\Delta a$ is less than a certain critical energy release rate, G_{cr}, the crack band cannot propagate. If G attains the critical value G_{cr}, the crack band is extended into the finite element. A value of G larger than G_{cr} is theoretically impossible, but due to discretization error a small excess over this value must be tolerated.

The calculation of the energy release, ΔU, may be carried out similarly to Rice's formula for the extension of the notch in an elastic material. The only generalization necessary is to take into account the fact that, in contrast to an arch extension, the material is not removed but merely permeated by parallel cracks, causing that only part of the energy stored in the material is lost by the crack band extension. A second generalization necessary for reinforced concrete is to take into account the effect of reinforcing bars crossing the finite element into which the cracks propagate.

The variation of the potential energy of an elastic body due to the extension of the crack band into a volume ΔV of the element in front of the crack band is independent from the path between the initial and final state and may, therefore, be separated [4] into two stages:

Stage I.- Volume ΔV of the element ahead of the crack band gets intersected by cracks in the direction of principal tensile stress (Fig. 1). At the same time, the stress and deformation state in the rest of the body is imagined to remain frozen. Accordingly, one must introduce surface tractions ΔT^{o}_{ci} acting on the boundary ΔS of volume ΔV, and in case of reinforced concrete, also forces Δf^{o}_{ci} transmitted from steel unto concrete within volume ΔV. These tractions and forces are calculated so as to replace the previous action of the volume ΔV of the concrete upon the rest of the body (Fig. 1c).

Stage II.- Forces ΔT^{o}_{ci} and Δf^{o}_{ci} are then released by gradually applying the

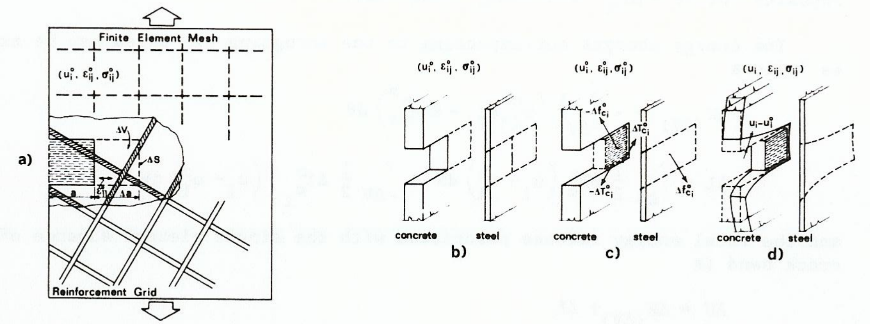

FIG. 1. (a) Crack Band Advance in Finite Element Mesh of Reinforced Concrete Panel; (b) Initial State; (c) Intermediate State; (d) Final State.

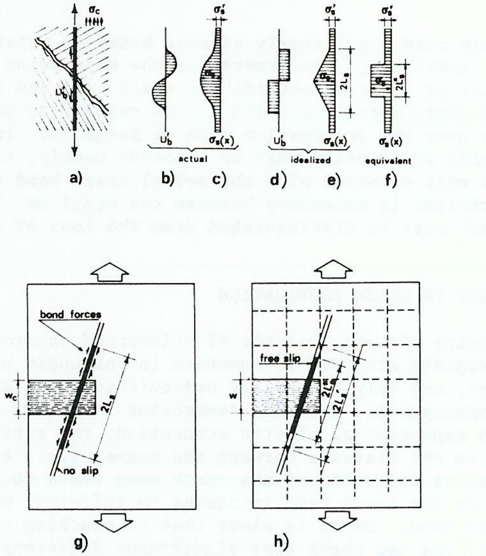

FIG. 2. (a) Reinforcement Crossing Crack Band and Bond Slip; (b),(c) Actual Bond Force and Bar Stress Distribution; (d),(e) Idealized Bond Force and Bar Stress Distribution Giving Equivalent Stiffness; (g) Actual Bond-Slip Length $2L_s$; (h) Equivalent Free-Slip Length for Actual Reinforcement Ratio ($2L^{*}_{s}$) and for Adjusted Reinforcement Ratio ($2L'_{s}$).

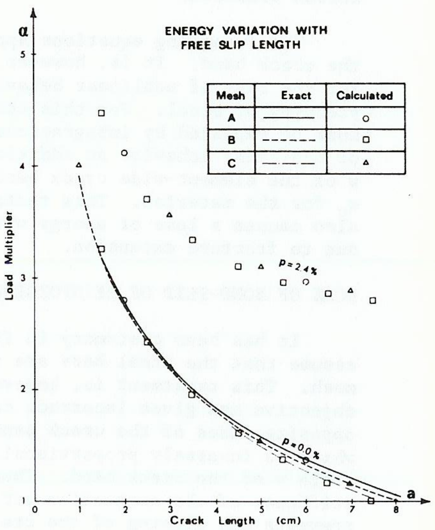

FIG. 3. Comparison of Numerical Results for Different Finite Element Meshes A,B,C, both for Plain Concrete (p = 0%) and Reinforced Concrete (p = 2.4%).

opposite forces $-\Delta T^o_{c_i}$ and $-\Delta f^o_{c_i}$ (Fig. 1d).

The energy changes corresponding to the foregoing two pages may be expressed as follows

$$\Delta W_{(\Delta V)} = -\int_{\Delta V} \frac{1}{2}\left(\sigma^{c^o}_{ij}\epsilon^o_{ij} - E'_c \epsilon^{o^2}_{11}\right) dV \tag{1}$$

$$\Delta L = \int_{\Delta S} \frac{1}{2}\, \Delta T^o_{c_i}\left(u_i - u^o_i\right) dS + \int_{\Delta V} \frac{1}{2}\, \Delta f^o_{c_i} \left(u_i - u^o_i\right) dV \tag{2}$$

and the total energy release associated with the single element advance of the crack band is

$$\Delta U = \Delta W_{(\Delta V)} + \Delta L \tag{3}$$

Here, u_i = displacements in cartesian coordinates x_i $(i = 1,2,3)$, σ_{ij} = cartesian stress components, ϵ_{ij} = cartesian components of the small strain tensor, superscript c refers to concrete, u^o_i, ϵ^o_{ij}, and $\sigma^{c^o}_{ij}$ = values of displacements, strains and stresses in volume ΔV before the advance of the crack band (before Stage I); $E'_c = E_c$ for plain stress and $E'_c = E_c/(1 - \nu^2_c)$ for plain strain, in which E_c = Young's modulus for plain concrete and ν_c = Poisson's ratio. The foregoing equations represent the basic energy relations for the propagation of crack bands in reinforced concrete.

The foregoing equations apply only for linearly elastic behavior outside of the crack band. It is, however, possible [2] to generalize the expression for ΔU for the case of nonlinear behavior of concrete outside the crack band and for the yielding of steel. For this end, the coefficients 1/2 in the expression for ΔU must be replaced by integrations over the deformation path of Stage II. In case of nonlinear behavior an additional restriction must be imposed; namely, the width w of the element-wide crack band must coincide with the actual crack band width w_c for the material. This restriction is necessary because the nonlinear behavior also causes a loss of energy which must be distinguished from the loss of energy due to fracture extension.

ROLE OF BOND-SLIP OF REINFORCEMENT IN CRACK PROPAGATION

It has been customary in finite element analysis of reinforced concrete to assume that the steel bars are rigidly attached to concrete in the nodes of the mesh. This treatment is, however, not only physically unjustified but also unobjective and gives incorrect convergence. The bars connecting the nodes on the opposite sides of the crack band represent an elastic connection, the stiffness of which is inversely proportional to the distance between the nodes, i.e. to the width w of the crack band. Thus, as the width of the crack band tends to zero, the stiffness of the connection across the crack band increases to infinity, which prevents any opening of the crack band. So it is clear that no cracking can be obtained in the limit. Moreover, one can check that significant differences of the results are caused by the lack of bond-slip than when meshes of various practically possible sizes are considered.

To obtain an objective and properly convergent formulation, one must take into account the bond-slip. The bond-slip in reality occurs over a certain length, L_s (Fig. 2). The most realistic treatment of the bond-slip would call for using separate nodes for concrete and steel connected by some nonlinear linkage elements representing forces transmitted by bond. However, this approach would be too cumbersome. In the spirit of the approximations involved in the smeared crack band model, it should be sufficient to introduce the bond slip in such a way that the stiffness of the connection provided by the steel bars crossing the crack band be roughly correct and independent of the finite element mesh.

Thus, to simplify the formulation, the actual curvilinear variation of the bond forces and the axial forces in the bars (Fig. 2) may be replaced by an idealized piece-wise variation of the bond force and the corresponding piece-wise linear variation of the actual axial force in the bars. The latter may further be replaced by a piece-wise constant variation of the axial force, such that the overall extension of the bar over distance of the bond-slip be roughly the same. The actual distance of the bond slip may be approximately determined by the expression

$$L_s = \frac{(\sigma_s - \sigma'_s)A_b}{U'_b} \tag{4}$$

in which A_b = cross-sectional area of the steel bar; σ_s = tensile stress in the bar at the point it crosses the crack band; σ'_s = tensile stress in the bar at the end of the slipping segement, i.e., at locations sufficiently remote from the crack band; and U'_b = the ultimate bond force as determined from pull-out tests. Certain reasonable estimates of σ_s and σ'_s can be made on the basis of the yield stress of steel and the tensile strengths of concrete [2]. Expression (4) gives the bond-slip length L_s as a fixed property characteristic of the steel-concrete composite.

For the purpose of finite element analysis the actual bond-slip length L_s may be replaced by some modified length L^*_s (Fig. 2f) such that the steel stress over this length is uniform and the slip of steel bar within concrete may be considered as free. The length L^*_s is determined from the condition that the extension of the steel bar over the length L_s is the same, as already noted. In this manner, the following expression for the equivalent free bond-slip length can be obtained [2]:

$$L^*_s = \frac{L_s}{2}\,\frac{p^*}{p(1 + pn - p^*n)} + k_b\,\frac{w}{2} \tag{5}$$

Here w represents the width of the element-wide crack band, k_b is a correction factor smaller than one but close to one (for which also a theoretical expression exists, Ref. 2), p is the reinforcement ratio (the ratio of the cross-section areas of reinforcement and concrete), n is the ratio of Young's modulus of steel to that of concrete, and p^* is a certain modified reinforcement ratio which may be conveniently chosen so as to make the length L^*_s equal to the distance between the adjacent nodes located across the crack band (L'_s in Fig. 2h).

SOME COMPUTATIONAL RESULTS

The performance of the proposed method may be illustrated by the numerical finite element results plotted in Fig. 3 . Considered is a rectangular reinforced concrete panel subjected to tensile forces at two opposite ends. A symmetric centrally located crack band normal to the applied loads is assumed to grow from the center of the panel symmetrically towards its sides. Using the energy criterion and the free bond-slip length the multiplier α of the applied loads that is necessary to cause the extension of the crack band of length a has been calculated [2]. As expected, the load multiplier needed for further crack extension decreases as the crack band length increases. Computations have been carried out [2] for three different rectangular meshes, the sides of which are in the ratio 4:1, labeled as A, B, C (Fig. 3). The grid used in the calculations was a uniform square grid and each square element was assumed to consist of two constant strain triangles. Both triangle elements forming one square were assumed to always crack simultaneously.

It may now be observed in Fig. 3 that the results for the three different meshes fall approximately on the same curve. It has been previously demonstrated [2] that in case of the classical strength criterion, the results of the calculaticfor these three meshes are widely different and deviate from each other as much as 100%. It is also noteworthy that coincidence of the results is obtained for plain concrete ($p = 0$) as well as reinforced concrete ($p > 0$). It has been also previously shown [2] that the results for reinforced concrete for these three meshes are far apart when the strength criterion is used, but also when the energy criterion without the bond slip of steel is considered.

The results presented in Fig. 3 and numerous further results given in [2,3] demonstrate that the proposed method is objective, i.e., independent of the chosen finite element mesh.

The solid curves indicated in Fig. 3 represent the exact solutions for a sharp crack according to linear fracture mechanics. From this comparison it is seen that the concept of an element-wide crack band may be also used as a convenient and effective approximation to the propagation of sharp cracks, gaining all the practical advantages of the smeared crack band model as compared to considering sharp inter-element cracks. One may now naturally ask what is then the physical difference between the smeared crack band and the sharp inter-element crack. Obviously, up to a certain rather wide crack band the difference is insignificant in these computations. The real difference arises only through the value of the energy-release rate which is to be considered in the calculation. In an on-going work that has not yet been completed, it is found that the critical energy release rate for a smeared crack band cannot be considered to be a fixed material property (unless the width of the band tends to zero), and must be regarded as a function of the band width as well as of the stiffness of the structure surrounding the front of the crack band. When the differences in the critical energy release rate between a smeared crack band and a sharp crack are considered, the results of computations cannot be, of course, identical. It is by means of the variation of the critical energy release rate that one can explain deviations from fracture mechanics predictions as observed on concrete specimens the size of which is not sufficiently large compared to the aggregate size.

EQUIVALENT STRENGTH CRITERION

Determination of the energy release ΔU needed for the propagation criterion requires two finite element calculations, one for the initial crack band length and one for the crack band extended by one element. It has been found [1,2] that calculations may be simplified by approximately estimating the energy release rate on the basis of the stress state in the uncracked finite element just in front of the crack band. The energy release rate becomes approximately critical when the normal stress orthogonal to the direction of cracks reaches the value

$$(\sigma_{22}^{0})_{cr} = \sigma_{eq} = c \sqrt{\frac{E_c' G_{cr}}{w}} \qquad (6)$$

in which σ_{eq} was named the equivalent strength, and c is a coefficient characteristic of a given element type. Generally, c is close to one. For a square element consisting of four linear strain triangles, $c = 0.826$, while for a square element consisting of only two constant strain triangles $c = 0.921$. It is interesting to observe that σ_{eq} increases as the width of the crack band decreases and tends to infinity as the element size approaches zero. Obviously, the equivalent strength criterion in Eq. (6) must give results that significantly differ from those for a constant strength limit. In an on-going work which will be reported at the conference, it has been found that the equivalent strength criterion, which is in Eq. (6) given only for plain concrete, may be extended to reinforced concrete. For this purpose, a corrective term which involves a reinforcement ratio must be added to the expression in Eq. (6).

CONCLUSIONS

The research results outlined in the foregoing summary have led to the

following conclusions:

1. The use of a constant strength limit for determining extension of a crack band in a finite element mesh is unobjective and has incorrect convergence behavior. The results may differ by as much as 100% when meshes of different size are used.

2. An objective and physically realistic criterion for crack band extension must be expressed in terms of the energy release rate by unit length of the crack band. Expressions for calculating the energy release rate in a finite element program have been formulated.

3. To achieve an objective and properly convergent propagation criterion for reinforced concrete, the bond-slip between steel reinforcement and concrete must be taken into account. This may be conveniently done in terms of the equivalent free bond-slip length, for which an expression is presented.

4. As an acceptable approximation, the energy criterion may be replaced by an equivalent strength criterion such that the strength limit depends on the width of the element-wide smeared crack band.

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