MECHANISMS AND MECHANICS OF FRACTURE OF CONCRETE

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ABSTRACT

First of all the structure of hardened cement paste and concrete is discussed. Based on characteristic structural features crack initiation and crack propagation in a porous composite material is explained. The different fracture mechanisms are used in a complex computer simulation program. By means of a computer experiment it is possible to study the different failure processes of hardened cement paste, normal, and high strength concrete. Finally a stochastic approach to describe failure of concrete is mentioned. This theoretical concept is also based on the structure of hardened cement paste and concrete. As an example the probabilistic concept is used to describe the influence of rate of loading on strength of concrete.

KEYWORDS

Hardened cement paste; concrete; composite material; porosity; fracture mechanisms; computer simulation method; stochastic approach.

INTRODUCTION

Today concrete is the most widely used building material. For a long time research on concrete properties remained rather phenomenological. It was common practice to start new test series if an unusual structure had to be built. Consequently test results often have not been interpreted in general terms or using a theoretical basis provided by materials science but they were used for the very special application only.

Results of a systematic study to apply fracture mechanics principles have first been published by Kaplan (1961). Ten years later Kesler and co-workers (1971) and Naus and Lott (1973) pointed out that linear fracture mechanics cannot describe the actual behaviour of concrete as it fails under load. They adopted the plastic strip model (Kesler and co-workers, 1971). The further development until the International Conference on Fracture in Munich has been summarized by Radjy and Hansen (1973). But until now the controversy on the question to which extent fracture mechanics may be applied to concrete, exists (Shah, 1979).

It is not the aim of this contribution to sum up present day knowledge in a comprehensive way. This will be published together with an annotated bibliography in the near future as a state-of-the-art report by a RILEM Technical Committee (50-FMC).

Some aspects of interrelation of fracture mechanisms and fracture mechanics shall be pointed out instead.

First of all relevant details of the structure of concrete will be discussed and characterized subdivided into four different levels. Fracture processes in hardened cement paste will be considered by using two levels (submicrolevel and microlevel) and two additional levels with larger scale (submacrolevel and macrolevel) are introduced to take crack formation and failure of concrete into consideration. Then some examples for the use of simulation methods will be explained and finally a stochastic model for fracture of concrete will be presented. In this way different trends in research and their potential practical implications can be described.

STRUCTURAL DETAILS WITH REFERENCE TO CRACK FORMATION AND FRACTURE MECHANICS

Hardened Cement Paste

The classical approach of fracture mechanics has been applied successfully to describe failure processes of many different homogeneous isotropic materials.

Concrete is a composite material with a porous matrix and a certain degree of anisotropy. The modulus of elasticity of the matrix and the aggregates usually differs considerably. As a consequence there is not one single dominating failure process and one has to be extremely careful if applying methods of fracture mechanics.

In this contribution the structure of concrete will be discussed in terms of different levels. The significance of characteristic features of each level for the failure mechanisms and for fracture mechanics will be pointed out.

As mentioned already the structure of hardened cement paste and concrete will be subdivided into four different levels. Relevant materials properties and processes will be characterized in terms of a sublevel whereas macroscopic crack propagation and failure mechanisms will be discussed by introducing a different level for both materials i.e. microlevel for hardened cement paste and macrolevel for concrete.

Level I (submicrolevel). It is well known that strength of hardened cement paste is sensitively influenced by porosity. As an example the compressive strength after 28 days is plotted as function of the water-cement ratio in Figure 1. These data are taken from Ruetz (1966).

At a given degree of hydration the total porosity is essentially governed by the water-cement ratio. By means of sorption measurements the pore size distribution of pores having a radius below 300 Å has been determined by several authors (Wittmann and Englert, 1967). By mercury porosimetry much larger pores can be determined. In Figure 2 a typical example redrawn from Diamond (1971) is shown. It is evident that with increasing water-cement ratio the total porosity increases and that the pore size distribution is shifted towards coarser pores.

In a real specimen of hardened cement paste there are always some microcracks present which are caused by chemical, capillary and/or drying shrinkage. In addition there are usually some compaction pores with a diameter of up to 1 mm. These structural defects cause high stress concentration and therefore initiate crack propagation.

Once a crack spreads, it has to pass through the porous structure. That means that

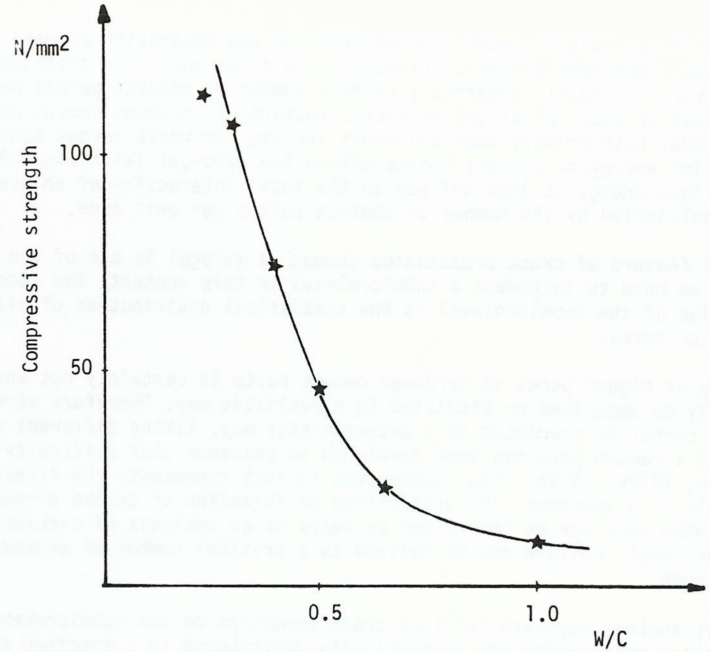


Fig. 1. Influence of W/C ratio on compressive strength of concrete (Ruetz, 1966).

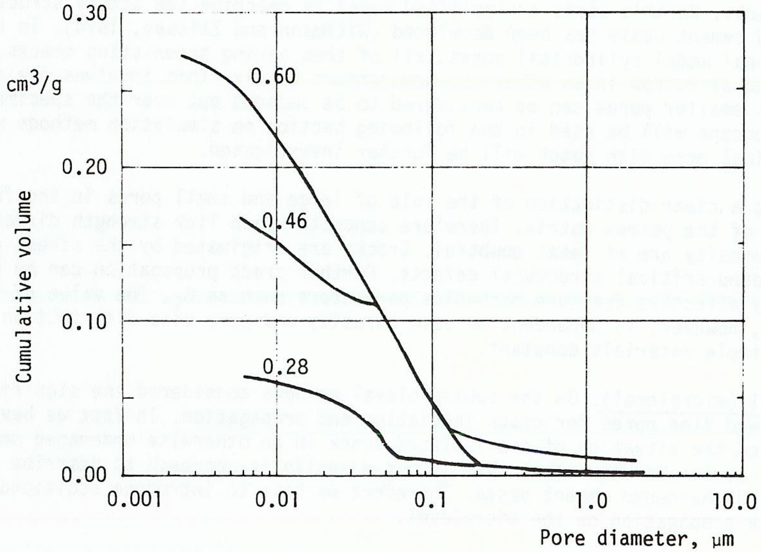


Fig. 2. Cumulative pore volume of hardened cement paste for three different water-cement ratios taken from Diamond (1971).

crack propagation cannot be understood as creation and separation of well defined solid surfaces. Dependent on the total porosity and the pore size distribution a travelling crack tip has to separate a certain number of contact points per unit area. From what we know so far on the microstructure of hardened cement paste, we may assume, that both primary chemical bonds and van der Waals forces contribute to the interaction energy of contact points within the xero-gel (Wittmann, 1977). The fracture surface energy is then defined as the total interaction of an average contact point multiplied by the number of contact points per unit area.

This special feature of crack propagation through a xerogel is one of the main reasons why we have to introduce a submicrolevel in this context. The second characteristic value of the submicrolevel is the statistical distribution of microcracks and compaction pores.

The geometry of bigger pores in hardened cement paste is certainly not uniform and it can hardly be described or simulated in a realistic way. Therefore stress concentrations cannot be predicted in a deterministic way. Taking different geometries into account a random pore has been developed to overcome this difficulty (Zaitsev and Wittmann, 1974a, 1974b). The random pore in fact represents the stress concentration field in a specimen. The probability of formation of cracks propagating from the random pore can be determined by means of an analysis of exceedance of a given stress level. Failure can be defined as a critical number of exceedance around the random pore.

There is yet another approach to study crack formation on the submicrolevel. The bigger critical pores which are statistically distributed in a specimen are surrounded by the porous phase. The actual strength of the porous phase varies locally to a great extent. The weakest zones (regions with the lowest number of contact points per unit area) along the pore surface may be assumed to react like preexisting cracks. On this basis a simplified model to describe the actual structure of hardened cement paste has been developed (Wittmann and Zaitsev, 1974). In this two-dimensional model cylindrical pores, all of them having preexisting cracks, are dispersed at random in an otherwise homogeneous matrix. This involves the assumption, that all smaller pores can be considered to be smeared out over the specimen. This basic concept will be used in the following section on simulation methods where a cylindrical pore with notch will be further investigated.

There is a clear distinction of the role of large and small pores in the failure process of the porous matrix. Therefore concepts which link strength directly with total porosity are at least doubtful. Cracks are originated by the stress concentration around critical structural defects. Further crack propagation can be characterized by effective fracture mechanics parameters such as G_c. The value of the effective G_c, however, is dependent on both porosity and pore size distribution and is not a simple materials constant.

Level II (microlevel). On the submicrolevel we have considered the significance of coarse and fine pores for crack initiation and propagation. In fact we have only described the situation of one isolated crack in an otherwise undamaged porous matrix. It can be shown that this is not a realistic approach to describe failure process of hardened cement paste. Therefore we have to introduce additional aspects of crack propagation on the microlevel.

Grudemo (1979, 1977, 1979) summarized his findings with simplifying but indicative sketches of the microstructure. Based on extensive studies by using electron microscopes and based on experimental studies of failure of hardened cement paste he concluded that the tortuous crack surface passes along cleavage surfaces of $Ca(OH)_2$ phases, ruptured zones of contact between gel particles, and surfaces of remaining unhydrous nuclei. But most important he indicated mechanisms of crack arresting.

Once a crack spreads according to the conditions discussed in terms of the submicrolevel it may be trapped by zones of low stress concentration or stopped by zones of high local resistance. By a further increase of load new cracks may be formed and before a critical crack runs through the specimen, the material is already significantly damaged by a certain density of microcracks.

By means of direct microscopical observation Higgins and Bailey (1976a, 1976b) were able to follow stable crack growth. In their study cracks were made visible by diffuse illumination. They provided direct evidence that microcracks are formed in a line ahead of the spreading crack. Most recently Mindess and Diamond (1980) used an elegant method which enabled them to observe stable crack propagation directly in a scanning electron microscope. In this way it is possible to detect crack branching and discontinuous cracking in the stressed zone. That means a lot of energy is absorbed before a critical crack runs through the material. This fracture energy cannot be related to the macroscopic fracture surface observed after failure. But it is also important to realize that a spreading crack does not propagate through undisturbed material but runs through a precracked damaged zone. Similar results are described by Yoshimoto and co-workers (1976).

We can summarize these findings and conclude that hardened cement paste does not fail by the development of one critical crack. Simple fracture mechanics cannot be applied to such a complex structure. On the microlevel we have to consider the interaction of several cracks and the formation of a damaged zone. It is obvious that the dimension of the damaged zone ahead of a critical crack depends on the specimen geometry and more precisely on the stress gradient ahead of the crack. This phenomenon possibly explains partly the differing values of fracture mechanics parameters as determined by using different techniques and different sizes and shapes of specimens. No attempt has been made so far to quantify this interrelation.

Recently elements of the submicrolevel and the microlevel have been combined to study crack formation and failure processes by means of simulation techniques by Zaitsev (1980). This seems to be a promising approach for future research. It should be possible to simulate and verify in more detail the processes described here in terms of the two levels which had been introduced to characterize fracture mechanisms in hardened cement paste.

Before we will discuss the composite structure of concrete, some values of K_{IC} as found in the literature shall be presented. In Figure 3 some data are compiled and plotted as function of water-cement ratio. Corresponding authors and references are indicated in the figure. It must be pointed out, however, that the physical meaning of K_{IC} is not yet quite clear and it certainly does not coincide with the classical interpretation based on homogeneous materials.

Concrete

We have just described some structural aspects of crack initiation and failure of a porous material such as hardened cement paste. It turned out that it is useful to introduce two different levels : submicrolevel and microlevel. A characteristic length of a big pore with notch as described on the submicrolevel will be of the order of some tenth of a millimeter. The damaged zone in front of a propagating crack as described on the microlevel probably will have a length in the order of magnitude of 10 mm.

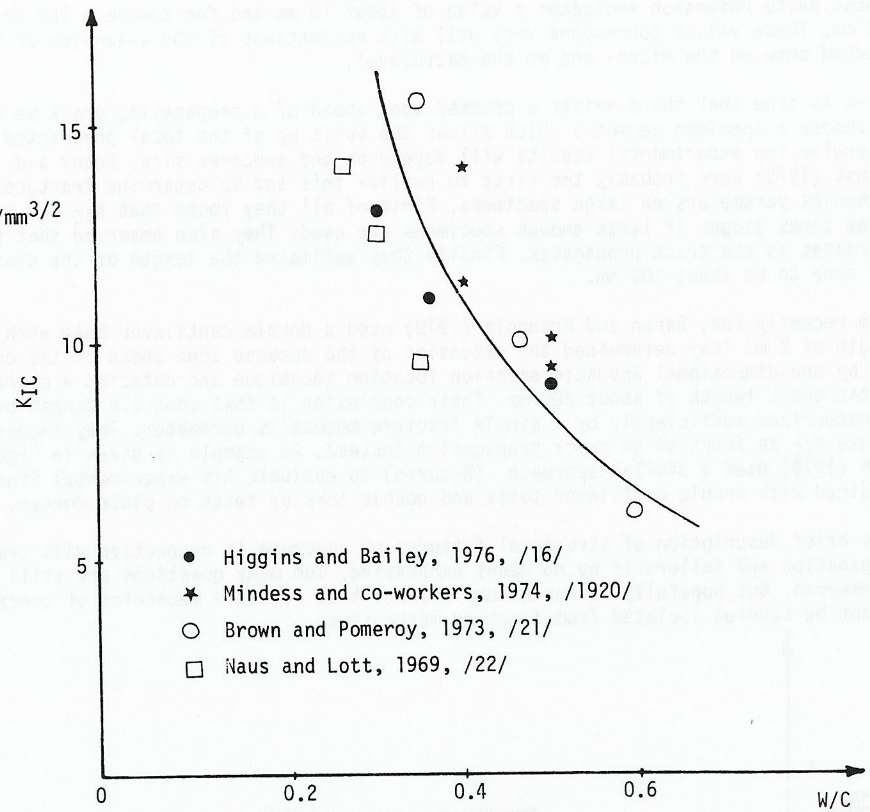


Fig. 3. K_{IC} as function of water-cement ratio according to different authors. The age of the specimens has been 14 days or older.

In a similar way we will try to describe the structure of concrete. Therefore we introduce again two levels. One level to characterize structural defects such as "a priori" cracks and interfaces. We will call this level submacrolevel. A second level, called macrolevel, will be used to deal with actual failure processes.

Structural defects in concrete (submicrolevel) may have an elongation of some 10 mm. The length of a damaged zone ahead of a spreading crack can be estimated to be 200 to 500 mm.

Level III (submicrolevel). The structure of concrete is built up of fine and coarse aggregates embedded in a porous binding matrix of hardened cement paste. On the submicrolevel we have to concentrate on the difference of physical and mechanical properties of hardened cement paste and aggregate as well as the properties of the interface between the two main components.

During the hydration of cement a considerable amount of heat is liberated. In mass concrete elements the heat of hydration can easily cause a temperature rise of 50K and more. Due to different coefficients of thermal expansion of matrix and aggregate temperature introduced interfacial cracks and cracks in the matrix can be created during the cooling period. In addition a temperature gradient is usually built up during the hardening process. A simple estimation reveals that in many cases tensile strength is overcome in the tensile zone. Thus we have a second type of thermally introduced cracks.

During the hardening process cement paste experiences chemical shrinkage (Czernin, 1977). If the surface of fresh concrete dries out capillary forces may cause considerable cracking (Wittmann, 1978). And finally the porous hardened cement paste shrinks due to moisture loss after demoulding. Next to the two types of thermally introduced cracks we have at least three different mechanisms which can lead to hygral cracking.

To which extend cracks are formed by the above mentioned mechanisms can be predicted if the mix proportion, the specimen geometry and the ambient conditions are known.

Partial segregation during the placing process and during compaction causes water filled pockets underneath coarse aggregates. In this way technological cracks or at least very weak zones are created. Technological cracks are horizontally orientated and therefore cause some anisotropiy of hardened concrete. The compressive strength is significantly lower if the stress is applied parallel to the technological cracks. Insufficient compaction of fresh concrete is a second type of technological structural defects.

We can conclude that in reality many concrete elements will be precracked because of thermal, hygral or technological reasons.

On the submacrolevel we have to deal next with the uncracked parts of the structure. Once a "a priori" crack spreads it has to pass through the matrix, along an interface or through an aggregate. It has been early recognized that the interface in normal concrete is comparatively weak (Alexander and co-workers, 1965). As the duration of hydration increases, however, the tensile strength of interfaces increases too and finally reaches the strength of the matrix. Rehm, Diem and Zimbelmann (1977) tried to increase tensile strength of concrete by ameliorating the interface. The micromorphology of the interfacial zone around aggregates has been studied in great detail by Barnes, Diamond and Dolch (1978, 1979). Essentially based on electron microscopy they developed a simplified morphological model and they were able to explain the relative weakness of the interface. Research on the interface in concrete has recently been summarized by Maso (1980).

Hillemeier and Hilsdorf (1977) determined fracture mechanics parameters of natural stone, hardened cement paste, and the interface between hardened cement paste and quartz. Some values (Hillemeier and Hilsdorf, 1977) are given in Table I.

In agreement with earlier findings, K_{IC} of the interface is smaller than the corresponding values of the two major components. This means that cracks will propagate preferentially along interfaces. In section three we will use these structural details from the submacrolevel to predict crack formation semi-quantitatively.

Level IV (macrolevel). On the submacrolevel we have discussed several mechanisms to cause cracks in an unloaded specimen and we have illustrated major structural defects. But as we realized while discussing the structure of hardened cement paste the characteristic features of the sublevel are necessary to understand the failure process but they are not sufficient. A preexisting crack may become critical under

TABLE I Fracture Toughness of Typical Componnents of Concrete (Hillemeier and Hilsdorf, 1977).

Material	Fracture toughness K_{IC}; $MN/m^{3/2}$
quartz	3,4
marble	1,9
hardened cement paste; w/c = 0,4	0,31
hardened cement paste-quartz	0,13

a given load. Then this crack will spread until it reaches an aggregate with sufficient strength to resist further growth. Either the crack has to run around the aggregate or it can penetrate. For both alternatives an increased load is necessary. And it is evident that before a stopped crack may further propagate under increasing load a number of cracks in the vicinity may become critical and get stopped. Thus a zone of cracks is first formed. The final separating crack does again not penetrate through the original material. The size of the precracked zone depends on both the loading history and the specimen geometry.

Bazant and Cedolin (1979) translated the concept of multicracking before failure in terms of finite element method. They investigated the development of a blunt crack band instead of studying the formation of one individual crack. In this approach cracks are assumed to be smeared out over a finite zone. This zone retains the capability of transmitting stresses parallel to the aligned crack direction but not normal to it. It seems that the concept of blunt crack band can be further developed by approaching even more realistically actual materials behaviour.

Another combination of finite element method and fracture mechanics has been published by Hillerborg, Modéer and Petersson (1976). These authors also take the existence of a precracked zone as described on the macrolevel into consideration. The actual behaviour is simplified, however, and a special relation (σ-w) describing the transmitted force as function of a fictitius crack width is introduced. This relation can be looked upon as being the equivalent of Dugdale's model for porous composite materials.

It has often been questioned if fracture mechanics parameters such as K_{IC} (Kaplan, 1961) are really meaningful for a material such as concrete. Several alternative experimental techniques to characterize crack resistance have been suggested. Possibly the crack opening displacement method (COD) is more adequate. Mindess, Lawrence and Kesler (1977) used the J-Integral as a fracture criterion.

Petersson (1980a, 1980b) also uses the concept of a precracked zone and he introduces a characteristic length in his fictitious crack model. This characteristic length should be comparable with the extension of the precracked zone. For hardened

cement paste Petersson indicates a value of about 10 mm and for concrete 200 to 300 mm. These values correspond very well with estimations of the extension of the cracked zone on the micro- and on the macrolevel.

If it is true that there exists a cracked zone ahead of a propagating crack we have to choose a specimen geometry which allows the built up of the total precracked zone. Otherwise the experimental results will depend on the specimen size. Entov and Yagust (1975) were probably the first to realize this and to determine fracture mechanics parameters on large specimens. First of all they found that K_{IC} can be three times bigger if large enough specimens are used. They also observed that K_{IC} increases as the crack propagates. Finally they estimated the length of the disturbed zone to be about 100 mm.

More recently Sok, Baron and François (1979) used a double cantilever beam with a length of 2 m. They determined the extension of the damaged zone ahead of the crack tip by one-dimensional acoustic emission location technique and detected a characteristic crack length of about 200 mm. Their conclusion is that concrete cannot be characterized sufficiently by a single fracture mechanics parameter. They suggest to use K_{IC} as function of crack propagation instead. An example is given in Figure 4. Shah (1979) used a similar approach (R-curve) to evaluate his experimental findings obtained with double contilever tests and double torsion tests on plain mortar.

This brief description of structural features of concrete in connection with crack propagation and failure is by no means exhaustive. Too many questions are still unanswered. But hopefully it has become evident that fracture mechanics of concrete cannot be studied isolated from fracture mechanisms.

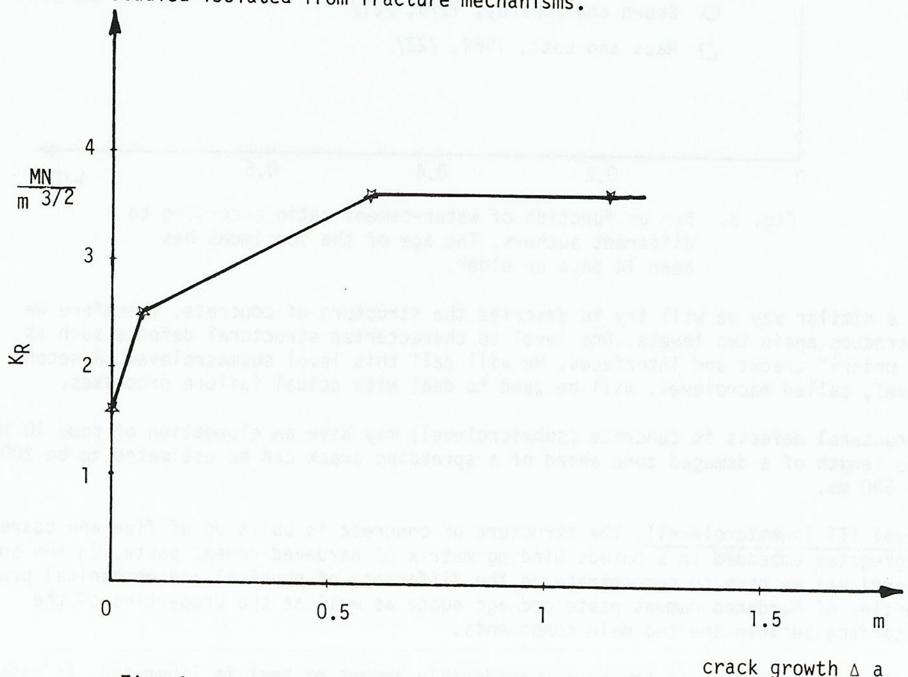


Fig. 4. Dependence of fracture toughness on crack growth (Sok, Baron and François, 1979).

COMPUTER EXPERIMENTS

Hardened Cement Paste

By making use of the concept which has been developed above while we discussed the submicrolevel of the structure of hardened cement paste, we start now to investigate one isolated pore in an isotropic matrix.

The matrix itself contains many small pores. The pore size distribution of the small pores and the total porosity depends essentially on the water-cement ratio of the hardened paste. It is assumed that the irregular border of the big pore can be represented by a spherical hole with one notch.

In Figure 5 this simplified description of the actual surrounding of a big pore in hardened cement paste is shown graphically. In the two-dimensional representation a cylindrical pore with radius r and a "in-plane" notch with initial length ℓ is shown.

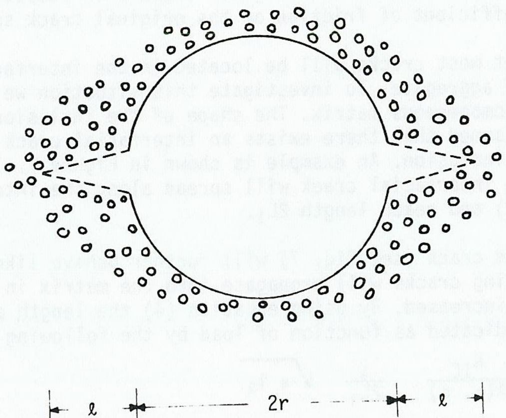


Fig. 5. Shematical representation of a cylindrical pore with one notch in a porous material.

If a material with one precracked pore, as shown in Figure 5, is loaded above a certain critical load, the crack length will increase as function of increasing load. In most practical cases concrete is loaded under compression. Therefore we assume that a compressive load is applied in the direction of the "a priori" crack. This is the most critical arrangement. It can be shown that under these conditions stable crack growth occurs (Wittmann and Zaitsev, 1974). If we relate crack length ℓ to the pore radius r :

$$\lambda = \frac{\ell}{r} \qquad (1)$$

we get the following implicit relation for related crack length λ as function of load q :

$$q = \sqrt{\frac{\pi E \gamma}{2r}} \sqrt{\frac{(1+\lambda)^7}{(1+\lambda)^2-1}} \qquad (2)$$

In equation (2) E stands for the elastic modulus and γ for the fracture surface

energy of the matrix. Both values are not defined in the classical sense of solid mechanics. The porosity of the matrix and the coupling of the gel particles have to be taken into accont (Ubelhack, 1976).

According to equation (2) the related crack length λ increases in a monotonous way. In a real specimen, however, there exists a certain probability that the growing crack interacts with cracks originating from other pores. In this case crack tips attract one another (Wittmann and Zaitsev, 1974) and finally cracks may coalesce.

By means of computer simulation methods a random structure of a porous material can be generated. In Figure 6 one two-dimensional realization is shown. Pores of the type as shown in Figure 5 are distributed at random within the specimen. The related crack length λ is assumed to be equally distributed within the range $0 < \lambda < 2$. The random distribution of λ is not shown graphically in Figure 6. The crack orientation is also equally distributed within the range $0 < \alpha < 2\pi$.

The realization of the porous structure of hardened cement paste as shown in Figure 6 can now be loaded in a computer experiment. That means that crack development arround each pore is studied as function of load. At comparatively low loads crack extension can be described by equation (2). In this case each pore is assumed to be in an infinite porous matrix.

Equation (2) can be rewritten in a more general way :

$$q = \frac{K^*_{IC}}{\sqrt{2r}} \quad f(\lambda) \qquad (3)$$

In this case K^*_{IC} represents an average value of the porous structure. Equation (3) describes crack propagation on the submicrolevel. Interaction of mutually approaching cracks is neglected.

If the load is increased above a certain level the average crack length increases to such an extent that the interaction of individual cracks cannot be neglected any more. It is assumed that two approaching cracks interact if both α_1 and α_2 are in the range $0 < \alpha_i < \pi/6$ or $0 < \alpha_i < -\pi/6$ or if $0 < \alpha_1 < \pi/6$ and $5\pi/6 < \alpha_2 < \pi$ or finally if $0 < \alpha_1 < -\pi/6$ and $\pi < \alpha_2 < 7\pi/6$. If two crack tips approach one another under the indicated geometrical arrangement they are treated like coplanar cracks. After introducing these simplifications it is possible to calculate crack interaction and final coalescence (Wittmann and Zaitsev, 1974).

In Figure 6 two stages of crack propagation as obtained by the computer experiment are shown. As can be seen gradually a crack pattern is built up and finally one crack runs all along the specimen. In the computer experiment this is defined to be failure of the porous material.

The gradual degradation of the porous structure and the formation of many microcracks before failure is simulated by means of a computer experiment. Finally a slightly inclined crack runs through the specimen. These essential features of the computer experiment agree well with the observed materials behaviour. The results shown in Figure 6 represent fracture mechanisms of the microlevel, that means stable crack growth, interaction of initially isolated cracks, gradual degradation of the composite structure, and finally the development of one coherent fracture surface. It is evident that the final fracture surface cannot be linked with the total energy consumed until failure.

It will be interesting to extend computer simulation methods to more complex states

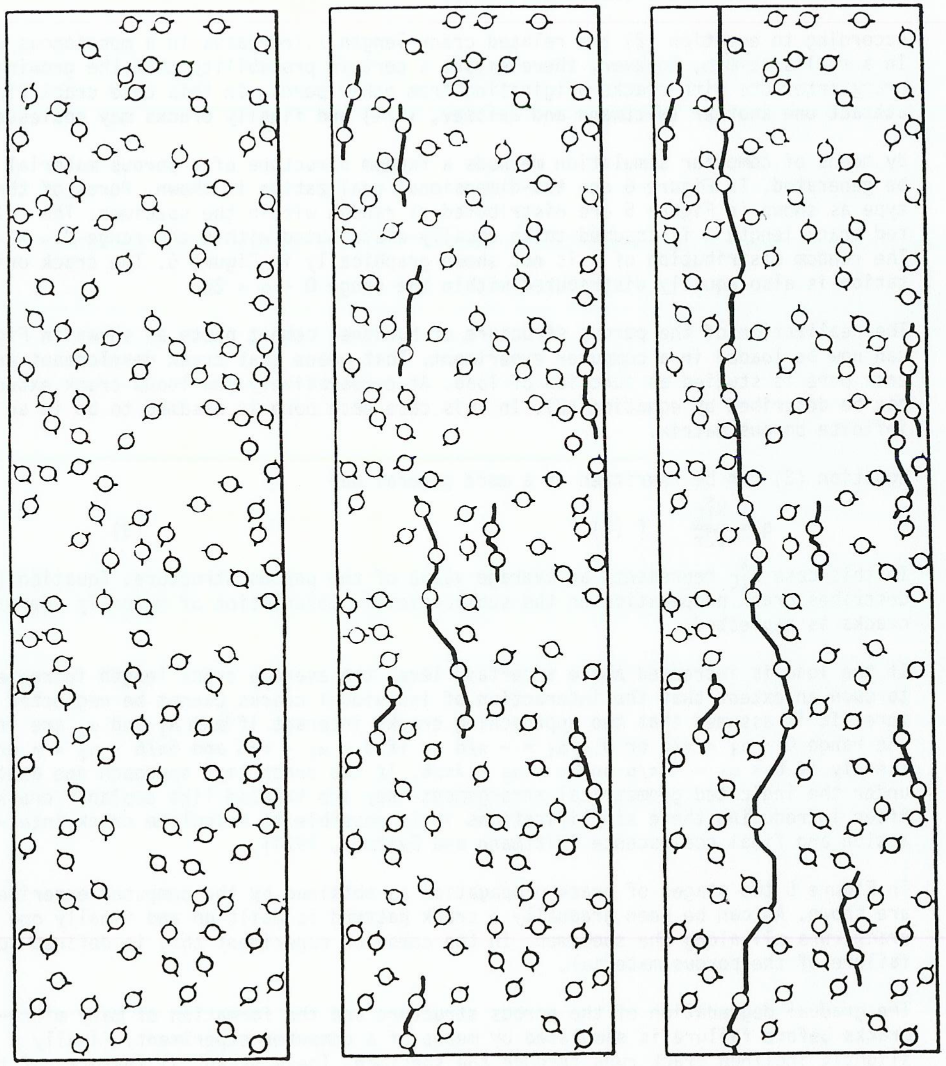


Fig. 6. Two-dimensional computer realization of a simulated porous structure. The crack pattern as obtained under increasing load in a computer experiment is shown.

of stress and to include varying and even more realistic assumptions on the porous structure of hardened cement paste.

Concrete

We have mentioned above, while discussing the submacrolevel, that in real concrete there exist "a priori" cracks. The orientation of these cracks with respect to an external load may be assumed to be random. It has been shown both experimentally and theoretically that stable branching cracks develop in a compressive stress field if there exists an inclined crack (Santiago and Hilsdorf, 1973; Wittmann, 1979; Zaitsev and Wittmann, 1977; Wittmann and Zaitsev, 1981; Zaitsev and Wittmann, to be published). The length of the two branching cracks ℓ_2 which run into the matrix at a given load q is related to the length of the original crack $2\ell_1$ by the following equation (Wittmann, 1979; Wittmann and Zaitsev, to be published) :

$$q = \sqrt{\frac{\ell_2}{\ell_1}} \; \frac{K_{IC}}{2A(\alpha,\rho)} \; \sqrt{\frac{\pi}{\ell_1}} \qquad (4)$$

In this equation $A(\alpha, \rho)$ has the following meaning :

$$A(\alpha, \rho) = \sin^2\alpha \cos\alpha - \rho \sin^3\alpha \qquad (5)$$

where α is the inclination of the original crack with respect to the applied load and ρ is the coefficient of friction of the original crack surfaces.

We have seen that most cracks will be located in the interface between hardened cement paste and aggregate. To investigate this situation we consider one isolated inclusion in a homogeneous matrix. The shape of the inclusion is chosen to be polygonial. It is assumed that there exists an interfacial crack with length $2\ell_1$ at one side of the inclusion. An example is shown in Figure 7. It can be shown that at a given load the interfacial crack will spread along the interface in an instable fashion (mode II) and reach length $2L_1$.

The newly created crack (see Fig. 7) will further behave like an inclined crack in a matrix. Branching cracks will propagate into the matrix in a stable way if the load is further increased. By using equation (4) the length of the two branching cracks can be indicated as function of load by the following equation :

$$q = \frac{K_{IC}}{A(\alpha, \rho)} \; \frac{1}{2L_1} \; \sqrt{\pi\, l_2} \qquad (6)$$

In real concrete, however, cracks which penetrate into the matrix will meet another aggregate very soon. In Figure 7 this situation is shown schematically too. When the crack reaches the second inclusion further crack growth is dependent on both the inclinations of the first and second interface. The conditions for opening (I) and shear (II) mode for crack propagation can be given as follows :

$$q_I^{IF} = \frac{2K_{IC}^{IF} \sqrt{\pi\, l_2/L_1}}{A(\alpha,\rho)\,\{3\cos\frac{\beta}{2} + \cos\frac{3\beta}{2}\} - 3C(\alpha,\rho)\,\{\sin\frac{\beta}{2} + \sin\frac{3\beta}{2}\}} \qquad (7)$$

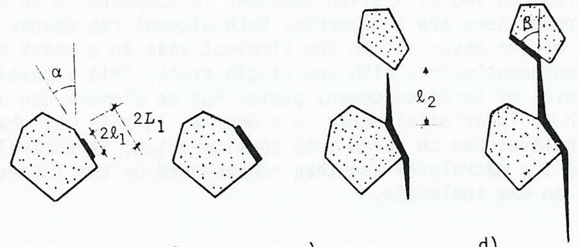
and

$$q_{II}^{IF} = \frac{2K_{IIC}^{IF} \sqrt{\pi\, l_2/L_1}}{A(\alpha,\rho)\,\{\sin\frac{\beta}{2} + \sin\frac{3\beta}{2}\} + C(\alpha,\rho)\,\{\cos\frac{\beta}{2} + 3\cos\frac{3\beta}{2}\}} \qquad (8)$$

In equations (7) and (9) $C(\alpha,\rho)$ has the following meaning :

$$C(\alpha,\rho) = \sin\alpha \cos^2\alpha - \rho \sin^2\alpha \cos\alpha \qquad (9)$$

It is important to note that further crack growth depends also on the sign of β because sign β appears in equations (7) and (8). In fact it turns out that crack propagation is favoured if the inclinations of α and β have the same sign.



a) b) c) d)

Fig. 7. Schematic representation of crack growth starting from an interface, (a-b) the initial crack grows unstable along the interface, (b-c) stable branching cracks run through the matrix, (c-d) the growing crack meets another inclusion.

But a crack, meeting an inclusion, has a third possibility; the crack may extend through the inclusion. Wheter a crack penetrates an inclusion or whether it is deviated along the interface depends on equations (7) and (8) and the condition for straight crack formation through the aggregate :

$$q_I^A = \frac{K_{IC}^A}{A(\alpha,\rho)} \quad \frac{1}{2L_1} \quad \sqrt{\pi l_2} \qquad (10)$$

With the formulae (7), (8), and (10) it is possible to calculate crack formation in an idealized two-dimensional composite material. By means of this set of equations the elements of crack formation on the submacrolevel are described.

On the macrolevel we have to look into the interaction of different units of the submicrolevel. Similar to the procedure which was used to study failure mechanisms of hardened cement paste we can generate a random structure of concrete. In Figure 8 one realization of a computer generated structure is shown. Each polygonial inclusion has one interfacial crack. The length of these cracks is randomly distributed.

If we apply a compressive load in a computer experiment first of all the most critical cracks start to grow. Finally some cracks coalesce and an inclined crack runs all through the specimen. The corresponding load is defined to be the failure load in the computer experiment. Results shown in Figure 8 are based on the assumption that aggregates are stronger than the matrix :

$$K_{IC} < K_{IC}^A \qquad (11)$$

This necessarily means that condition (10) does not become critical and as a consequence all cracks in Figure 8 run around the aggregates. This behaviour is typical for normal concrete.

If we assume instead that strength of matrix and aggregates are comparable :

$$K_{IC} \approx K_{IC}^A \qquad (12)$$

some cracks may penetrate through aggregate particles. This condition is characteristic for high strength and lightweight concrete.

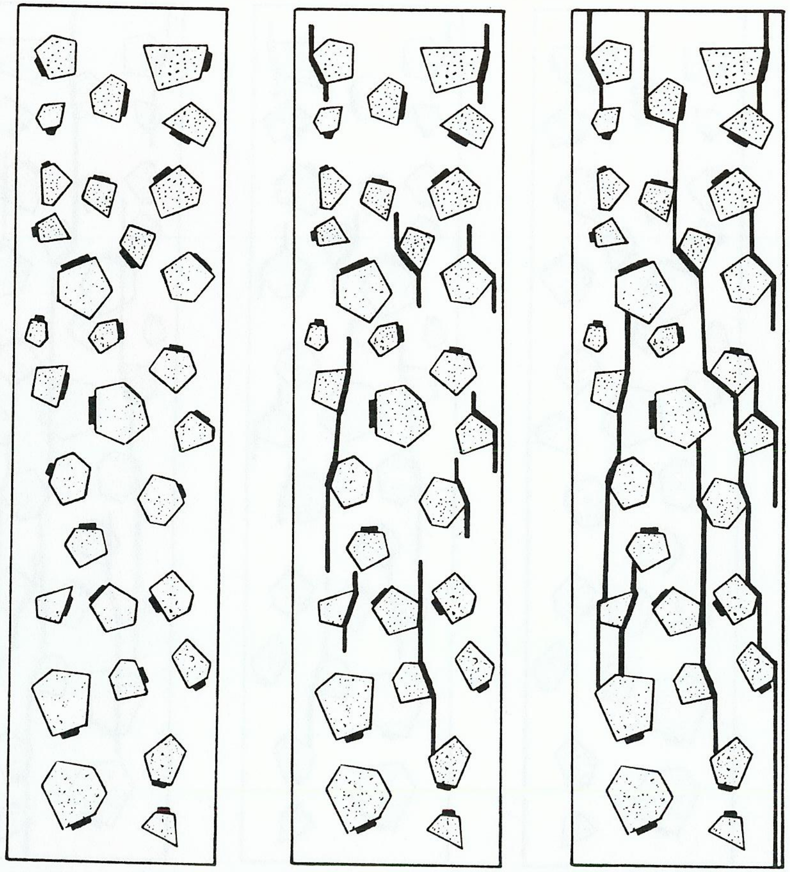


Fig. 8. Crack formation as calculated for normal concrete.

An example of a computer experiment under the condition (12) is shown in Figure 9. As can be clearly seen, not all cracks follow the interface. The final fracture is much less inclined if compared with the fracture surface of normal concrete. There is another feature of high strength concrete which is realistically reproduced by the computer experiment : there is less crack arresting and hence the material reacts generally more brittle.

By means of computer experiments the behaviour under high sustained load or under impact loading conditions can be studied too. Results obtained supplement conventional experimental findings and at the same time they form a valuable contribution to a more fundamental and generalized understanding of materials behaviour.

PROBABILISTIC APPROACH

Probabilistic Concept and Different Structural Levels

Aggregates in concrete are distributed at random. In addition microcracks, other

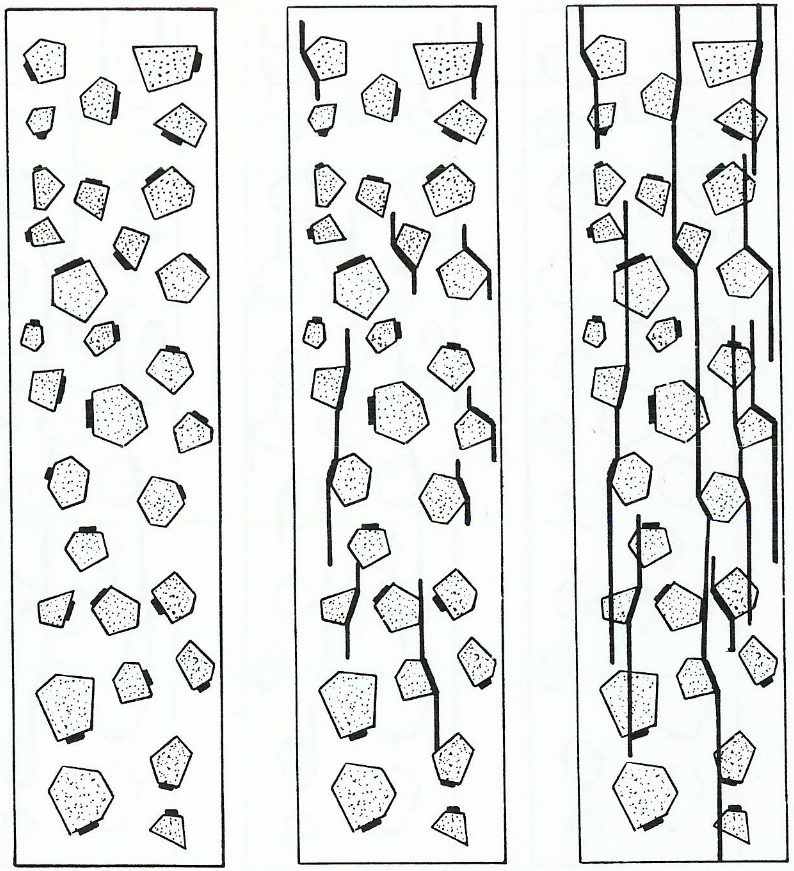


Fig. 9. Crack formation as calculated for high strength concrete.

structural defects, and weak zone are statistically spread over a given specimen. In a computer generation of the concrete structure as shown in Figures 8 and 9 these statistical aspects are taken into consideration. To study the influence of the statistical nature of concrete structure on the variability of macroscopic properties such as strength it is possible, in principle at least, to generate many random structures and determine their failure load in a computer experiment.

In connection with a realistic assessment of the reliability of concrete structures the variability of materials properties has recently gained much interest (Zech and Wittmann, 1978; Alou and Wittmann, 1980). Therefore a more direct method to investigate the statistical nature of concrete properties is needed. Hori (1959) was probably the first to study statistical aspects of fracture of concrete on a thorough theoretical basis. Later Yokobori and Sawaki (1973) developed a stochastic theory of fracture of solids. Mihashi and Izumi (1977) adopted the probabilistic approach and described failure of concrete in terms of a stochastic process.

The statistical concept of Mihashi and Izumi (1977) is based on simplifying assumptions on the structure and can be linked directly with the different structural levels as introduced above. A given specimen is supposed to be built up of m groups and in each group there are n elements. Each element represents the typical structural details of the material. In the simplest case an element consists of a unit volume of homogeneous matrix with one single crack. This situation corresponds with the submicrolevel of hardened cement paste. But an element can also contain an aggregate with an interfacial phase in a matrix. In this way characteristic features of the submacrolevel can be taken into consideration. Structural details of the microlevel and the macrolevel are then represented by the interaction of the elements defined on the sublevels.

Theoretical Basis

If we assume for a moment that a specimen is loaded with constant rate of loading ($\dot{\sigma}$ = const.), we find for the non-fracture probability $P(\sigma)$ the following equation (Mihashi and Wittmann, 1980) :

$$P(\sigma) = \exp\left\{ - \frac{m\,L}{(\beta+1)\dot{\sigma}}\, \sigma^{\beta+1} \right\} \qquad (13)$$

The size of a specimen is taken into consideration by m and L represents the statistical distribution of an equivalent length of a structural defect. β is a materials constant.

The probability density distribution $g(\sigma)$ of strength is found to be :

$$g(\sigma) = \frac{m\,L}{\dot{\sigma}}\, \sigma^{\beta} \quad \exp\left\{ - \frac{m\,L}{(\beta+1)\dot{\sigma}}\, \sigma^{\beta+1} \right\} \qquad (14)$$

And finally the average value of strength of a material with the chosen structural defects is given by :

$$\bar{\sigma} = \left\{\frac{(\beta+1)\dot{\sigma}}{m\,L}\right\}^{\frac{1}{\beta+1}} \quad \Gamma\left(\frac{\beta+2}{\beta+1}\right) \qquad (15)$$

In addition to the mean value the variance $V^2(\sigma)$ can be deduced from the probability density function. In the present case this leads to :

$$V^2(\sigma) = \left\{\frac{(\beta+1)\dot{\sigma}}{m\,L}\right\}^{\frac{2}{\beta+1}} \quad \left\{\Gamma\left(\frac{\beta+3}{\beta+1}\right) - \Gamma^2\left(\frac{\beta+2}{\beta+1}\right)\right\} \qquad (16)$$

We will discuss the influence of rate of loading on mean value and coefficient of variation of strength of a given material here exclusively. From equation (15) we find a simple power function relating strength and rate of loading :

$$\frac{\bar{\sigma}}{\bar{\sigma}_0} = \left(\frac{\dot{\sigma}}{\dot{\sigma}_0}\right)^{\frac{1}{\beta+1}} \qquad (17)$$

The coefficient of variation can be obtained by using equations (15) and (16) :

$$\frac{V(\sigma)}{\bar{\sigma}} = \frac{\left\{\Gamma\left(\frac{\beta+3}{\beta+1}\right) - \Gamma^2\left(\frac{\beta+2}{\beta+1}\right)\right\}^{\frac{1}{2}}}{\Gamma\left(\frac{\beta+2}{\beta+1}\right)} = \text{const.} \qquad (18)$$

It is obvious from equation (18) that the coefficient of variation does not depend on rate of loading.

Comparison with Experimental Results

The variability and strength of mortar bars has been studied experimentally by Zech and Wittmann (1980). Their results essentially verified equations (17) and (18). In the meantime test series with considerably higher numbers of specimens have been carried out (Mihashi and Wittmann, 1980) and these results provide a basis for a more critical comparison. Mortar specimens with two different water-cement ratios have been tested in compression (Series C : w/c = 0,45; Series D : w/c = 0,65). The strength has been determined at six different rates of loading with ensembles of approximatively 30 specimens. The observed increase of strength at higher rates of loading is graphically shown in Figure 10. From the inclination of the straight lines in Figure 10 the materials parameter β can be easily determined (see equation (17)). Values of 27,4 and 25,2 have been found for Series C and D respectively.

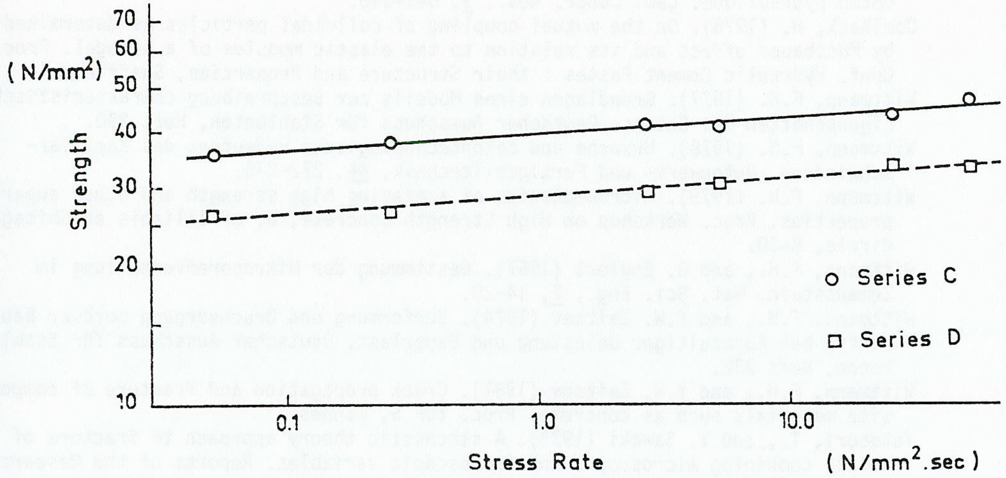


Fig. 10. Mean strength as function of stress rate.

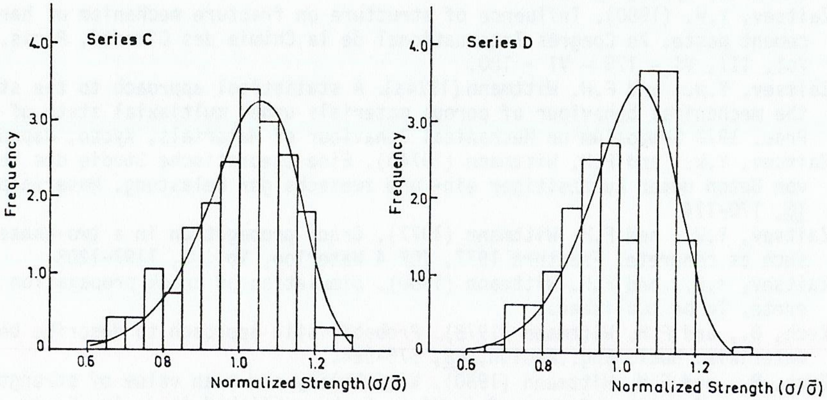


Fig. 11. Frequency distribution of normalized compressive strength of two mortar series.

Within the accuracy of the present experiments the coefficient of variation remained constant over the range of rates of loading under consideration. That means that equation (18) is found to be valid. The frequency distribution of the normalized strength has been determined from all test results of one series. Results are shown in Figure 11. It is obvious that the obtained strength values are not distributed at normal. The solid line in Figure 11 is obtained by using equation (14). Thus we may conclude that a Weibull-type distribution function is a reasonable assumption for compressive strength of mortar and concrete.

CONCLUDING REMARKS

It has been shown that classical fracture mechanics cannot be applied to concrete. The porous matrix and the composite structure of the material necessitate a careful investigation of fracture mechanisms and a revised definition of failure criteria. Fracture mechanics of concrete has to be developed on the basis of characteristic structural features of the material. It is suggested that the structure be subdivided into four different levels. On each level different fracture mechanisms can be discussed. To a certain degree structural properties can be changed independently on the four levels by concrete technology. In this way dominant fracture mechanisms which finally cause failure can be controlled and/or predetermined externally.

It is evident that fracture toughness cannot be characterized sufficiently by a single parameter. Information provided by R-curves is certainly more complete but the actual significance of results obtained by this comparatively new method for the behaviour of a loaded concrete element has to be investigated further (Irwin and Paris, 1977). The gradual development of a damaged zone under a given stress field and its relevance for failure of concrete are far from being understood.

Khrapkov and co-workers (1977) applied fracture mechanics to study crack formation in massive concrete elements. Until now, however, fracture mechanics of concrete cannot be introduced in a more general way into structural design.

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