INSTABILITY PROBLEMS IN DUCTILE FRACTURE

BY

MICHAEL P. WNUK

The Technological Institute, Northwestern University, Evanston, Illinois
(on leave from South Dakota State University, Brookings, South Dakota)

ABSTRACT

When laboratory tests are performed on small scale specimens made of metals exhibiting low yield stress and high fracture toughness their ductile behavior inhibits or totally impedes growth of brittle-like fractures. When, however, the same metals are used to manufacture piping systems or pressure vessels of large dimensions, the so called "size effect" is observed; the initially stable cracks develop into an unstable fracture which rapidly propagates across the thickness of the component leading to a total loss of structural integrity. Conditions under which such transition from stable to unstable crack propagation may occur in ductile metals are studied in this work. An extension of the early version of the final stretch model to other geometrical and loading configurations is suggested, while the restrictions on the amount of plasticity which precedes the onset of crack growth and accompanies spread of stable ductile fracture up to the point of global failure are removed. The differential equations defining the material resistance developed during the early stages of ductile fracture process are derived from the concept of "final stretch". The model suggests a certain near-tip distribution of displacements associated with a quasi-static Mode I crack such that the resulting strains are logarithmically singular at the crack tip.

The final results are of closed form and they are analogous to the numerical data obtained by other researchers on the basis of the incremental plasticity theory. Similarities between the present results and the solutions due to Paris and coworkers, Rice and coworkers as well as the most recent data obtained by Shih's group at the General Electric Company are pointed out.

KEYWORDS

Ductile fracture, crack propagation, tearing modulus, resistance curve, stable and unstable fracture, structural integrity, stability.

Existing linear elastic fracture mechanics and J-integral analyses are well suited for safety assessments of high strength-low toughness materials. These analyses apply only to the onset of crack growth, which is usually tantamount to crack instability and structural failure in that class of materials. However, this is not the case for the low strength, higher toughness grades when crack instability may be preceeded by extensive stable crack growth under rising load. Here, a substantial margin of safety may exist even when the onset of crack growth is imminent.

This paper describes research leading to a ductile fracture mechanics methodology designed to treat two-dimensional large scale yielding and stable crack growth problems. The line plasticity model of a moving crack of Wnuk (1972-1979) is used to obtain predictions concerning the material toughness associated with the preliminary crack extension (R-curve) and to calculate the critical parameters, i.e., load and the crack size, at which a transition to unstable brittle-like fracture will occur. One important finding of this work is that parameters truly reflecting the state of crack tip process zone are not functions of the extent of stable crack growth when the mode of fracture (full shear or flat) remains fixed. The "final stretch" suggested by Wnuk in 1972 or the "crack tip opening angle" could be used, for example, as such material characteristics, constant over the subcritical range of crack growth, and obtainable from a series of relatively simple tests which involve direct measurements of the material toughness and the slope of the resistance curve at the onset of stable crack growth. The possibility exists, therefore, that useful, stable growth parameters can be evaluated from the state of the crack tip at the onset of crack extension. The plastic stress relaxation occurring at the tip of a quasi-statically progressing crack, and the redistribution of strains ensuing within the end-zone adjacent to the crack front renders a stabilizing effect on a spreading fracture. In fact, the change in the nature of plastic strains encountered near the crack front is distinctly reflected by the dominant term which changes from an (l/r) type, observed for a stationary crack, to a log (l/r) form, as the crack tip is approached.

The analytical investigation of extending cracks in near-yield or post-yield situations is difficult mathematically. Some progress has been made in the Mode III or anti-plane strain case by McClintock and Irwin (1965). Although the tensile or Mode I case has received a great deal of attention, Rice (1968, 1975, 1978, 1979), Cherepanov (1974), Wnuk (1972, 1974), Amazigo and Hutchinson (1977), Paris et al. (1977), the analytical solutions produced to date are usually restricted by the requirement of plastic strain field being contained within the surrounding dominant elastic field, i.e., to the so called "small scale yielding" situation. Fracture occuring under the large scale yielding condition has been discussed in some detail by Wnuk (1979) and Smith (1980). Both investigations were based on a highly idealized line-plasticity model modified to account for a moving crack. Here we intend to explore the physical assumptions underlying this model and to present a brief deriviation of the governing equations for 2D and 3D geometries consistent with the final stretch concept. Implications of these results will also be discussed.

Numerical investigations of stable growth done in Europe by Kfouri and Miller (1976), de Koning (1977), Andersson (1973), Tilley (1978) and in this country by Shih et al. (1978), Kanninen et al. (1977) and many other authors seem to point out a number of parameters suitable for the characterization of stable fracture ensuing upon the onset of fracture in a fully plastic range. One such parameter, as suggested by studies of de Koning (1977), Andersson (1973), Shih and et al (1978), Kanninen et al. (1977) is the crack tip opening angle (CTOA). It may be shown that the basic physical concepts underlying Wnuk's model of finite stretch (1972, 1974) and the criterion of critical

opening (say δ) observed at a fixed distance from the crack tip (say Δ), as used by Rice and Sorensen (1978) in their studies of a quasi-static plane strain crack monotonically extending in an elastic-plastic medium, are essentially same. Both ideas are indeed equivalent to the concept of the critical crack opening angle. Somewhat unexpectedly, and despite the entirely different analytical approaches, the end results of both papers, i.e., that of Wnuk (1974) and of Rice and Sorensen (1978) turned out to have the same mathematical form (the pertinent equations in both papers are identical within the accuracy of a numerical constant).

Experiments of Griffis and Yoder (1976) and Clarke et al. (1976) and also the later studies aimed at direct measurement of the CTOA by the crack infiltration technique, cf. Garwood (1977) and Willoughby (1978), seem to indicate that the wedge-shaped tip of a slowly growing crack retains the angle measured at its apex substantially constant during slow crack growth. This observation supports the assumption of a constant crack opening angle. Some researchers have indicated that under monotonic loading the nominal J integral continues to rise steeply with crack advance. It would be, therefore, overly conservative to base a limiting strength prediction on J_{IC}. Accordingly, the experimentally determined curve of J plotted vs. an increment of crack growth, Δa, has been suggested as a means for predicting the stable/unstable transition occurring in the ductile fracture process. Hutchinson and Paris (1977) stated limitations of validity of such J-resistance curve through an inequality

$$R_o \propto b \gg J/(dJ/da) \qquad (1.1)$$

Here, R_o and b are certain linear dimensions. The first one denotes the radius in which the "HRR" (Hutchinson-Rice-Rosengren) stress and strain fields at a distance r from the crack tip are dominant, i.e.,

$$\left.\begin{array}{l} \underset{\sim}{\sigma} \propto (J/r)^{N/(1+N)} \\ \underset{\sim}{\varepsilon} \propto (J/r)^{1/(1+N)} \end{array}\right\} \quad 0 \leq r \leq R_o \qquad (1.2)$$

where N is the work-hardening exponent in the power-law hardening material, $\tau = \gamma^N$, in which τ and γ denote the von Mises equivalent shear stress and shear strain measures. The quantity R_o was shown to scale approximately with the maximum radius of the plastic zone for the small scale yielding situation (ssy), i.e., $R_o \simeq .16\ EJ/\sigma_o^2$ in where σ_o denotes the uniaxial yield stress, cf. Rice and Sorensen (1978). In the large scale yielding situation (lsy) R_o scales approximately with overall size of the specimen, measured by the uncracked ligament size b, as stated in the expression (1.1). The physical assumption underlying inequality (1.1) is the requirement that the amount of proportional straining of the HRR type due to increments in J, dominates over the non-proportional straining due to increments in a, i.e.,

$$\frac{dJ}{J} \gg \frac{da}{R_o} \qquad (1.3)$$

This is satisfied when the quantity R_o is large enough to fully envelop the crack tip process zone (say Δ) which provides the length-scale for stable cracking, i.e., $da \approx a$. Since this process zone size Δ is typically of the same order of magnitude as the crack tip opening displacement at the onset of growth, i.e., $\Delta \approx \delta_{ini} \simeq .65\ J_{IC}/\sigma_o$ (the factor .65 is a result of numerical work based on finite element approach by Rice and Sorensen (1978) and McMeeking (1977)), the requirement (1.1) can be re-interpreted in terms of a tearing modulus

$$T_J = (E/\sigma_o^2)\ (dJ/da) \tag{1.4}$$

suggested by Paris, or a tearing modulus

$$T_\delta = (E/\sigma_o)\ (\hat{\delta}/\Delta) \tag{1.5}$$

suggested by Shih et al. (1977).

Applying Wnuk's model of final stretch to a Mode I crack Smith (1980) have shown that both the moduli, as defined by eqs. (1.4) and (1.5), become identical provided that slow crack growth occurs in a solid of low strength and high tearing resistance (at $T_J \geqslant 50$), and that the extent of this growth is small vs. the initial crack size and other dimensions of a specimen. Under these restrictions the quantity dJ/da can indeed be identified with the <u>initial</u> slope of the J-resistance curve, $(dJ/da)_i$, and regarded a material property as suggested by Paris and coworkers (1977). In contrast, the second tearing modulus given above, T_δ, remains invariant to the amount of crack extension and, therefore, it provides a more fundamental material characteristic associated with ductile fracture. This observation is confirmed both by the existing experimental and numerical data derived by finite elements approaches to a quasi-static crack problem. Thus the condition of failure based on the CTOA parameter or, equivalently, on the final stretch concept appears to be not only physically sound but also free of the restrictions that need to be imposed to validate Paris' concept of a constant slope of the J-resistance curve.

Briefly, the validity of the J-controlled stable crack growth is affected by the strength level (σ_o/E) and the tip opening parameter (CTOA or $\hat{\delta}/\Delta$), given as the ratio of the critical opening $\hat{\delta}$ observed at a characteristic near tip distance Δ. Low values of strength level (σ_o/E) and the tip opening parameter $(\hat{\delta}/\Delta)$ are required to validate any theory of quasi-static tensile crack based on a J-concept. Low strength/high toughness implies a steeply rising J versus change in crack length curve and an impossibility of attainment of the terminal instability of fracture developed under conditions of well contained yielding. This considerably restricts the scope of applications of the presently existing analytical solutions all of which describe in fact the small scale yielding stress and strain fields associated with an advancing crack.

Analysis given here, which is based on the final stretch concept suggested by the author in 1972, provides equations governing quasi-static extension of a tensile crack contained in either partially or in a fully yielded specimen. These equations are

$$dR/da = (R/a) + (\phi')^{-1}\left\{M - \phi - \Phi\right\};\quad R = R(a),\ \phi = \phi(R/a),\ \Phi = \Phi(\Delta, a)$$
$$\phi' = d\phi/d(R/a) \tag{1.6}$$

for a resistance curve represented in the (R,a) plane, and

$$dJ_R/da = \varkappa\left\{M - \Phi(\Delta, a)\right\} \tag{1.7}$$

for a material resistance described by the contour integral dependent on the instantaneous crack length a, $J = J_R(a)$. The length R scales with the maximum extent of the

plastic zone developed ahead of the crack front, while functions ϕ and Φ are related respectively to the crack tip opening displacement $u_{tip}(a)$ and the gradient of the displacement $u(x_1, a)$ evaluated at a small distance $x_1 = \Delta$ from the crack tip. Two constants $M\ (=(\pi E_1/8\sigma_Y)\ (\hat{\delta}/\Delta))$ and $\varkappa\ (=8n\sigma_Y^2/\pi E_1)$ incorporate the effective yield stress σ_Y, the elastic material properties, E and ν, an empirical factor n $(=J/2\sigma_Y u_{tip})$, the final stretch $\hat{\delta}$ and the size of the process zone Δ over which the final act of fracture takes place (one may think of Δ as a step size for a quantum-like crack extension). The constant M is shown to be identical within a numerical factor with Shih's tearing modulus $T_\delta = (E/\sigma_o)\ (\hat{\delta}/\Delta)$, while the product $M\varkappa\ (=n\sigma_Y(\hat{\delta}/\Delta))$ is shown to be proportional to the tearing modulus suggested by Paris, $T_J = (E/\sigma_o^2)\ (dJ_R/da)$, if the slope dJ_R/da is interpreted as the <u>initial</u> slope of the J-resistance curve.

The differential equations defining the material resistance functions R (a) and the J_R (a), as given above, are valid for arbitrary crack and loading configurations. Their use is illustrated by a number of examples involving traction free (case a) and pressurized (case b) cracks either under plane stress or plane strain condition (2D problem) or in an axi-symmetrical configuration of a disc-shaped crack (3D problem). To make the problem susceptible to a mathematical treatment a modified model of DBCS type is employed, but it should be emphasized that the basic physical concept underlying these considerations, i.e., the constancy of the final stretch or, equivalently, the invariance of the crack tip opening angle (CTOA) during slow crack growth, remains valid in a general sense, irrespective of a particular choice of the computational approach. The solutions based on the DBCS model suggest the following expressions for the dimensionless functions, $\phi(x)$ and $\Phi(\Delta, a)$; in which x denotes the ratio R (a)/a:

<u>2D Crack</u>

$$\phi(x) = \begin{cases} \log(1+x) & \text{case a} \\ x\left\{1 + 2\log\left(\sqrt{1+x} + \sqrt{x}\right)\right\} & \text{case b} \end{cases} \tag{1.8}$$

$$\Phi(\Delta, a) = \begin{cases} \frac{1}{2}\log\left[\dfrac{2eax(2+x)}{\Delta(1+x)^2}\right] & \text{case a} \\ \frac{1}{2}\log\left(\dfrac{4eax}{\Delta}\right) + \sqrt{x(1+x)} & \text{case b} \end{cases} \tag{1.9}$$

<u>3D Crack</u>

$$\phi(x) = \begin{cases} x(1+x)^{-1} & \text{case a} \\ x\left\{1 + x + \sqrt{x(2+x)}\right\} & \text{case b} \end{cases} \tag{1.10}$$

$$\Phi(\Delta, a) = \begin{cases} \frac{1}{2}\log(8a/\Delta) - \left[1 + \left(x(1+x)(2+x)\right)^{-1}\right] & \text{case a} \\ (1+x)\left[1 + x + \sqrt{x(2+x)}\right]\left\{\frac{1}{2}\log\left(\dfrac{8a}{\Delta}\right) - 1 - \left[x(1+x)(2+x)\right]^{-1}\right\} & \text{case b} \end{cases} \tag{1.11}$$

The corresponding J_R resistance curves are defined by certain non-linear differential equations which are derived from the forms (1.8 - 1.11) and the governing equation (1.7). These equations can be considerably reduced if two limiting cases are considered; they are (1) contained yielding $(x \ll 1)$, and (2) large scale yielding $(x \gg 1)$. We shall use the abbreviations "ssy" to denote the first one and "lsy" to designate the second one. When the appropriate limiting procedures are carried out for very small and very large R/a ratios, one obtains these representations

2D Crack, case a

$$\frac{dJ_R}{da} = \begin{cases} (4\sigma_Y^2/\pi E_1)\log(J_{ss}/J_R) & \text{ssy} \\ n\sigma_Y(\hat{\delta}/\Delta) - (4n\sigma_Y^2/\pi E_1)\log(2ea/\Delta) & \text{lsy} \end{cases} \qquad (1.12)$$

2D Crack, case b

$$\frac{dJ_R}{da} = \begin{cases} \mathcal{H}\left[M - \frac{1}{2}\log(4eJ_R/\mathcal{H}\Delta)\right] & \text{ssy} \\ \mathcal{H}\left[M - \frac{1}{2}\log(a/\Delta) - x\right] & \text{lsy} \end{cases} \qquad (1.13)$$

in which x and J_R are related in this fashion

$$x\log(4ex) = \mathcal{H}^{-1}(J_R/a) \qquad (1.13a)$$

3D Crack, case a

$$\frac{dJ_R}{da} = \mathcal{H}_1 \begin{cases} \left[M - \frac{1}{2}\log\left(\frac{8a}{\Delta}\right) + \frac{1}{4}(J_R/\mathcal{H}_1 a)\right] & \text{ssy} \\ \left[M - \frac{1}{2}\log\left(\frac{8a}{\Delta}\right) + 1\right] & \text{lsy} \end{cases} \qquad (1.14)$$

3D Crack, case b

$$\frac{dJ_R}{da} = \mathcal{H}_1 \begin{cases} \left[M - \frac{1}{2}\log\left(\frac{8a}{\Delta}\right) + (1/2x) + (1/\sqrt{2x}) + 5/4\right]_{x = J_R/\mathcal{H}_1 a} & \text{ssy} \\ \left[M - x\log\left(\frac{8a}{\Delta}\right) + 2x^2\right]_{x = \sqrt{J_R/2\mathcal{H}_1 a}} & \text{lsy} \end{cases} \qquad (1.15)$$

The constants $\mathcal{H}$, $\mathcal{H}_1$, and J_{ss} are defined as follows

$$\mathcal{H} = 8n\sigma_Y^2/\pi E_1 \qquad E_1 = \begin{cases} E & \text{for plane stress} \\ E(1-\nu^2)^{-1} & \text{for plane strain} \end{cases} \qquad (1.16)$$

$$\mathcal{H}_1 = 8n\sigma_Y^2(1-\nu^2)/\pi E_1$$

$$J_{ss} = (2n\sigma_Y^2\Delta/\pi E_1)\exp\left\{(\pi E_1/4\sigma_Y n)(\hat{\delta}/\Delta) - 1\right\}$$

Expressions quoted here enable one to study the instabilities occurring in ductile fracture process. A non-dimensional parameter, named stability index (see Fig. I)

$$\lambda = (\pi E_1/8\sigma_Y^2)\left\{dJ_R/da - \partial J_A/\partial a\right\} \qquad (1.17)$$

provides a useful measure of the "degree of stability" of any given stress state associated with a growing crack. As is seen from the formula (1.17) the quantity λ is positive for a stable crack, as only then the demand for the energy flow into the process zone, dJ_R/da, exceeds the actual rate of energy supply, $\partial J_A/\partial a$ (J-integral labeled

with an index "A" denotes the intensity of the external field).

One of the important byproducts of the theoretical considerations of this sort is the refinement of Paris' concept of tearing modulus and its applications in assessment of ductile fracture toughness. The methodology of ductile fracture derived from analyses of quasi-static crack extension in fully plastic range, i.e., either under the near-yield or post-yield condition, is currently being applied in design of piping systems and thick-walled pressure vessels in reactor technology.

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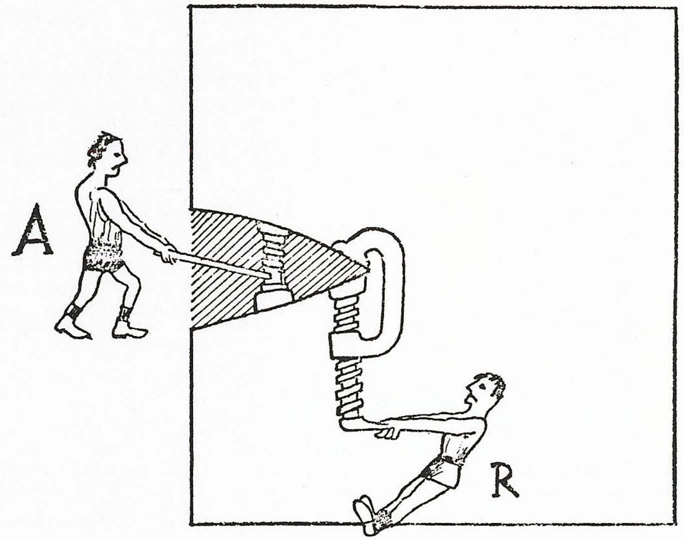

Fig. I An artist's view of the equilibrium state which exists at the crack tip. Note that the greater is the effort expended by gentleman A, who represents the external field, the stronger will be the resistance put up by gentlemen R, who symbolizes the material response. For a stable crack one would expect the demand for the energy flow into the crack tip, dJ_R/da, to exceed the available rate of energy supply, $\partial J_A/\partial a$, or shortly, $\lambda > 0$.