FRACTURE MECHANICAL CALCULATIONS, TEST METHODS AND RESULTS FOR CONCRETE AND SIMILAR MATERIALS

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ABSTRACT

A calculation model, based on fracture mechanics and the finite element method, is presented. In the model the fracture zone in front of a crack is represented by a fictitious crack that is able to transfer stress. The stress-transferring capability of the fictitious crack decreases when the crack width increases.

Calculation results are presented which show how the results from fracture mechanical tests are affected by specimen dimensions.

The shape of the complete tensile stress-strain-curve is introduced as a fracture mechanical parameter and a new testing machine is presented by which it is possible to carry out stable tensile tests with concrete. New test methods for determining the fracture energy and the tensile strength are also presented.

KEYWORDS

Fracture mechanics; cracking; concrete; finite element method; fracture energy; tensile strength; complete stress-strain-curve.

THE FICTITIOUS CRACK MODEL

When a notched specimen of a linear elastic material is subjected to load, the stress in front of the notch will, theoretically, approach infinity. Of course this is impossible for a real material. In the case of concrete, or other non-yielding materials, a zone of microcracks will develop in front of the notch and this reduces the stress-concentration. However, if the zone is small compared with the dimensions of the specimen and the crack depth, linear elastic fracture mechanics may be useful, otherwise other calculation methods have to be used.

A few investigations (Entov and Yagust, 1975; Sok and Baron, 1979) have been carried out in order to study strains in front of notches in large concrete specimens. The results of these tests indicate that microcracks develop hundreds of millimeters in front of the notch before the maximum load is reached. This implies that the fracture zone is never small compared with normal specimen dimensions and also that linear elastic fracture mechanics cannot be used for such a material. In order to carry out relevant calculations one has to consider the properties of the fracture zone.

The discussion in this paper deals mainly with the opening mode, mode I. In this mode the fracture zone develops in a tensile stress field perpendicular to the crack and consequently it is possible to obtain a great deal of information about the fracture zone from a stable tensile stress-strain curve, see Fig. 1.

In Fig. 1 the stress continuously increases up to the maximum load and therefore the material must be uniformly strained on this part of the curve. The non-linearity of the increasing part of the curve is due to microcracks. When the stress reaches its maximum value, one cross sectional area is unable to carry more load and it is fair to assume that the development of microcracks will be concentrated on a small material volume close to this cross sectional area when the specimen becomes further deformed.

When a fracture zone starts developing the stress decreases and consequently no more fracture zones will be created. As the properties of the fracture zone are unaffected by specimen dimensions, this means that the decreasing part of the σ-ε-curve in Fig. 1 is not only related to material properties but also to the specimen length. A better way to describe the deformation properties of a non-yielding material therefore is to use two relations; one relation between stress and relative strain up to the maximum stress, see Fig. 2a, and one relation between stress and the absolute deformation of the fracture zone after the maximum stress is reached, see Fig. 2b. These relations are unaffected by specimen dimensions and they can therefore be dealt with as material properties.

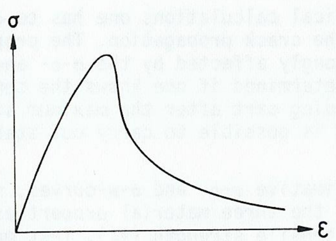

Fig. 1. A stable stress-strain curve for concrete. σ= stress, ε=strain.

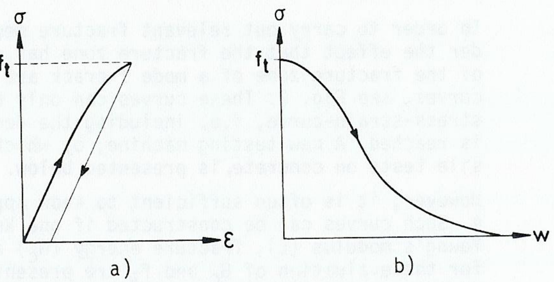

Fig. 2. a) A relation between stress and relative strain (ε) outside the fracture zone.
b) A relation between stress and absolute deformation (w) of the fracture zone.

In order to describe complicated fracture processes one has to use approximative models. In the model described here the real fracture zone (Fig. 3a) is represented by a crack that can transfer stress (Fig. 3b). The stress transferring capability of the crack is dependent on its widening according to the σ-w-curve in Fig. 2b. This curve describes the stress transferring capability of a fracture zone in direct tension but a reasonable assumption is that the same curve can be used for mode I-cracks. As the stress transferring crack is not a real crack but more of a fictitious crack the model is called the Fictitious Crack Model (FCM).

Models with stress transferring zones have been used before (Barenblatt, 1962; Dugdale, 1960). In the Dugdale model, which is suitable for yielding materials, the attractive stress across the fictitious crack is always constant and equals the yield stress. Consequently this model is a special case of FCM. In the Barenblatt model the attractive stress is caused by cohesive forces and therefore the fracture zone in this model is of another magnitude than is normal for FCM.

FCM normally cannot be treated analytically but numerical methods have to be used. The most suitable method seems to be the Finite Element Method (FEM). When FEM is used the stresses acting across the crack are replaced by nodal forces, see Fig. 3c. These forces decrease continuously according to the σ-w-curve when the width of the fictitious crack increases. When the first principal stress reaches the tensile strength in the top node, see Fig. 3c, this node is opened and forces start acting across the crack at this point. In this way it is possible to follow the crack growth through the material. The model is discussed in detail by Hillerborg, Modéer and Petersson (1976) and Modéer (1979).

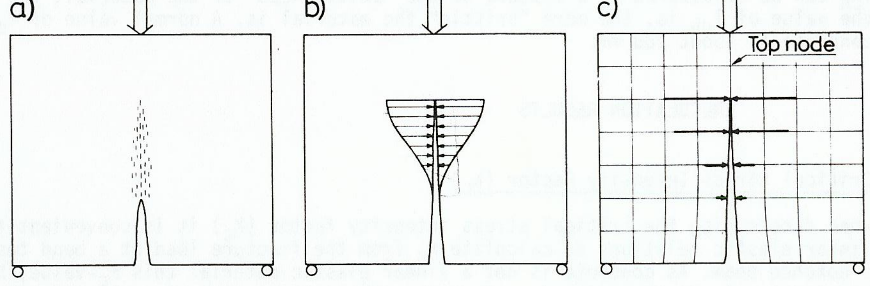

Fig. 3. a) The fracture zone in a real material.
b) In FCM the fracture zone is represented by a crack that can transfer stress.
c) FEM-representation of FCM.

Normally the σ-ε-curve for non-yielding materials is almost linear up to the maximum stress and therefore a good method of approximation is to replace the curve with one straight line, see Fig. 4a. The position of this line is defined by the tensile strength (f_t) and Young's modulus (E).

To obtain the true, descending σ-w-curve one has to carry out stable tensile tests. This is difficult for concrete and similar materials, the reason will be discussed below. This means that one normally has to assume a shape of the σ-w-curve. For concrete a good approximation seems to be one straight line, see line 1 in Fig. 4b.

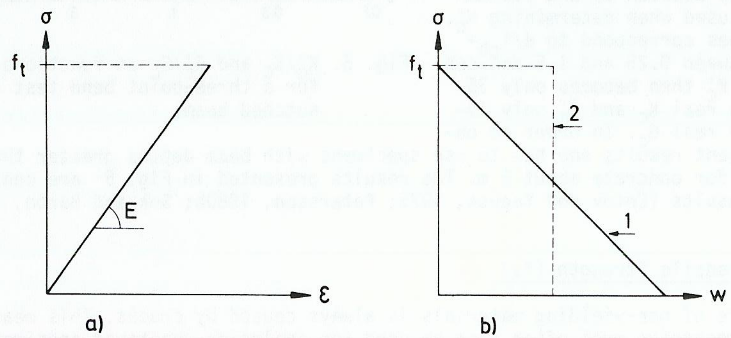

Fig. 4. a) A straight-line approximation of the σ-ε-curve for non-yielding materials.
b) Straight-line approximations of the σ-w-curve for plain concrete (1) and fibre-reinforced concrete (2).

The position of the line is defined by the tensile strength and the area under the curve. This area represents the amount of energy necessary to create one unit of area of a crack and consequently it equals the fracture energy (G_c). For fibre-reinforced concrete an approximation according to Dugdale, see line 2 in Fig. 4b is often better.

A material parameter used in this paper is the characteristic length (l_{ch}), which is defined by:

$$l_{ch} = G_c E / f_t^2 \quad \ldots\ldots (1)$$

l_{ch} can be considered as a measure of the "brittleness" of the material; the lower the value of l_{ch} is, the more "brittle" the material is. A normal value of l_{ch} for concrete is about 200 mm.

CALCULATION RESULTS

Critical Stress Intensity Factor (K_c)

When determining the critical stress intensity factor (K_c) it is convenient to use linear elastic relations to calculate K_c from the fracture load at a bend test on a notched beam. As concrete is not a linear elastic material this K_c-value, here denoted as K_c', is approximative. By using FCM it is possible to calculate the difference between K_c' and the real K_c ($K_c = \sqrt{G_c E}$) and in Fig. 5 the ratio of K_c'/K_c is shown as a function of beam depth (d).

The ratio of G_c'/G_c is also shown in the Fig. ($G_c' = (K_c')^2/E$). The real value of G_c corresponds to the area under the σ-w-curve. The curves in Fig. 5 are relevant for a three-point bend test on a notched beam with a notch depth/beam depth ratio of 0.5. The σ-w-curve is approximated to one straight line.

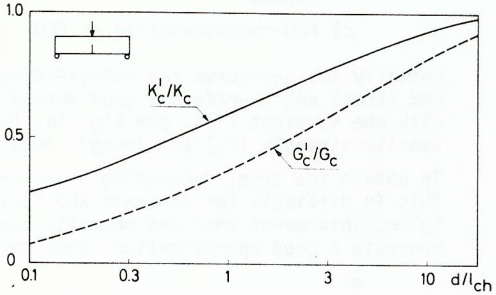

Fig. 5. K_c'/K_c and G_c'/G_c as functions of d/l_{ch} for a three-point bend test on a notched beam.

A normal value of l_{ch} for concrete is 200 mm. Normally specimens with beam depths between 50 and 300 mm have been used when determining K_c'. These values correspond to d/l_{ch}-values between 0.25 and 1.5 and consequently K_c' then becomes only 35-65% of the real K_c and G_c' only 15-40% of the real G_c. In order to obtain relevant results one has to use specimens with beam depths greater than 10x l_{ch}, i.e. for concrete about 2 m. The results presented in Fig. 5 are confirmed by test results (Entov and Yagust, 1975; Petersson, 1980b; Sok and Baron, 1979).

Flexural Tensile Strength (f_f)

The failure of non-yielding materials is always caused by cracks. This means that fracture mechanics must often also be used for analysing unnotched specimens.

One frequently used test carried out on unnotched specimens is the determination of the flexural tensile strength (f_f) in a three-point bend test. Results from many investigations indicate that f_f is strongly affected by specimen depth, but no one has been able to produce a satisfying explanation for this. By FCM it is possible to analyse the bend test and in Fig. 6 it is shown how the ratio f_f/f_t (f_t = tensile strength), theoretically, depends on beam depth. In the calculations the σ-w-curve is approximated to one straight line. The upper curve in the Fig. represents beams subjected to no shrinkage stresses, i.e. wet specimens, while the lower curve represents beams with a certain degree of shrinkage stresses. In the Fig. test results are also presented and the agreement between the calculated curves and the test results is good, in spite of the simplified assumptions.

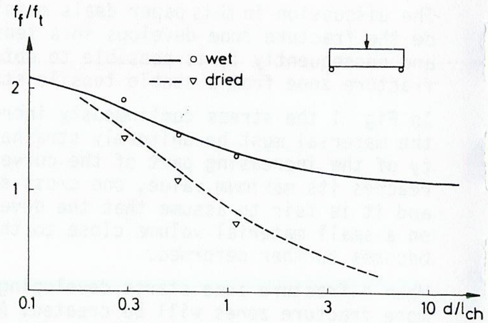

Fig. 6. f_f/f_t as a function of d/l_{ch}.

METHODS FOR DETERMINATION OF FRACTURE MECHANICAL PROPERTIES

Introduction

In order to carry out relevant fracture mechanical calculations one has to consider the effect that the fracture zone has on the crack propagation. The properties of the fracture zone of a mode I-crack are strongly affected by the σ-ε- and σ-w-curves, see Fig. 2. These curves can only be determined if one knows the complete stress-strain-curve, i.e. including the descending part after tha maximum stress is reached. A new testing machine, by which it is possible to carry out stable tensile tests on concrete, is presented below.

However, it is often sufficient to know approximative σ-ε- and σ-w-curves, see Fig. 4. Such curves can be constructed if one knows the three material properties Young's modulus (E), fracture energy (G_c) and tensile strength (f_t). Test methods for the evaluation of G_c and f_t are presented below. E can be determined by standard test methods and these will not be discussed in this paper.

Fracture Energy (G_c)

Almost all values of G_c available in literature are determined indirectly from the fracture load at bend tests on notched beams. This method is unsuitable as the results are strongly affected by the specimen dimensions, see Fig. 5. A better way

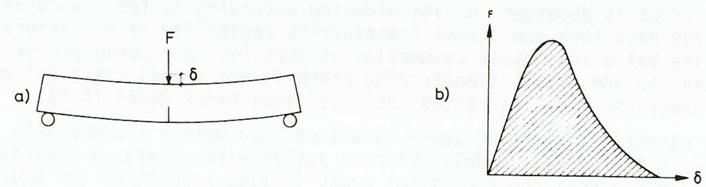

Fig. 7. a) A three-point bend test on a notched beam for G_c-determination.
b) A stable F-δ-curve obtained from a test according to Fig. 7a.

of determining G_c is to use a direct method. From a three-point bend test on a notched beam, see Fig. 7a, it is possible to obtain a stable load-deflection-curve (F-δ-curve), see Fig. 7b, if one uses a beam with suitable dimensions and a stiff testing machine. The area under the curve represents the amount of energy that is consumed by the crack when it grows through the beam (corrections have to be made for the energy supplied by the weight of the beam). As the cross-sectional area of the beam is known it is possible to calculate G_c. This direct method of determining G_c does not seem to be affected by the specimen dimensions or the crack depth (Petersson , 1980a).

G_c-values determined by this direct method are presented by Petersson (1980b) for different concrete qualities. The parameters that most significantly affect the values of G_c seem to be the water-cement-ratio, the quality of aggregate and the age of the concrete.

Tensile Strength (f_t).

It is complicated to determine the tensile strength by direct tensile tests on prismatic concrete specimens. When the specimen is adapted to the testing machine by ordinary grips, these give rise to stress concentrations and multi-axial stresses and the determined strength becomes less than the real tensile strength. Also when the specimen is glued into the testing machine, complicated stress fields occur at the ends of the specimen due to shear deformations in the glue (Hughes and Chapman, 1965) and this reduces the fracture load. In order to avoid direct tensile tests it is therefore normal to carry out indirect tests, for example bend tests or splitting tests. However, these tests are strongly affected by specimen dimensions, see Fig. 6, and are therefore unsuitable.

In Fig. 8 a new type of grip is presented by which it is possible to determine the tensile strength by direct tests on prismatic specimens. The grips consist of steel plates on which wedge-shaped rubber inserts are glued. It is essential that the inserts are orientated according to the Fig. Each part of the grip is pressed against the specimen at two points; one force near the end of the specimen and one smaller force close to the free part of the specimen. This means that the stress perpendicular to the specimen becomes small at the critical cross section (i.e. at point A in the Fig.) and here the stress field will be almost one-dimensional.

Due to the variable stiffness of the wedge-shaped rubber inserts the main part of the load is applied close to the ends of the specimen and consequently the stress grows smoothly to its maximum value which is shown in the Fig. This implies that the stress-concentrations at point A will be reduced to a minimum.

Results from tensile tests carried out by the grips in Fig. 8 are presented by Petersson (1980b).

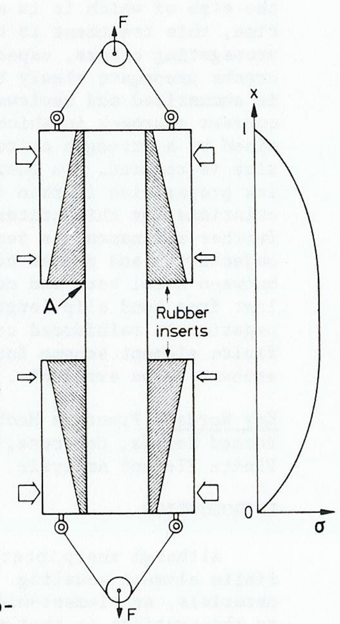

Fig. 8. Grips for tensile tests. The wedge-shaped rubber inserts reduce the stress concentration to a mimimum.

The Complete Stress-Strain-Curve of Concrete

When the maximum load is reached in a tensile test, a fracture zone develops through the specimen. The fracture zone consumes energy when the specimen is further deformed while energy is released due to elastic unloading of the material outside the fracture zone and of the testing machine. The fracture becomes stable if the fracture zone, at each moment of the test, is able to consume all the energy that is released. In order to obtain a stable fracture it is necessary to reduce the energy release by using small specimens and stiff testing machines. Normally available testing machines are too weak for determining the complete stress-strain-curve for concrete and this is probably the reason why only a few results are presented in literature (Evans and Marathe, 1968; Heilman, Hilsdorf and Finsterwalder, 1969; Hughes and Chapman, 1966).

In Fig. 9 a new type of testing machine is shown by which it is possible to carry out stable tensile tests. Three aluminium columns are fixed between two concrete blocks and cylindrical heating elements are attached to the columns. The specimen is fixed in special holders between the concrete blocks. The aluminium columns expand when they are heated and then the specimen becomes subjected to a tensile load. The load is registered by strain gauges which are attached to one of the holders and the deformation is registered by inductive deformation transducers which are fixed to the specimen. The aluminium columns are insulated with mineral wool in order to keep the temperature around the specimen constant.

In Fig. 10 and Fig. 11 examples of the complete stress-deformation-curves are shown for some concrete qualities. When measuring the deformations a 40 mm effective gauge length was used. Each curve represents a mean value of three specimens. The aggregate is crushed quartzite and the ratio cement paste/aggregate (by volume) is 0.5. The age of the concrete was 28 days when the tests were carried out.

In Fig. 10 curves representing two different water-cement-ratios are compared. The curves differ in the beginning but when the deformation exceeds 50 μm they become almost identical. As can be seen from the Fig. the material is able to carry load even when it is considerably deformed and consequently concrete is far from being a linear elastic material.

The curves in Fig. 11 represent mortar and concrete (max. particle size = 2 mm and 8 mm respectively). The water-cement-ratio for both the materials is 0.7. The main difference between the curves is that concrete seems to be able to transfer stress

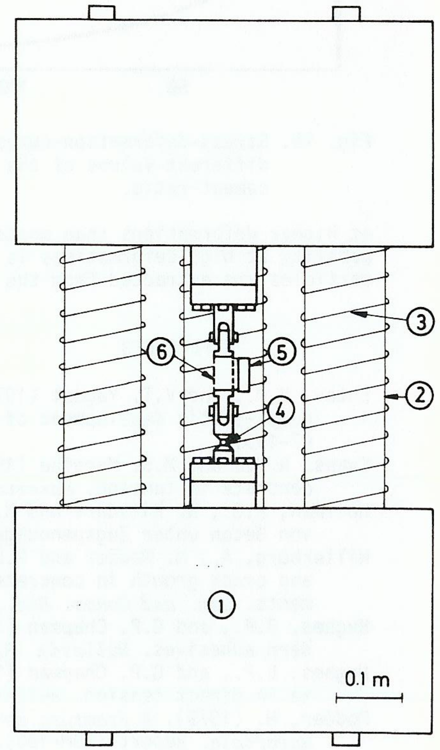

Fig. 9 . A stiff testing machine for determining the tensile stress-strain-curve. 1=concrete block, 2=aluminium column, 3=heating element, 4=strain gauges for load registrations, 5=inductive displacement transducer for displacement registrations, 6=specimen.

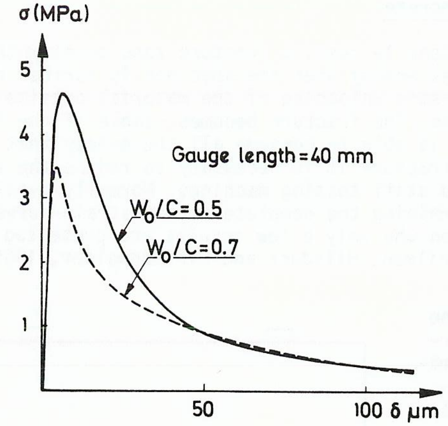

Fig. 10. Stress-deformation-curves for different values of the water-cement-ratio.

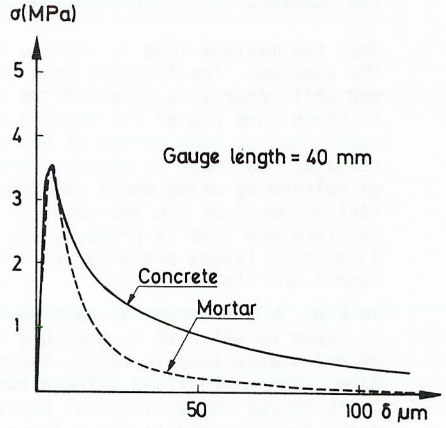

Fig. 11. Typical stress-deformation curves for concrete and mortar respectively.

at higher deformations than mortar. This indicates that the stress-transferring capability at high deformations is due to friction forces when the coarse aggregate particles are extracted from the cement paste.

REFERENCES

Entov, V.M., and V.T. Yagust (1975). Experimental investigation of laws governing quasi-static development of macrocracks in concrete. *Mechanics of Solids, 10,* 87-195.

Evans, R.H., and M.S. Marathe (1968). Microcracking and stress-strain curves for concrete in tension. *Materiaux et Constructions*, No 1, 61-64.

Heilman, H.G., H. Hilsdorf and K. Finsterwalder (1969). Festigkeit und Verformung von Beton unter Zugspannungen. *Deutcher Ausschuss für Stahlbeton*, Heft 203.

Hillerborg, A., M. Modéer and P.E. Petersson (1976). Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cem. and Concr. Res., 6,* 773-781.

Hughes, B.P., and G.P. Chapman (1965). Direct tensile test for concrete using modern adhesives. *Bulletin RILEM*, No. 26, 77-80.

Hughes, B.P., and G.P. Chapman (1966). The complete stress-strain curve for concrete in direct tension. *Bulletin RILEM*, No. 30, 95-97.

Modéer, M. (1979). *A Fracture Mechanics Approach to Failure Analyses of Concrete Materials.* Report TVBM-1001, Division of Building Materials, University of Lund, Sweden.

Petersson, P.E. (1980a). Fracture energy of concrete: Method of determination. *Cem. and Concr. Res., 10,* 78-89.

Petersson, P.E. (1980b). Fracture energy of concrete: Practical performance and experimental results. *Cem. and Concr. Res., 10,* 91-101.

Sok, C., and J. Baron (1979). Mecanique de la rupture appliquee au beton hydraulique. *Cem. and Concr. Res., 9,* 641-648.