EXPERIMENTAL DETERMINATION OF FRACTURE TOUGHNESS PARAMETERS K_{Ic} AND J_{Ic} FOR AGGREGATIVE MATERIALS

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ABSTRACT

The object of the paper is to determine the fracture toughness parameters K_{Ic} and J_{Ic} for some aggregative materials. Values of the J-integral are calculated from load-displacement curves, following the procedure suggested by Begley and Landes for steel alloys. Some recurring experimental incoherences are explained applying Buckingham's Theorem for physical similitude and scale modeling to Fracture Mechanics. Thus a non-dimensional parameter can be defined (the test brittleness number), which governs the fracture-sensitivity phenomenon. The fracture parameters K_{Ic} and J_{Ic} are connected by a fictitious Young's modulus E*, which is lower than the real modulus E and represents the stiffness of the damaged material near the crack tip before the extension. When the specimen sizes are so small that the material becomes fracture insensitive, then E* appears higher than E.

KEYWORDS

J-integral; fracture sensitivity; physical similitude; Buckingham's Theorem; brittleness number; fictitious Young's modulus.

INTRODUCTION

The most worrying problem connected with fracture testing is the uncertainty of the reproducibility for the crack propagation phenomenon, varying the shape and/or the size of the cracked structure. While the techniques to obtain the fracture toughness parameters have rapidly been developed and, as far as the metallic materials are concerned, have already been standardized (ASTM E 399-74), it is still not completely clear today how far it is possible to extrapolate the laboratory results to the project of structures of large sizes and complex shape.

This report aims at presenting the experimental determination of the following critical fracture toughness parameters:

1) stress-intensity factor K_{Ic};
2) integral J_{Ic};

relating to a Carrara marble, a mortar and two concretes with aggregate of different maximum size. The main purpose is to study the statistical fluctuations of the results and to clarify the connection between the above mentioned experimental parameters. Namely K_{Ic} is a statical parameter, directly attainable by the structure and crack geometry and by the fracture load, while J_{Ic} is an energetic parameter representing the work necessary to have a unit fracture surface. It will be shown how the "force" K_{IC} and the "energy" J_{IC} are connected by a "stiffness" E*, probably representing Young's modulus of the damaged material near the crack tip before the extension.

In the last few years a great effort has been made to verify the range of applicability of the J-integral to the metallic materials, but a similar effort has not been made to clarify limitations and advantages of J applications to aggregative materials. The J values have hereby been calculated following the procedure suggested by Begley and Landes (1972) for the high strength steel alloys.

Finally some recurring experimental incoherences will be explained based on the Dimensional Analysis. Applying Buckingham's Theorem for physical similitude and scale modeling, a non-dimensional parameter (the brittleness number s) can be defined, which depends on the material fracture toughness, on the ultimate strength and on a linear size, characteristic for the considered specimen or structure. In this case the Dimensional Analysis is justified by the coexistence of two different structural crises, induced by generalized forces with different physical dimensions ($\sigma = FL^{-2}$; $K_I = FL^{-3/2}$). Further research in this direction could lead to more definitive considerations on the concrete notch-sensitivity, with reference to specimen and aggregate sizes.

REVIEW OF PREVIOUS WORKS DEALING WITH FRACTURE TESTING OF AGGREGATIVE MATERIALS

The first research, applying Fracture Mechanics to concretes and referring to Griffith's Theory, is by Kaplan (1961). Kaplan performed three and four point bending tests and determined the crack extension force G_{IC}. The doubts and problems of today were already present in that pioneer work. For instance Kaplan, who performed his testing with specimens of different sizes, observed in wonder that G_{IC} varied strongly with the specimen size. Missing non-linear plastic effects, Kaplan charged this variability to the slow crack growth, which reveals itself prior to the unstable crack propagation.

From then on results by several Authors have shown up disagreements and inconsistences. The critical fracture parameters have often appeared as functions of specimen size and crack length, instead of material constants. Romualdi and Batson (1963) came to the conclusion that G_{IC} must probably be an increasing function of the crack length, giving up considering G_{IC} as a material constant. Welch and Haisman (1969), unlike Romualdi and other following Authors, claimed the role of "material constant" for K_{IC} and G_{IC}, and especially asserted their independence of the ultimate strength σ_u.

The dependence of the notch sensitivity on the specimen sizes was explicitly pointed out in three fundamental papers of 1976 (Walsh; Higgins and Bailey; Schmidt). Walsh was the first who doubted the "slow crack growth", proposed by several Authors as cause of the K_{IC} variability. He charged the incoherences of the results, reported in literature, to the fact that the specimens used were too small.

Recently Hillemeier and Hilsdorf (1977) have experimentally determined the fracture toughness of the single concrete compounds, i.e. of cement paste, aggregate and interface paste-aggregate. Their intention to connect the partial fracture toughnesses with the global concrete ductility appears extremly interesting.

MATERIAL AND SPECIMEN DESCRIPTION

Four series of three point bending tests were performed using 18 specimens (10 x 10 x 30 cm) of Carrara marble, 15 specimens (15 x 15 x 60 cm) of mortar, 21 specimens (15 x 15 x 60 cm) of a concrete with maximum aggregate size 9.52 mm and 21 of a concrete with maximum aggregate size 19.10 mm.

Mortar had the weight ratios (water/cement) = 0.4 and (sand/cement) = 2.05. The maximum size of sand was 2.38 mm. The first concrete (1) had ratios: (water/cement) = 0.5, (sand/cement) = 2.46 and (aggregate/cement) = 2.89. The second concrete (2): (water/cement) = 0.6, (sand/cement) = 2.16 and (aggregate/cement) = 2.34. After casting the specimens were mixed in a high speed mixer in order to avoid voids and inhomogeneities.

For every notch depth three similar specimens were prepared. For every material three specimens were not notched, in order to determine the ultimate strength σ_u and Young's modulus E (Table 1). The notch was performed on the smooth side of the specimen, corresponding to a lateral side of the casting mould, by a circular saw of 2 mm thickness.

TABLE 1 Experimental Results

Material	Marble	Mortar	Concrete 1	Concrete 2
specimen width b (cm)	10	15	15	15
ultimate strength σ_u (kg cm^{-2})	124.53	41.55	36.42	46.44
Young's modulus E (kg cm^{-2} x 10^4)	12.76	3.78	4.39	3.86
stress-intensity factor K_{IC} (kg cm$^{-3/2}$)	109.16	66.23	54.30	76.61
standard deviation $\Delta K_{IC}/K_{IC}$	16.96 %	5.30%	25.29%	10.80%
crack extension force G_{IC} (kg cm^{-1})	0.063	0.192	0.065	0.058
standard deviation $\Delta G_{IC}/G_{IC}$	44.44%	31.25 %	40.81%	16.77 %
J-integral J_{IC} (kg cm^{-1})	0.051	0.230	0.055	0.061
standard deviation $\Delta J_{IC}/J_{IC}$	29.41%	30.40 %	27.30%	36.06 %
$(J_{IC} - G_{IC})/J_{IC}$	− 23.53%	+ 16.52%	− 18.18%	+ 4.92%
fictitious Young's modulus E* (kg cm^{-2} x 10^4)	23.36	1.91	5.36	9.62
brittleness number $K_{IC}/\sigma_u b^{1/2}$	$>$0.32	0.41	$>$0.52	$>$0.48

The notched specimens were subjected to a monotonic loading process by a hydraulic testing machine. The load and displacement transducer outputs were plotterized on a cartesian plane. The loading rate was in any case such that fracture was achieved in about one minute.

EXPERIMENTAL DETERMINATION OF THE FRACTURE TOUGHNESS PARAMETERS

The critical values K_{IC} of the *stress-intensity factor* have been obtained applying, for each single test, the formula suggested by ASTM E 399-74. Such values have then been averaged, for each material and for each crack depth, and represented in Fig. 1. The mean value and the standard deviation for each material are reported in Table 1. Marble shows a fracture toughness higher than that shown by the cement materials. Among the latter the least tough was concrete 1, perhaps because of the high ratios (sand/cement) and (aggregate/cement) of its composition. The standard deviation for mortar was the lowest (5.3 %), for marble and concrete 2 was acceptable (16.96 % and 10.80 % respectively), while for concrete 1 was very high (25.29 %). Observing Fig. 1 the suspicion may arise that the mentioned deviations, more than by true statistical fluctuations, are caused by systematic errors. The factor K_{IC}, as a function of the relative crack depth, shows very similar and clear trends for marble and concretes: its values are increasing for low depths a/b and decreasing for higher depths. The tops of these diagrams are for a/b = 0.25 ÷ 0.30.

The values of *J-integral* were calculated from load-displacement curves, following the procedure suggested by Begley and Landes (1972) for steel alloys. At a given deflection the area under the load-displacement curves was recorded. Such energy at constant displacement was plotted as a function of crack length and fitted by straight lines with the method of least squares (Fig. 2). The slope of these lines is equal to the variation in absorbed energy per unit variation in crack length and thus enables the determination of J as a function of deflection (Fig. 3). It is supposed that J is directly a function only of deflection and not of crack depth. The last step of Begley-Landes' Method is the determination of the mean fracture deflection (Fig. 4) and, then, of the critical value J_{IC} (Fig. 3). In other words the generalized crack extension force J_{IC} was determined simulating the crack extension phenomenon by cracks of different length. For mortar J_{IC} shows a value much higher than that of the other materials (Table 1). Observe how the standard deviation in fracture deflection, fluctuating for the four test series between 13.87 % and 16.73 %, is reflected and amplified in the J_{IC} value (Table 1).

Finally also the G_{IC} parameter was determined by the Direct Method (Kaplan, 1961). The critical values K_{IC}, G_{IC}, J_{IC} and the standard deviations, relating to the four materials, are summarized in Table 1. A comparison between K_{IC} and J_{IC}, which have different physical dimensions, can be synthetically performed by considering the real Young's modulus E and the fictitious Young's modulus $E^* = K_{IC}^2/J_{IC}$ (Table 1). For mortar the fictitious modulus E* is about the half of E, while for marble and concretes E* is much higher than E. For mortar E* could represent the stiffness of the damaged material near the crack tip before the extension. In the case of the remaining materials E* could sound as an alarm and reveal a too small specimen size in order to obtain a real fracture collapse. In fact it is very improbable that a crack produces a stiffening of the material at its tip.

FRACTURE SENSITIVITY AND APPLICATION OF DIMENSIONAL ANALYSIS

Based on the Dimensional Analysis, Carpinteri (1980) studied specimen and crack size effects on fracture testing of aggregative materials. Such effects are due to the co-existence of two structural crises, induced by generalized forces with different physical dimensions, and to the finiteness of specimen sizes. The application of Buckingham's Theorem allows the definition of a non-dimensional parameter s, which governs the fracture sensitivity phenomenon. Some recurring experimental incoherences are thus explained, such as the increase or decrease of fracture toughness K_{IC} increasing the crack length, the increase of K_{IC} increasing the specimen sizes, the variability of K_{IC} varying the test geometry.

Let q_0 be the failure load, for ultimate strength overcoming and/or for crack propagation, in a cracked structure. In the simplest case of homogeneous, isotropic and linear elastic material it is possible to show:

$$q_0/\sigma_u^\alpha b^\beta = \varphi_1(s,\ a/b)\ \ \varphi_2(t/b,\ l/b),$$

where φ_1 is a function of the brittleness number:

$$s = \frac{K_{IC}}{\sigma_u\, b^{1/2}},$$

and of the relative crack depth a/b. The actual function φ_1 depends on the material stress-strain laws and could be exactly determined only by experience. However an upper bound to φ_1 can be utilized, for

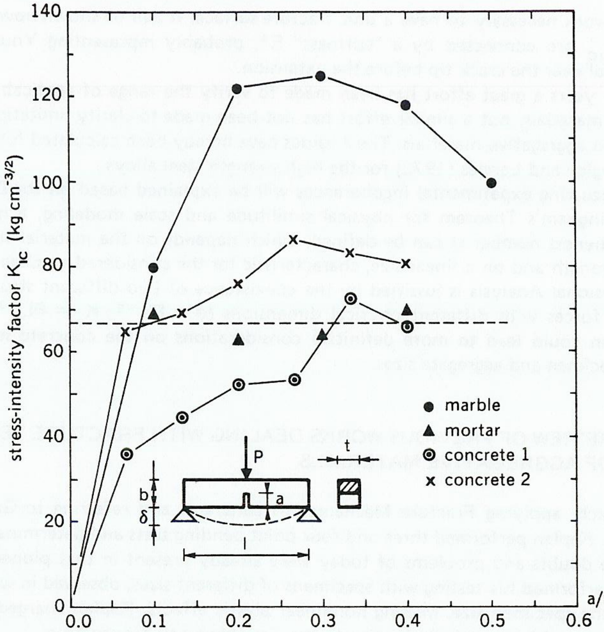

Fig. 1. Fracture toughness K_{IC} against relative crack depth.

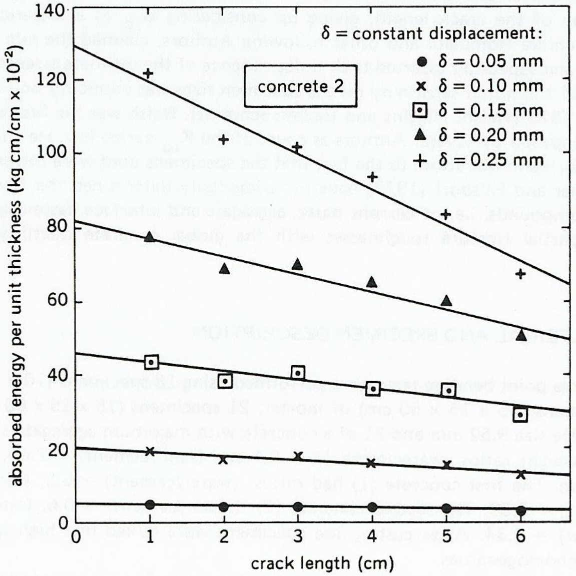

Fig. 2. Absorbed energy at constant displacement against crack length (concrete 2).

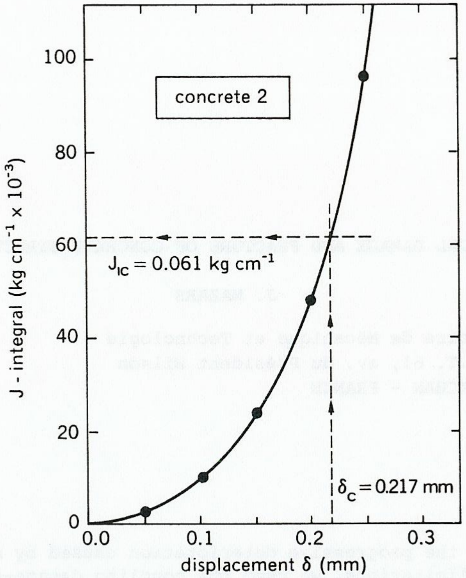

Fig. 3. J-integral as a function of displacement (concrete 2).

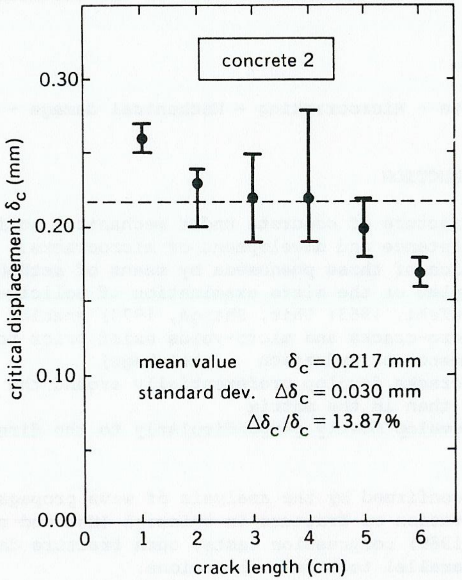

Fig. 4. Fracture deflection against crack length (concrete 2).

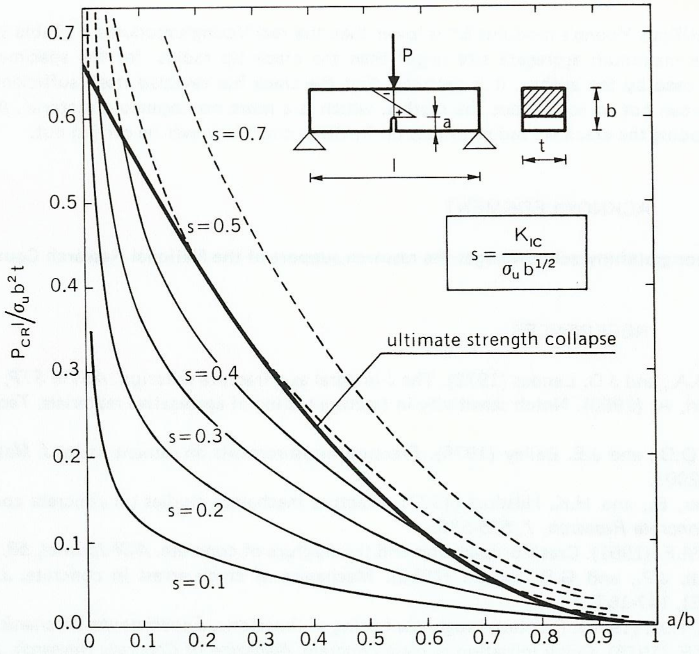

Fig. 5. Fracture load and ultimate strength load as functions of relative crack depth.

example in the case of the three point bending test. The non-dimensional fracture load against the relative crack length, varying the brittleness number s, is reported in Fig. 5, together with the ultimate strength load. It is evident how the fracture tests completely lose their meaning for $s \gtrsim 0.50$. For $s \lesssim 0.50$ the fracture tests are significant only with cracks of intermediate length. Diagrams of Fig. 5 would represent the real φ_1 only if the ultimate strength collapse were independent of the fracture collapse. Really the two collapses are interacting; that is, by decreasing the specimen sizes, fracture collapse becomes ultimate strength collapse, passing through mixed collapses.

The bell-shaped diagrams of K_{IC} (Fig. 1), obtained for marble and concretes, reveal that the crisis has prevailingly occurred for ultimate strength overcoming. It is interesting to observe how the top of these diagrams corresponds to the value a/b, for which the fracture curve s = 0.50 is tangent to the ultimate strength curve (Fig. 5). Therefore it was possible to determine the test brittleness number for mortar, and to set a lower bound to the numbers of the remaining tests (Table 1).

CONCLUSIONS

A fracture collapse seems to have occurred only for mortar, because of the low ratio K_{IC}/σ_u of such material and the sufficiently large specimen sizes. This assertion is supported by four symptoms in mortar tests:

1) the regularity of the strength ratio (and its statistical fluctuations) as a function of the crack length;
2) the low standard deviation of K_{IC} and the lack of a bell-shaped variation (Fig. 1);
3) the toughness parameter J_{IC}, considering also the plastic effects, is higher than G_{IC}, holding only in the linear elastic field (Table 1);

4) the fictitious Young's modulus E* is lower than the real Young's modulus E (Table 1).

Being the maximum aggregate size larger than the crack tip radius, for the specimens of mortar and concrete used by the author, it is probable that the crack has revealed itself sufficiently sharp. Perhaps the same can not be said about the marble, which is a more homogeneous material. A dimensional analysis including the crack tip radius effects on fracture, could however be carried out.

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