CALCULATING THE LOAD BEARING CAPACITY OF A STRUCTURE FAILING BY DUCTILE CRACK GROWTH

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ABSTRACT

A simple procedure is described which allows the load bearing capacity of a structure undergoing ductile crack growth to be calculated. The important factors in the analysis are the plastic limit load of the cracked structure, the initial crack length and the crack growth resistance toughness.

An example is evaluated using test data from A533B steel plate and associated welds which for the material properties considered illustrates the following points.

(1) Simple structures can only fail by ductile mechanisms if they are loaded to near their collapse limit at the instantaneous crack length.

(2) Initiation criteria are therefore unsatisfactory in determining the flaw tolerance of such structures.

(3) The load bearing capacity of a weld metal which is stronger than the surrounding plate may exceed that for the plate, even when the resistance to cracking is lower in the weld than in the plate.

KEYWORDS

Load bearing capacity; plastic limit load; crack growth resistance toughness; A533B plate and welds; initiation criteria.

INTRODUCTION

Most modern ferritic steel structures are designed to operate at temperatures where cracks propagate by ductile mechanisms. In this regime the load capacity of a structure is generally thought to be much higher than that to initiate ductile cracks, yet until recently there has been no technique generally available to analyse structural integrity beyond the initiation of cracking. The developments of the CEGB procedures into the ductile crack growth regime now enable such an analysis to be performed.

THE PROCEDURES

The CEGB procedures (Milne, Loosemore and Harrison, 1978) require two parameters to be evaluated, S_r and K_r, which may be defined as follows

$$S_r = \frac{\sigma}{\sigma_I(^a/t)}, \quad K_r = \frac{K_I(\sigma, ^a/t)}{K_{IC}} \qquad (1)$$

where σ is the applied stress, a is the crack size in a structure of thickness t, $\sigma_I(^a/t)$ is the plastic collapse stress of the structure, K_I is the linear elastic applied stress intensity factor, and K_{IC} is the fracture toughness. These two parameters are entered as a co-ordinate point on a failure assessment diagram, at A in Fig. 1a, and the position of this point relative to the failure assessment line defines how safe the structure is. A reserve factor on applied stress, F, can be evaluated as indicated in Fig. 1a, and if K_{IC} is defined at the initiation of ductile cracking, values of F > 1 indicate that cracking will not occur.

The diagram plotted in Fig. 1a uses the Bilby Cottrell and Swinden (1963) model to interpolate between the two limiting criteria, plastic collapse and linear elastic failure. The Bilby Cottrell Swinden model does, of course, confine yielding to a strip coplanar with the crack, and hence is a rather unrealistic representation of plastic zone development in a real structure. However the effects of plasticity are more dependent upon the size of the plastic zone than its shape. Moreover the use of independently obtainable collapse solutions in the CEGB procedures turns the model into a semiempirical one the accuracy of which depends upon the solutions used rather than the detail of the model. Data from a variety of test and structural geometries have been used to validate the procedures, see for example Harrison and Milne (1979).

To extend these procedures into the ductile crack growth regime the parameters S_r and K_r have to be generalised to include the effects of crack growth (Milne 1979). Thus after Δa of crack growth

$$S_r = \frac{\sigma}{\sigma_I(\frac{a+\Delta a}{t})}, \quad K_r = \frac{K_I(\sigma, \frac{a+\Delta a}{t})}{K_\Omega(\Delta a)} \qquad (2)$$

The parameter $K_\Omega(\Delta a)$ is the ductile crack growth resistance toughness after Δa of growth, obtainable from J resistance curves as $K_\Omega(\Delta a) = \sqrt{EJ_R(\Delta a)}$ for plane stress, where E is Youngs Modulus, or $K_\Omega(\Delta a) = \sqrt{EJ_R(\Delta a)/(1-\nu^2)}$ for plane strain where ν is Poisson's ratio. A locus of coordinate's points S_rK_r is then plotted on the failure assessment diagram at constant applied load L but increasing postulated crack extension, Δa, as demonstrated by the curve ABC in Fig. 1b. The reserve factor, F, is evaluated as before for each coordinate point, and of course it is now variable with Δa. Alternatively F may be calculated from the formula (Chell and Milne 1979)

$$F = \frac{2}{\pi S_r}\cos^{-1}\exp -\left\{\frac{\pi^2}{8}\frac{S_r^2}{K_r^2}\right\} \qquad (3)$$

For the case represented by ABC in Fig. 1b, F is always greater than 1 but goes through a maximum at B. The curve ABC has therefore been scaled by this maximum value of F, F_B and replotted. The resultant curve, A'B'C', is equivalent to a reassessment of the original curve but at a new load given by the product $L \times F_B$, and is tangential to the assessment line at B'. This tangency point is identical to the tangency point obtained in a conventional J resistance analysis, as has

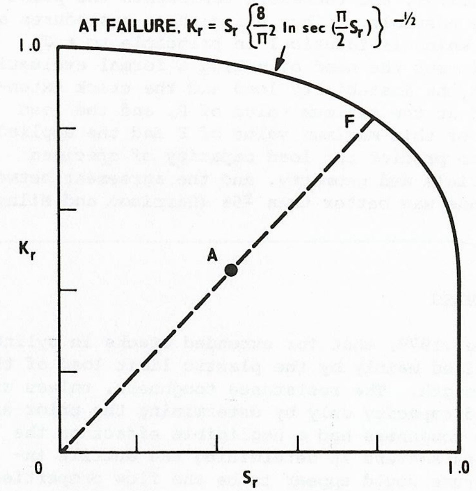

FIG. 1a: Evaluation of Tolerance to Crack Initiation using the Failure Assessment Diagram

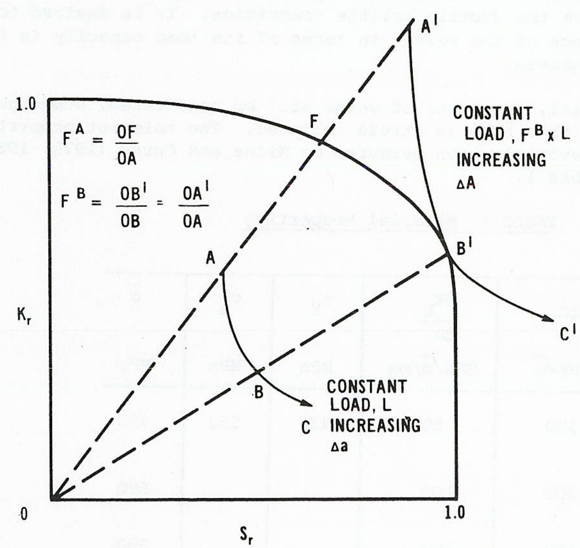

FIG. 1b: Evaluation of Tolerance to Crack Growth using the Failure assessment diagram

been demonstrated by Chell and Milne (1979), and therefore represents the point beyond which the structure will become unstable. Thus using these procedures an instability analysis can be performed which is identical in principle to a J resistance instability analysis but without the need of making a formal evaluation of J. Regardless of the applied load, the instability load and the crack extension at instability are always defined at the maximum value of F, and the load capacity is then given by the product of this maximum value of F and the applied load. This procedure has been used to predict the load capacity of specimen tests from a variety of sources, materials and geometry, and the agreement between the predicted and measured maximum loads was better than ±5% (Harrison and Milne 1979).

DEPENDENCE ON INPUT VARIABLES

It was demonstrated by Chell and Milne (1979) that for extended cracks in cylinders, the load capacity tended to be controlled mainly by the plastic limit load of the cylinder at the instantaneous crack length. The resistance toughness, unless unrealistically low, influenced the load capacity only by determining the prior extent of crack growth. The initiation toughness had a negligible effect on the instability load. Thus the dominant parameters in determining the ductile instability load of such a simple structure would appear to be the flow properties of the material and the initial length of the crack. This has a particular significance in welded structures as is evident from the following example.

Consider a cylindrical seam welded pressure vessel of 10.13m radius made of 76.2mm thick A533B plate. The vessel is considered to operate at a pressure of 1.314MPa, and a temperature well above the ductile brittle transition. It is desired to evaluate the defect tolerance of the vessel in terms of its load capacity in the presence of ductile crack growth.

As well as the plate material, two types of welds will be considered, submerged arc (SA) and manual metal arc (MMA) both as stress relieved. The relevant properties of these materials have previously been measured by Milne and Curry (1979, 1980) and these are listed in Table 1.

TABLE 1 Material Properties

	K_{IC} MPa$\sqrt{m}$	$\frac{dK_\Omega}{da}$ MPa$\sqrt{m}$/mm	σ_Y MPa	σ_u MPa	$\bar{\sigma}$ MPa
A533B Plate	300	50	433	553	490
SA Weld	200	100			590
MMA Weld	100	100			590

Note that the crack growth resistance curves measured by Milne and Curry were nonlinear, but that since their slope dK_Ω/da is constant in Table 1 they are taken as linear. This assumption will have little influence on the results. In addition the flow properties of the weld metal have not been measured, but the hardness measurements of Milne and Curry (1979) suggest that the flow stress $\bar{\sigma}$ of both weld

metals would be 20% greater than that of the plate. This is an important factor in the results.

The analysis was performed for an extended crack on the inner surface of the vessel, see Appendix, and the results are interpreted in terms of the reserve factor on applied pressure, F^p. The results are plotted in Figs. 2a b and c against crack size, for the plate, SA and MMA weld in turn. In each case F^p is plotted against crack extension for initial cracks of 5, 10, 20, 30 and 40mm and from these curves initiation and maximum load loci were obtained and compared with the plastic collapse locus. From these figures it is possible to determine the load capacity at a given initial crack size or the crack tolerance at a given load. The general trends are as follows:
For any given initial crack length;

(1) The load to initiate cracking is lowest in the MMA weld and highest in the plate, with the SA weld metal coming in between.

(2) The load capacity is highest for the SA weld and lowest for the plate with the MMA weld coming in between.

The defect tolerance of the vessel at initiation is defined on the initiation loci at $F^p = 1$. As expected the lowest K_{IC} produces the smallest crack for initiation. From the small dashed curves in Figs. 2, this is 28mm for the MMA weld, 38.5mm for the SA weld and 43.5mm for the plate. This however gives little insight into the real defect tolerance of the vessel. For example, at these initial defect sizes the load tolerance of the plate is given by $F^p = 1.01$, while for the SA and MMA weld metals it is 1.5 and 2 respectively. Thus the 43.5mm crack is of near critical size in the plate while the 28mm crack in the MMA weld can sustain twice the load before it becomes critical, as demonstrated by the small dashed F^p - Δa curves in Figs. 2.

The crack tolerance of the vessel, after allowing for growth, is also defined at $F^p = 1$, but this time by the maximum load loci. In Fig. 2b this coincided with a crack depth of 51.75mm at C on the maximum load locus. A crack of this depth could only result from loading the cylinder to $F^p = 1$ if it already contained a crack of 48mm, i.e. along the path ABC in Fig. 2b. Hence 48mm is the critical initial defect size for the SA weld metal as this defines the initial crack size at and beyond which the vessel will become unstable when loaded to its operating pressure. The critical initial crack sizes for the MMA weld is 47.5mm, Fig. 2c, and for the plate it is about 45mm, Fig. 2a. Thus, the plate material has a lower defect tolerance than the two weld metals despite being the toughest material.

DISCUSSION AND CONCLUSIONS

The foregoing results demonstrate the following important points. For the material properties and the geometry considered, the load tolerance of the structure is always close to the plastic collapse load at the instantaneous crack length. This may be well above the load to initiate cracking. Consequently initiation criteria are unsatisfactory in determining the flaw tolerance of even this simple structure. Indeed they may not even produce a satisfactory ranking of the relative integrity of the various contending materials for which the structure may be made.

For simple structures to fail by ductile mechanisms, they must be loaded to near their collapse loads at the instantaneous crack length. Because the amount of prior crack growth depends upon the materials resistance toughness, this may or may not be significantly below the collapse limit at the initial crack size.

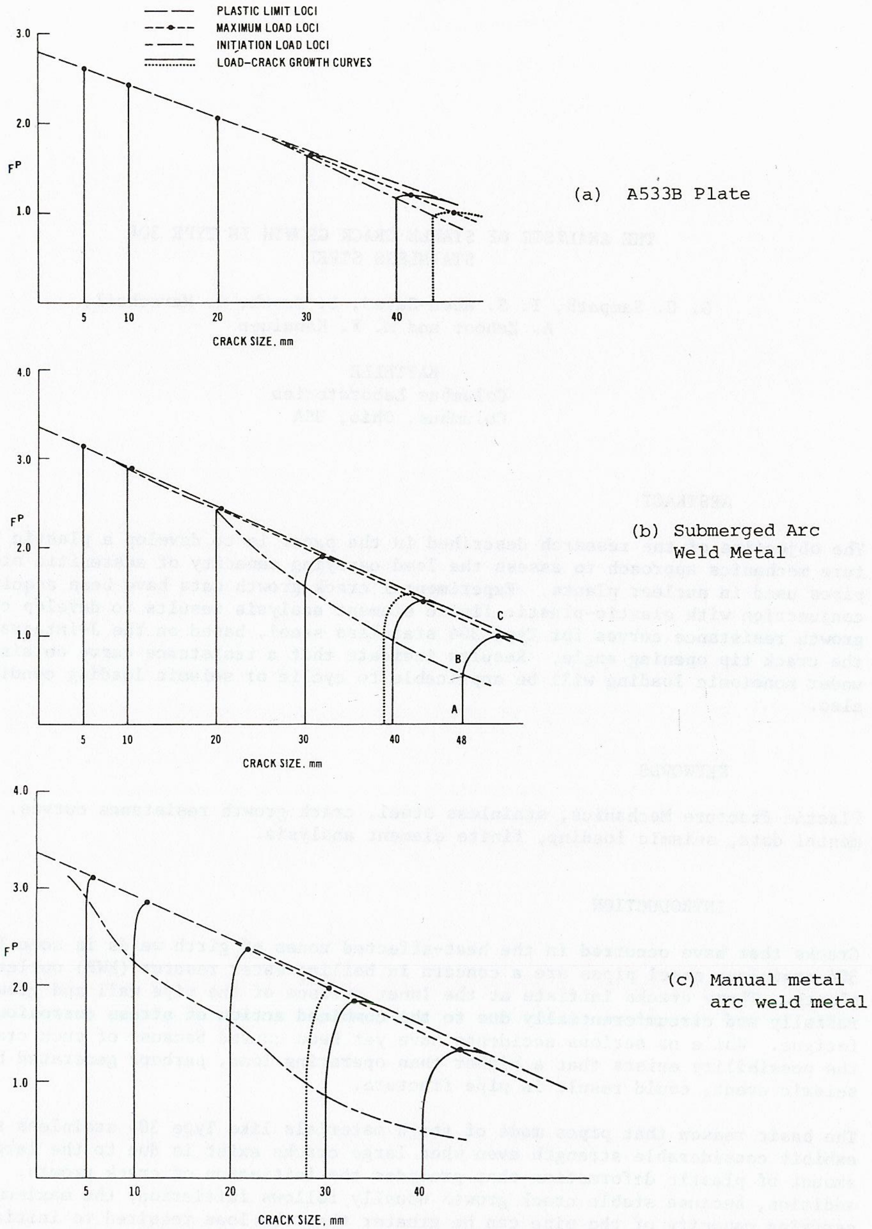

FIG.2: Load Factor, F^p, Versus Crack Size for the 3 Metals of Interest

The load tolerance of a structure failing by ductile cracking is therefore dependent on the initial crack size and the flow properties of the material as well as the crack growth resistance toughness. Consequently the load capacity of weld metal which is stronger than its parent plate may exceed that for the plate even when the resistance to cracking is lower in the weld metal than in the plate.

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APPENDIX

Calculation of S_r.

The collapse stress, σ_I, was evaluated by equating forces over the uncracked ligament, taking into account the extra force due to the pressure on the crack face.

Thus $$S_r = \frac{\sigma}{\sigma_I} (^a/t) = \frac{P(R_i+a)}{t} \cdot \frac{1}{\bar{\sigma}(1-{}^a/t)}$$

where R_i is the internal radius of the vessel and $\bar{\sigma}$ is the flow stress, Table 1. This was calculated for each initial flaw size, 5, 10, 20, 30 and 40mm and for subsequent increments of flaw size, Δa, in half millimetre steps.

Calculation of K_r.

For the K solutions the cracked geometry was considered to be simulated by the single edged notched pin loaded tension geometry, again including the force due to the pressure on the crack surfaces.

Thus $K_I(\sigma, {}^a/t) = Y({}^a/t).P.\left(\frac{Ri}{t} + 1\right)\sqrt{t}$, where $Y({}^a/t)$ is the compliance calibration function.

For a linear crack growth resistance curve

$$K_\Omega = K_{IC} + \Delta a.\frac{dK_\Omega}{da}, \text{ hence}$$

$$K_R = \frac{K_I(\sigma, {}^a/t)}{K_\Omega} = \frac{Y({}^a/t).P.(Ri/t+1)\sqrt{t}}{K_{IC}+\Delta a \; dK_\Omega/da}$$

This ratio was evaluated with $\Delta a = 0$ for each initial flaw size as above, and for subsequent increments of Δa in half millimetre steps.

The load factor was calculated from equation 3 as required.

Table A1 lists the relevant calculations for the SA weld metal and a 30mm initial crack.

TABLE A1

a (mm)	$^a/t$	K_I (MPa$\sqrt{m}$)	K_Ω (MPa$\sqrt{m}$)	S_r	K_r	F^P
30	0.394	111.7	200	0.490	0.559	1.524
30.5	0.400	113.4	250	0.496	0.454	1.720
31	0.407	116.9	300	0.501	0.390	1.830
31.5	0.413	120.2	350	0.508	0.343	1.885
32	0.420	121.7	400	0.513	0.304	1.911
32.5	0.426	126.9	450	0.522	0.282	1.898
33	0.433	131.0	500	0.529	0.262	1.884
33.5	0.440	135.0	550	0.534	0.246	1.869
34	0.446	137.7	600	0.540	0.230	1.852