AN ELASTIC-PLASTIC FRACTURE MODEL OF PLANE STRESS AND ITS FINITE ELEMENT ANALYSIS

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ABSTRACT

In the present paper, a plane stress elastic-plastic fracture model—the strip necking zone model is proposed on the basis of the experimental results given by Schaeffer, Liu and Ke. It may be regarded as an extension of Dugdale Model. With this model, the calculations by finite element method incorporating plastic deformation theory or plastic incremental theory are both carried out to determine the crack opening displacement. The calculated results are compared with the Burdekin's design curve and some published experimental results. The analysis indicates that the nominal strains vary with different gauge lengths. Therefore Burdekin design curve is safe when the nominal strains are taken at some definite gauge lengths. The calculated values of the crack opening displacements in this paper are in good agreement with the published experimental results.

KEYWORDS

Plane stress; mechanic model; necking; elastic—plastic; finite element method.

INTRODUCTION

For thin plate of ductile material there occurs a large scale yielding or general yielding before the initiation of crack. Therefore, the theory of linear elastic fracture mechanics is no longer valid and it is necessary to develop a theory of elastic-plastic fracture mechanics for the plane stress condition.

As commonly known, in low stress fracture condition. the load at crack initiation obtained by Dugdale model [1] incorporating COD criterion is in good agreement with many experimental results. However if the crack length is not long enough, the initiation of the crack usually occurs in large scale yielding or general yielding. After a limited amount of plastic flow has taken place along the line of the crack, the plastic region is not only confined to the strip zone ahead of the crack but also extended to two shear bands with an angle of about 50 degrees to the line of crack. It is evident that Dugdale model can no longer be applicable. In the case of large scale yielding or general yielding, the fracture analysis in high strain area is mainly dependent on the experimental results. The Burdekin design curve [2,3] , which gives the relation between crack tip opening displacement δ , nominal strain e, and crack size a, is deduced from the experiments obtained by British Welding Institute. However, during the loading process, wide plates with different crack geometry may produce entirely different types of yielding, and then significantly different relations between δ and e would be obtained [4] . Burdekin's formula is only valid in the case of small cracks with a uniform plastic strain zone without any disturbance by ligament yielding [5] .

In the present paper, a plane stress elastic-plastic fracture model —the strip necking zone model is proposed on the basis of the experimental results given by Schaeffer, Liu and Ke [6] . By using this model, the crack opening displacements of thin plates are calculated by finite element method incorporating plastic deformation theory or plastic incremental theory. The calculated results are compared with the Burdekin design curve and some published experimental results [5,7] .

AN ELASTIC PLASTIC FRACTURE MODEL OF PLANE STRESS PROBLEM

In the experimental work performed by Schaeffer, Liu and Ke, the crack opening displacement is measured on the thin steel sheets with crack. Through observation, they found out that for a strain hardening material, the local strain at the crack tip would be very large. When a limited amount of plastic flow has taken place ahead of the crack it forms a strip necking zone there. The strip necking zone is imbedded in a plastic region, which is surrounded by the elastic region as shown in Fig.1 [6.8] .

The typical uniaxial tensile stress-strain curve of a specimen is shown in Fig.2(a). The stress reaches its maximum value σ_B , called ultimate stress, at point B. Beyond this point necking begins to occur. In the case of plane stress, it is assumed that the stress components σ_z , τ_{xz} , τ_{yz} are zero within the plate. In the strip necking zone, σ_y , the stress component perpendicular to the crack, is regarded as the ultimate stress σ_B . We suppose that beyond the point B the stress is taken as constant σ_B as shown in Fig.2(b).

Therefore, a plane stress elastic-plastic fracture model—strip necking zone model is proposed in this paper. It is imagined that ahead of the crack there is a very narrow strip necking zone, with the ultimate stress σ_B acting on it. The plastic region is outside the strip necking zone and is surrounded by elastic region. The width of the strip necking zone is taken to be zero, then the strip necking zone is considered as the elongation of actual crack. The actual crack together with strip necking zone is treated as an effective crack shown in Fig.3. The stress and strain at the effective crack tip are both continuous and finite. So is the displacement in the direction of crack line. Above and below the strip necking zone, the displacement in the direction perpendicular to the crack is discontinuous.

The strip necking zone model may be regarded as an extension of Dugdale Model. These two models are equivalent for the perfectly plastic materials. In Dugdale model which is applicable only for perfectly plastic material, plastic zone is limited within a narrow strip ahead of crack, while the bulk region around the strip yielding zone is considered as elastic. However, in the strip necking zone model which is also applicable for strain hardening materials, no limitation is imposed on the size of plastic zone. In other words, around the strip necking zone, there is a real elastic-plastic region, which should be analysed by elastic-plastic theory.

ANALYSIS ON PLANE STRESS ELASTIC-PLASTIC FRACTURE MODEL BY PLASTIC DEFORMATION THEORY USING FINITE ELEMENT METHOD

For power hardening materials submitted to the stress-strain relation $\varepsilon = \alpha \sigma^n$, the strain field with dominant singularity at the crack tip is given by Hutchinson, Rice and Goldman [9-11] ,

$$\varepsilon_{ij} = \alpha \, \varepsilon_o \, K_\varepsilon \; r^{-(n/(n+1))} \tilde{\varepsilon}_{ij}(\theta) \tag{1}$$

where r is the distance from crack tip, and K_ε is the strain intensity factor. The displacement field near crack tip is given as follows

$$u_i = \alpha \, \varepsilon_o \, K_\varepsilon \; r^{(1/(n+1))} \, \tilde{u}_i(\theta) \tag{2}$$

According to the plane stress elastic-plastic fracture model proposed in the present paper, the determination of the length of strip necking zone should satisfy the condition of eliminating stress or strain singularity at the ends of effective crack under the action of external load σ together with a tensile stress σ_B distributed along that zone. It means that K_ε should be zero.

Let { u } represents the column matrix of nodel displacement and { F } the column matrix of nodel load, then the non-linear equilibrium equation is given as

$$[K(\{ u \})] \; \{u\} = \{ F \} \tag{3}$$

where [K({ u })] is the overall stiffness matrix. In the computation, triangular elements are used, and the ratio of the smallest element size to the length of crack is 1/128. The solution of (3) is obtained by initial stress method. The calculation process is as follows:

(a) A value of ρ , the length of strip necking zone ahead of the crack tip, is assumed and the ultimate stress σ_B acts on it as shown in Fig.3. The actual crack together with the necking zone ρ is treated as an effective crack.

(b) Equation (3) is solved by initial stress method under a specific tensile stress σ_1 .

(c) Using the calculated displacements u_i of the points with different distance r on the effective crack, a plot of $u_i/r^{(1/(n+1))}$ versus r is obtained.

(d) If the value of $u_i/r^{(1/(n+1))}$ approaches zero as r decreases to zero, thus k_ε approaches zero, we may continue to do the next step (e). Otherwise the value of the tensile stress σ_1 should be altered and steps (b)-(d) are repeated till k_ε approaches zero.

(e) Calculate the actual crack tip opening displacement δ , the opening displacement δ_0 at the middle of the centre crack, the nominal strains e at some different gauge lengths and the distribution of stress and strain.

(f) Then repeat the above steps (a)-(e) by using the selected values of ρ .

ANALYSIS ON PLANE STRESS ELASTIC-PLASTIC FRACTURE MODEL BY PLASTIC INCREMENTAL THEORY USING FINITE ELEMENT METHOD

In plastic incremental theory the non-linear equilibrium equation is given as

$$[K(\{u\})]\ \{du\} = \{df\} \tag{4}$$

The outline of calculation process is as follows: If the strip necking zone length is taken to be A_0A_3 as shown in Fig.4, where the ultimate stress σ_B acts on the zone A_0A_3, then the equivalent nodel forces R_{A_0}, R_{A_1} and R_{A_2} at nodes A_0, A_1 and A_2 should be

$$R_{A_0} = \sigma_B l_1 t/2 \ ; \ R_{A_1} = \sigma_B (l_1+l_2)t/2;$$

$$R_{A_2} = \sigma_B (l_2+l_3)t/2 \tag{5}$$

where t is the plate thickness, l_i is the size of the element. As an increment of loading {df} is applied, the nodal displacement increment {du} is obtained from (4) by using tangential stiffness method. Therefore, the stress increments, stresses and the equivalent nodal forces at the nodes $A_0, A_1, \ldots$ can be calculated. When the equivalent force at A_0 reaches the value R_{A_0}, it means plastic necking flow occurs at the zone A_0A_1, thus the strip necking zone length is regarded as A_0A_1, and the displacement constraint at node A_0 should be removed. A new loading increment is then applied until the equivalent nodal force at node A_1 reaches value R_{A_1}. Then, again the displacement contraint at node A_1 should be removed, and the strip necking zone length is regarded as A_0A_2. As the load increases continuously, the corresponding strip necking zone length can be calculated incrementally. As a result, the actual crack tip opening displacement δ , nominal strain e and the stress or strain field can be obtained.

RESULTS AND DISCUSSION

According to the strip necking zone model proposed wide central cracked plate is first studied by using finite element method incorporating the plastic deformation theory. The plate calculated here is 920 mm wide, 25 mm thick and the length of central crack is 2a = 115 mm. The size of the specimen is approximately the same size of that used in the experimental work at British Welding Institute. The material parameters used are as follows: the hardening coefficient n is equal to 7.75; the yield strength and the ultimated strength are 47hb and 59.4hb respectively. The crack tip opening displacement δ and the nominal strain e are calculated and plotted in non-dimensional form given in Fig.5 for different gauge length. The $\Phi\ (=\delta/2\pi e_y a)$ — e/e_y curve indicates that the the nominal strains vary obviously with the gauge length. The Burdekin design curve is on the safe side for the case of fracture analyses within areas of low nominal strain, $e/e_y < 1.5$. However, within the areas of high nominal strain, $e/e_y > 1.5$, Burdekin design curve is not safe when the gauge length y is so long that the ratio a/y is less than ¼.

The crack opening displacement at the middle of the centre crack, δ_0 and the nominal strain e are plotted in non-dimensional form in Fig.6 for different ratio of crack length to gauge length. These calculated results are in reasonably good agreement with the experimental results given in [5] . The experimental values of $\Phi_0 (= \delta_0/2\pi e_y a)$ are somewhat higher than the calculated results and may reach a maximum of 20%. This is probably due to the difference of the ratio of the crack length 2a to plate width 2b. The ratio, a/b of test specimens used in [5] is in the range, a/b = ½ ~ ¼, while the ratio, a/b used in our calculation is ⅛ .

The curves relating the applied tensile stress σ to strip necking zone length ρ are given in Fig.7 in non-dimensional form (σ/σ_y - $\rho/(a+\rho)$). The curves relating σ/σ_y to Φ are given in Fig.8. These results are compared respectly with Dugdale model solution for infinite plate and with the finite element solution of Dugdale model for finite plate calculated in this paper. Under the same applied tensile stress, the strip yielding zone length obtained by Dugdale model is larger than the strip necking zone length calculated in this paper as shown in Fig.7. Since Dugdale model is based upon an elastic-plastic material with no strain hardening, the region around the strip yielding zone is considered as elastic. Fig.8 shows that the deviation of Φ will increase obviously when σ/σ_y approximates 1. The value Φ , calculated by using Dugdale model for infinite plate, approaches to infinity if σ/σ_y increases to 1. Therefore, it is evident that Dugdale model can not be applied in this case. However, when σ/σ_y is less than 0.5 the Dugdale model solution agrees well with the calculated results in the present paper.

As shown in Fig.9, the plastic zone, obtained in this calculation, is not only confined to a narrow strip ahead of the crack tip but also extended to two other shear bands with an angle of about 50 degree to the crack.

In addition, Φ - e/e_y curve shown in Fig.10 is obtained by using plastic incremental theory for the same specimen plate. The calculated results obtained by using these two plasticity theories for strip necking zone model are approximately identical. By using incremental theory, we have also calculated the crack opening displacements for four central crack specimens under uniform tension [7] . The specimens are 120 mm wide, 3 mm thick, with central crack length of 10 mm or

50 mm. The material behavior data are listed in Table 1. The calculated results and the experimental results [7] as given is Table 2 are in quite well agreement. Corresponding to COD at initiation, the calculated applied stress σ_i Gross differs by no more than 10% for specimen 9 or 12 and 2% for specimen 7 or 8.

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TABLE 1 Mechanical Characteristics

Specimen	Yield Strength σ_y (hb)	U.T.S. σ_B(hb)	% Elongation at Maximum Load	Strain Hardening Exponent
9 12	28.5	42	23	0.2
7 8	38	46	10	0.095

TABLE 2 Calculation Results and Test Results

Specimen	2a mm	COD at Initiation Δ_{Ii} (mm) [7]	σ_i Gross (hb)[7]	σ_i Net (hb)[7]	σ_i Gross (hb)Cal.	σ_i Net (hb)Cal.
9	10	1.6	28.4	31.0	31.1	33.9
12	50	1.7	18.8	32.2	20.3	34.8
7	10	1.0	38	41.5	38.8	42.3
8	50	1.18	25.3	43.4	25.3	43.4

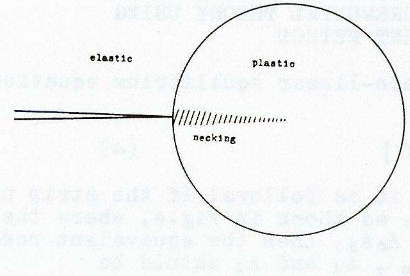


Fig.1. The various regions of deformation around a crack

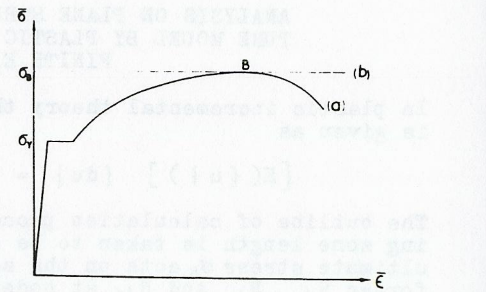


Fig.2. Typical uniaxial tensile stress-strain curve

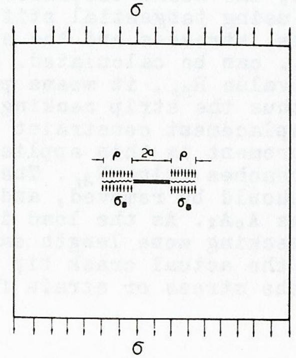


Fig.3. Centrally cracked plate under uniform tension

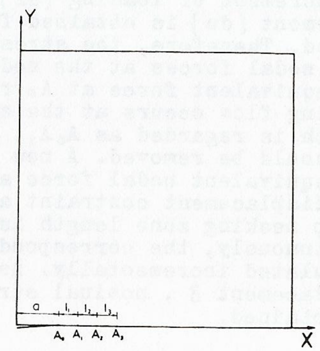


Fig.4. One quarter of cracked plate used in the analysis

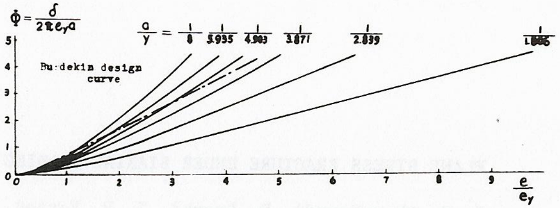


Fig.5. The relation between non-dimensional CTOD and non-dimensional nominal strain by deformation theory

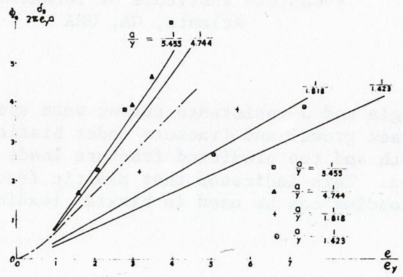
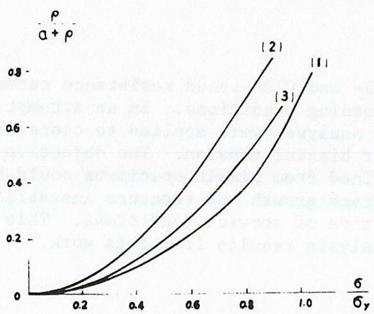
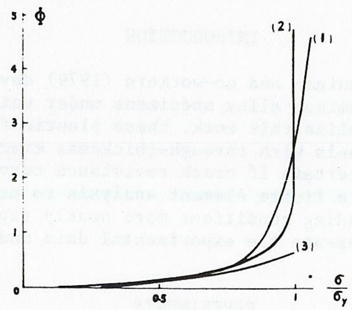


Fig.6. The non-dimensional COD and nominal strain relationships compared with the experimental results given by [5]





(1) Using the strip necking zone model
(2) Using the Dugdale model in infinite body with σ_y acting along strip zone ρ
(3) Using the Dugdale model incorpolating finite element method with σ_B acting along strip zone ρ

Fig.7. Non-dimensional necking zone length versus the ratio of σ/σ_y

Fig.8. Non-dimensional COD versus the ratio of σ/σ_y

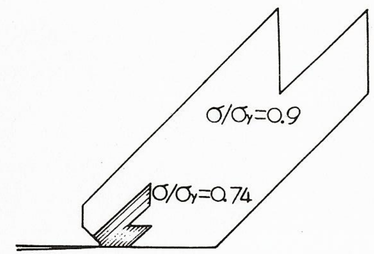


Fig. 9. Plastic zone at crack tip

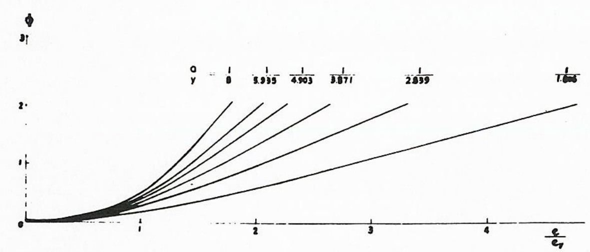


Fig. 10. The relation between non-dimensional CTOD and non-dimensional nominal strain by incremental theory