REGULARITIES OF SIMILARITY AND FATIGUE DAMAGE ACCUMULATION UNDER IRREGULAR LOADING

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ABSTRACT

Methods for calculating a machine parts' fatigue life under irregular loading were developed considering probabilistic aspects (Kogaev, 1969, 1977, 1979). These methods are based on the statistical theory of fatigue failure similarity and on the criterion of fatigue damage accumulation at irregular loading (Gusenkov 1979), briefly described in this paper.

On the basis of some experimental results it was shown that statistical theory of fatigue failure similarity is a very good instrument for calculating the fatigue strength of machine parts with different size, shape and type of loading. A corrected linear damage accumulation rule and a deformation kinetics criterion were proposed for the purpose of fatigue life calculations at irregular loading.

KEYWORDS

Stress concentration, size effect, similarity criterion of fatigue, nonpropagating cracks, irregular loading.

The influence of stress concentration, size effect, shape of cross section and type of loading on fatigue strength at the moment of fatigue crack initiation in the range of 10^5-10^7 cycles is well described by statistical theory of fatigue failure similarity (Kogaev 1977, 1979). This theory is based on Weibulls (1939) hypotheses of the strength of the weakest link. Integration upon volume in Weibulls equation was substituted by the integration upon cross sectional area. Only the first principal stress in the area of stress concentration, where stress distribution was taken as a linear function, determined by relative stress gradient G was taken into account, where

$$G = \frac{1}{\sigma_{max}} \left[\frac{d\sigma}{dx} \right]_{x=0} \qquad (1)$$

here $\sigma_{max} = \sigma_e \cdot K_T$ - maximum stress in the area of stress concentration, corresponding to the fatigue limit of the specimen or machine part σ_e ; K_T - theoretical stress concentration factor; x - the depth from the surface. The value of stress gradient G may be determined by the expression, derived from Neiber's equation for round specimens with grooves subjected to bending moment

$$G = \frac{2(1+\varphi)}{\rho} + \frac{2}{d} \qquad (2)$$

where

$$\varphi = \frac{1}{4\sqrt{\frac{t}{\rho}} + 2} \qquad (3)$$

t , ρ - depth and radius of curvatures of the groove; d - diameter of specimen.
From the statistical theory of fatigue failure similarity was received the similarity criterion of fatigue

$$\theta = \frac{L/G}{(L/G)_0} . \qquad (4)$$

Here L is the perimeter of the cross-sectional area of the part adjacent to the area of maximum stress; at rotating bending of the round specimen $L = \pi d$; $(L/G)_0$ is the ratio L/G for round plane (without stress concentration) laboratory specimen of small diameter d_0= 7,5 mm. If the dimensions are in mm, $(L/G) = 0,5\,\pi d_0^2$ = = 88,3 mm^2; $\theta = \frac{1}{88,3} \cdot \frac{L}{G}$.
The criterion θ has the following meaning: if a specimen, model or machine part have different sizes, shapes or types of loading, but the same value of θ , their fatigue strength distribution functions are practically the same.
This statement follows from the equation of fatigue failure similarity

$$\log(\xi - 1) = -\nu \log\theta + U_p \cdot S \qquad (5)$$

where

$$\xi = \frac{\sigma_{max}}{0{,}5\sigma_0} = \frac{\sigma_e \cdot K_T}{0{,}5\,\sigma_0} . \qquad (6)$$

Here σ_0 is the mean value of the fatigue strength for a laboratory specimen of diameter 7,5 mm without stress concentration; ν is a new material constant, characterizing the sensitivity for stress concentration and size-effect and changing in the range ν = 0,04 - - 0,20; U_p is the normal distribution quantile corresponding to the probability P , S = standard deviation of the random value $x = \log(\xi - 1)$. Mean value of x is

$$\bar{x} = -\nu \log\theta . \qquad (7)$$

As an example of the application of this equation, let's consider the experimental results, received by Massanet (I955) and presented in Fig. I in coordinates $\log(\xi - 1) - \log\theta$. In this work were tested a great number of types of specimens from carbon steel, shown in Fig. I,b. Sheet specimens were tested with width 45-70 mm, with a hole 0,5 - 32 mm in diameter at uniaxial loading (solid circular points in Fig. I), round specimens without stress concentration with diameter d = 5-32 mm at uniaxial loading (triangular points), round specimens d = I6 mm with a circular groove with radius ρ = = 0,I25 - 3,I25 mm at uniaxial loading (hollow round points), rectangular specimens without stress concentration with dimensions

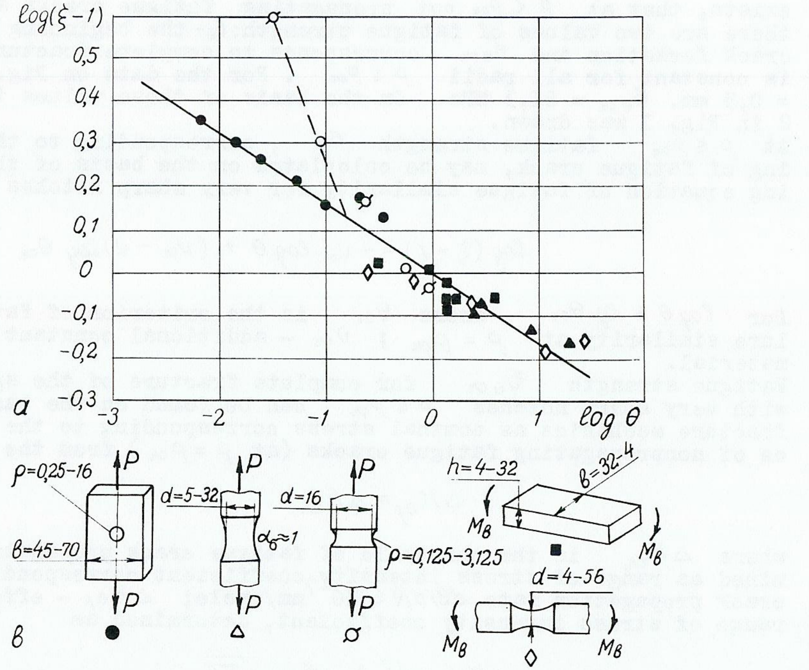

Fig. I. Log $(\xi - 1)$ vs log θ for results of Massonet (I955)

from 4x32 mm^2 till 32x4 mm^2 at bending (square point), round plane specimens with diameter d = 4 ÷ 56 mm at bending (rhomb point). For all this variety of sizes, shapes and types of loading of the specimens, experimental points lie on one straight line I, drawn according equation (7) at σ_0 = 255 MPa, ν = 0,I4 (dotted line with two points will be discussed later).
Therefore, on the basis of these parameter's values it is possible to calculate the fatigue strength of all tested specimens with the aid of the following equation derived from the eq (7) and (6):

$$\sigma_e = \frac{0{,}5\sigma_0}{K_T}\left(1 + \theta^{-\nu}\right) . \qquad (8)$$

Calculated values of fatigue strength differ from the experimental ones not more than by II% on the whole mass of data. This accuracy must be considered good for practical purposes, if we take into account that in the work of Massanet (I955) values of σ_e are found by testing 6-8 specimens. It was statistically shown (Kogaev, I977), that in the case of determining fatigue strength with probit or staircase methods the maximum error of the calculated values of σ_e did not exceed 5%.
Two points lying on the line 2 in Fig. I correspond to round specimens d = I6 mm with grooves of very small root radius ρ =0,I25 and 0,320 mm at uniaxial loading. It was shown by many authors that the critical radius of curvature in stress concentration zone ρ_{cr}

exists, that at $\rho < \rho_{cr}$ non propagating fatigue cracks appear and there are two values of fatigue strength: σ_e – the beginning of fatigue crack formation and σ_{ecr} corresponds to complete fracture, which is constant for all radii $\rho \leq \rho_{cr}$. For the data on Fig. I ρ_{cr} = = 0,5 mm, σ_{ecr} = 81,3 MPa. On the basis of these values the line 2 in Fig. I was drawn.
At $\rho \leq \rho_{cr}$ fatigue strength σ_e, corresponding to the beginning of fatigue crack, may be calculated on the basis of the following equation of fatigue similarity for very sharp notches

$$\log(\xi - 1) = -\nu_{cr} \log\theta + (\nu_{cr} - \nu)\log\theta_{cr} \qquad (9)$$

for $\log\theta \leq \lg\theta_{cr}$, where θ_{cr} is the criterion of fatigue failure similarity at $\rho = \rho_{cr}$; ν_{cr} – additional constant of the material.
Fatigue strength σ_{ecr} for complete fracture of the specimens with very sharp notches $\rho \leq \rho_{cr}$ can be found on the basis of fracture mechanics as nominal stress corresponding to the appearance of nonpropagating fatigue cracks (at $\rho = \rho_{cr}$) from the equation

$$\Delta K_{ef} = \Delta K_{th} \qquad (I0)$$

where ΔK_{th} is the threshold of fatigue crack propagation determined as range of stress intensity coefficient corresponding to crack propagation rate $d\ell/dN \leq 10^{-7}$ mm/cycle; ΔK_{ef} – effective range of stress intensity coefficient, determined as

$$\Delta K_{ef} = \sqrt{2}\, A \cdot \Delta\sigma_{nom}\sqrt{\pi\ell} \qquad (II)$$

where A – coefficient; from Harris (I967) equation for bending of round specimens with diameter $2R$ and a circumferential crack with depth ℓ it may be found that for ℓ/R < 0,I5 it is possible to take A = I,I (with an error not larger that 4%).
Coefficient $\sqrt{2}$ in (II) was proposed by Eisenstadt (I977) for taking into account the influence of compression stress of the cycle on fatigue crack propagation.
For specimens with small depth notches it is reasonable to take ℓ = = $t + \rho_{cr}$ where t is the depth of the notch. With this suggestion and A = I,I and $\Delta\sigma_{nom} = \sigma_{ecr}$ from (I0) and (II) it can be found

$$\sigma_{ecr} = \frac{\Delta K_{th}}{2{,}76\sqrt{t + \rho_{cr}}} \qquad (I2)$$

The idea of summing notch depth and crack length in the equation for crack growth rate was used by several authors (Broek, Dowling, I979, Smith and Miller, I977 and others).
In Fig. 2 are presented the experimental results of Nisitani (I97I). In this work rotating bending round specimens were tested with circumferential grooves of different depth t = 0,005 ÷ 0,5 mm with radii ρ = 0,0I – I,0 mm and also plane specimens without stress concentration from carbon steel.
Line I corresponds to the fatigue strength of specimens with $\rho > \rho_{cr}$ = 0,4 mm and was drawn according to eq. (7) at ν = 0,I2, σ_0 = 2I8 MPa.
Line 2 corresponds to the fatigue strength of the specimens with

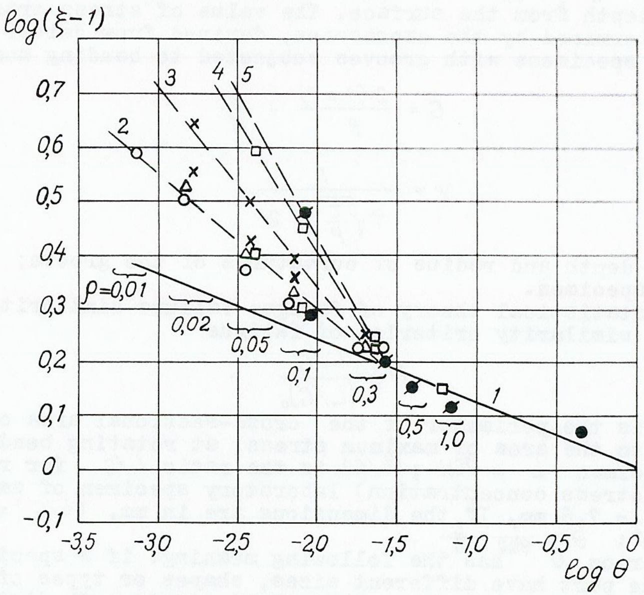

Fig. 2. Log $(\xi - 1)$ vs log θ for results of Nisitani (I97I).

$\rho < \rho_{cr}$ and the moment of fatigue crack initiation and was drawn according to eq (9) at ν_{cr} = 0,26, $\log\theta_{cr}$ = –I,6 (ρ_{cr} = = 0,4 mm).
Lines 3,4,5 correspond to the fatigue strength at the moment of complete fracture of the specimens with $\rho < \rho_{cr}$ and t = 0,02 mm, 0,I0 mm and 0,5 mm. These lines were drawn on the basis of σ_{ecr}, calculated upon eq. (I2) at ΔK_{th} = I0,7 MPa $\sqrt{m}$. In work Nisitani (I97I) value of ΔK_{th} was not determined. This value was calculated on the basis of the relationship between ΔK_{th} and yield strength σ_y of carbon steels, received according to the results of many investigators. Fig. 2 clearly shows, that equations (7), (9), (I2) give very good results at calculations of fatigue strength of the specimens.
On Fig. 3 are shown results of fatigue testing in rotating bending of the round specimens of 8,6 and I5 mm in diameter with circumferential grooves. Specimens were made from armco-ferrum. For the specimens d = 8,6 mm was received ρ_{cr} = 0,24 mm, σ_{ecr} = 88 MPa; for d = I5 mm, ρ_{cr} = 0,40 mm, σ_{ecr} = 78 MPa. Line I in Fig. 3 corresponds to eq. (5) at σ_0 = 230 MPa, ν = 0,I. Line 2 corresponds to the specimens with $\rho < \rho_{cr}$ and d = 8,6 mm. Line 3 corresponds to $\rho < \rho_{cr}$ and d = I5 mm.
Therefore in this work it was discovered that ρ_{cr} increases and σ_{ecr} decreases with increasing d. These regularities must be checked in future investigations if we take into account the possibility of random variations of σ_{ecr} and ρ_{cr} if fatigue curve is obtained on testing 6–8 specimens, as it was in this work.

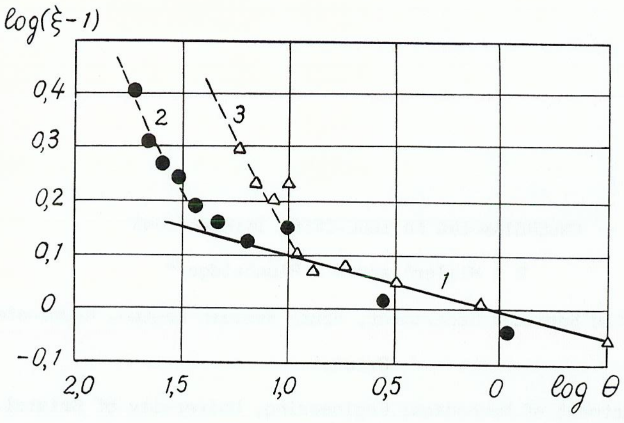

Fig. 3. Log ($\xi - 1$) vs log θ for armcoferrum.

For fatigue-life calculations at irregular loading was proposed a corrected linear damage accumulation rule, which is expressed by the following equations (Kogaev, 1977)

$$\sum \frac{n_i}{N_i} = a_p \qquad (I3)$$

where

$$a_p = \frac{\sum \sigma_{ai} t_i - 0{,}5\,\sigma_e}{\sigma_{a\,max} - 0{,}5\,\sigma_e} \qquad (I4)$$

Here $\sigma_{a\,max}$, σ_{ai} are maximum and i - level stress amplitude in a loading block; $t_i = n_{i\theta}/n_i$; $n_{i\theta}$, n_i are the numbers of cycles, corresponding to i - amplitude and total in loading block. According to (I4) a_p = 0,I ÷ I,0.
Many experimental results show that linear damage accumulation rule can give overestimation of fatigue life to ten times. But the corrected linear rule (eq.(I3),(I4)) with the probability 95% gives such overestimation not more than 2,5 times.
If in the irregular loading process there are substantial peak loadings, there appears a cyclic elastic-plastic strain field which can change during accumulation of loading cycles. In that case it is convenient to use deformation kinetics criterion (Gusenkov,1979) in the form

$$\int_{e=0}^{e_f} \frac{de}{\varepsilon_f} + \int_{N=1}^{N_f} \frac{dN}{N} = 1 \qquad (I5)$$

where N - number of loading cycles; N_i - number of cycles, determined at given strain amplitude on fatigue curve, received at constant strain amplitude; N_f - number of cycles to failure (initiation of crack); e - strain accumulated in one direction in the process of static and cyclic loading; e_f - strain accumulated in one direction at the moment of fracture (initiation of crack); ε_f - plasticity at static loading till fracture. Random and program fatigue testing have shown, that the sum of integrals in eq. (I5) varied from 0,59 till I,44. These results prove the applicability of criteria (I4), (I5) for the practical purposes of fatigue life calculations.

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