FRACTURE ANALYSES OF ANGLE - PLY LAMINATES

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ABSTRACT

In this paper the development of special laminated "crack-elements" and "hole-elements", with embedded analytical asymptotic solutions, and based on a complementary energy principle with subsidiary constraints, is reported. Application of these special finite elements in fracture analyses of angle-ply laminates is discussed. Detailed results and discussions are presented for the cases of a through-thickness crack and a through-thickness hole in a four-ply, (±45)s, laminate.

KEYWORDS:

Composite Materials, Fracture of Nonhomogeneous Media, Hybrid Elements, Complementary Energy, Crack and Holes in Flawed Laminates.

INTRODUCTION:

In general, angle-ply laminates are composed of a stack of layers each of which contains uniaxial fibers and have, therefore, preferred directions. Because of the non-homogeneous structure of these laminated composite materials, several failure mechanisms such as: Fiber-matrix separation in each layer, separation or debonding of layers, and non-self-similar growth of cracks in one of several, or all of the layers, may exist under the action of general external loading. In this paper we pay attention to the fracture-mode of failure, and its possible interaction with delamination, of laminates with through-the-thickness cracks and fastener holes.

Specifically, we consider a symmetric angle-ply laminate of the form $(\pm\beta_{1h_1}/\pm\beta_{2h_2}/\pm\ldots/\pm\beta_{nh_n})$ where β_n and h_n are, respectively, the orientation and thickness of the n^{th} angle-ply component of the laminate. We consider the cases of cracks and holes through the thickness of the laminate, and consider the crack-axis to be parallel to the fiber orientation of the Kth-ply. Thus, even though the crack is located symmetrically with respect to the preferred material direction of the Kth-ply, it is, in general, oriented unsymmetrically with respect to the material directions of all the other plys, $n \neq K$. Thus, in general, mixed mode crack-tip conditions, which vary from layer to layer, may exist in the laminate. Likewise, in the case of a hole, in general, stress-concentrations, which vary from layer to layer, may exist in both direct as well as shear stresses.

It is the objective of the research reported herein to develop efficient (finite element) analysis procedures to determine the mixed mode stress-intensity factors in the case of cracks, the stress-concentration factors in the case of holes, and their variation in the thickness-direction of the laminate, within each layer as well as from layer to layer. The method described herein is based on a complementary energy principle with subsidiary constraints such that: (*i*) each layer of the laminate is treated as an anisotropic medium and the material properties in the laminate global coordinates vary from layer to layer, (*ii*) the 3-dimensional stress-state, including the interlayer normal and shear stresses, is accounted for; (*iii*) the analytical asymptotic solution for stresses near the crack (the stress-solution being singular) and near a hole, in each layer, are embedded in special "crack" and "hole" elements near the crack-front; (*iv*) the assumed equilibrated stress-solution in the laminate is such that the interlayer traction reciprocity condition is satisfied a priori, but the planar stresses in the plane of the laminate are allowed to be discontinuous at the interlayer interface, (*v*) each finite element consists of the entire series of layers in the laminate, (*vi*) the inter-element traction reciprocity is enforced, a posteriori, in the variational formulation, through a Lagrange multiplier technique, and (*vii*) the procedures developed herein lead to a direct evaluation of the mixed-mode stress-intensity factors (and their variation) near the crack-front and of the stress-concentration factors (and their variation) near the hole.

In the present paper, a brief outline of the analysis procedure is first given. Detailed results for the problems of (*i*) a through-thickness crack, and (*ii*) a through-thickness hole in a 4-ply ($\pm 45^o$)s laminate are given, and their implication in the prediction of failure of the laminate is discussed. The present results are compared with other independent solutions when available, and the relative merits of the present procedure are noted.

OUTLINE OF ANALYSIS PROCEDURE:

Consider a system of cartesian coordinates (x_1,x_2,x_3) such that x_3 is the thickness-coordinate of the laminate, and, in the case of a crack, x_1 and x_2 are coordinates along and normal to the crack-axis, respectively, centered at the crack-tip; while in the case of a hole (x_1 and x_2) are coordinates centered at the center of the hole. We also consider cylindrical polar-coordinates (r,θ,x_3), in both the cases, such that $x_1=r\cos\theta$ and $x_2=r\sin\theta$. We consider K layers in the laminate, and the kinematic and mechanical variables in each layer are denoted by a superscript i, such that i=1,2,...K. We assume that the interlayer interfaces can be denoted by x_3=constant, and denote by x_3^i the x_3 coordinate of the interface between the i^{th} and (i+1)th layers, [illegible] x_3^o is the coordinate of the bottom surface of the laminate. In the complementary energy principle, one has to assume an equilibrated stress-field in each layer, i=1,2,...K, such that:

$$\underline{\nabla}\cdot\underset{\sim}{\sigma}^i \equiv \mathrm{div}\,\underset{\sim}{\sigma}^i = 0 \tag{1}$$

$$\sigma^i_{22} = \sigma^i_{21} = \sigma^i_{23} = 0 \text{ at } x_2=0\ x_1<0 \text{ (in the case of a crack)} \tag{2}$$

$$\sigma^i_{rr} = \sigma^i_{r3} = \sigma^i_{r\theta} = 0 \text{ at } r=a \text{ (in the case of a hole)} \tag{3}$$

where it is assumed that the body forces are zero, and that $x_1<0$ in Eq. (2) represents the presently considered case of an edge crack in the laminate. Further, the stress-tensor $\underset{\sim}{\sigma}^i$ in each layer is expressed either in cartesian components or cylindrical polar coordinates alternatively, and that the gradient opertor $\underline{\nabla}$ is thus appropriately expressed in either of the two alternative coordinate systems. The interlayer traction reciprocity conditions for the presently considered laminate are:

$$\sigma^i_{33} = \sigma^{i+1}_{33};\ \sigma^i_{3\theta} = \sigma^{i+1}_{3\theta};\ \sigma^i_{r3} = \sigma^{i+1}_{r3} \text{ at } x_3=x^i_3 \tag{4}$$

$$\text{or } \sigma^i_{3\alpha} = \sigma^{i+1}_{3\alpha}\ (\alpha=1,2,3) \text{ at } x_3=x^i_3 \tag{5}$$

Thus in the present case of a multilayer laminate, wherein each layer is modeled as an anisotropic medium at the macroscopic level, the in-plane stresses $\sigma^i_{\alpha\beta}$ are such that, in general, a priori,

$$\sigma^i_{\alpha\beta} \neq \sigma^{i+1}_{\alpha\beta}\ (\alpha,\beta=1,2 \text{ or } r,\theta) \text{ at } x_3=x^i_3 \tag{6}$$

In the present case, the laminate is modeled by two types of finite elements; (*i*) special "crack" or "hole" elements immediately near the crack-front or the hole, and (*ii*) "regular" elements away from the region of a crack or a hole. In general, a stress-field satisfying Eq. (1), as well as Eqs. (4) or (5), a priori, is chosen in each element, as:

$$\underset{\sim}{\sigma}^i = \underset{\sim}{\sigma}^{iS} + \underset{\sim}{\sigma}^{iR} \tag{7}$$

wherein the additional superscripts denote "special" and "regular" fields, respectively. In the case of "crack" and "hole" elements, both $\underset{\sim}{\sigma}^{iS}$ and $\underset{\sim}{\sigma}^{iR}$ are introduced, while in the far-field elements, only $\underset{\sim}{\sigma}^{iR}$ is introduced. It is noted that in the present case, the functions $\underset{\sim}{\sigma}^{iS}$ satisfy, in addition to Eqs. (1,4 or 5), the boundary conditions Eq. (2) or Eq. (3), as well, a priori. The functions $\underset{\sim}{\sigma}^{iR}$ which do not satisfy Eq. (2) or (3) a priori, are forced to satisfy these equations, in the case of "crack" or "hole" elements, through a Lagrange multiplier technique, analogous to a point-matching technique, given by Atluri and Rhee (1978).

The details of the procedure in deriving the appropriate $\underset{\sim}{\sigma}^{iS}$ are omitted here, but are indicated elsewhere (Nishioka and Atluri, 1980a; 1980b). However, the end results are as follows:

(*i*) case of a crack:

$$\sqrt{2\pi}\,\sigma^{is}_{11} = \frac{K^i_1(x_3)}{\sqrt{r}}F_{111}(\theta) + \frac{K^i_2(x_3)}{\sqrt{r}}F_{211}(\theta) + K^i_{4,33}(r)^{3/2}\frac{2}{3}\cos(\frac{3\theta}{2})$$

$$-K^i_{5,33}(r)^{3/2}\frac{2}{3}\sin(\frac{3\theta}{2}) + K^i_{3,3}(r)^{1/2}\left\{2\mathrm{Re}[S^i_3(\cos\theta+S^i_3\sin\theta)^{1/2}]\right\}$$

$$\sqrt{2\pi}\,\sigma^{is}_{22} = \frac{K^i_1(x_3)}{\sqrt{r}}F_{122}(\theta) + \frac{K^i_2(x_3)}{\sqrt{r}}F_{222}(\theta) + K^i_{4,33}(r)^{3/2}\left\{-\frac{2}{3}\cos(\frac{3}{2}\theta)\right.$$

$$\left.-C\sin\theta(|\cos\theta|)^{1/2}\right\} + K^i_{5,33}(r)^{3/2}\left\{\frac{2}{3}\sin(\frac{3}{2}\theta) + \frac{2}{3}C(|\cos\theta|)^{3/2}\right\}$$

$$+ K^i_{3,3}(r)^{1/2}\left\{-2\mathrm{Re}[(1/S^i_3)\ (\cos\theta+S^i_3\sin\theta)^{1/2}] + 2C(|\cos\theta|)^{1/2}\mathrm{Re}(i/S^i_3)\right\}$$

$$\sqrt{2\pi}\,\sigma^{is}_{12} = \frac{K^i_1(x_3)}{\sqrt{r}}F_{112}(\theta) + \frac{K^i_2(x_3)}{\sqrt{r}}F_{212}(\theta)$$

$$\sqrt{2\pi}\,\sigma_{13}^{is} = \frac{K_3^i(x_3)}{\sqrt{r}} F_{313}(\theta) - K_{4,3}^i (r)^{1/2} \cos(\theta/2) + K_{5,3}^i (r)^{1/2} \sin(\theta/2)$$

$$\sqrt{2\pi}\,\sigma_{23}^{is} = \frac{K_3^i(x_3)}{\sqrt{r}} F_{323}(\theta) + K_{4,3}^i \left\{ (r)^{1/2}(-\sin\theta/2) + C\left|x_1\right|^{1/2} \right\}$$

$$+ K_{5,3}^i (r)^{1/2}(-\cos\theta/2)$$

$$\sqrt{2\pi}\,\sigma_{33}^{is} = \frac{K_4^i(x_3)}{\sqrt{r}} \cos(\theta/2) + \frac{K_5^i(x_3)}{\sqrt{r}} \sin(\theta/2) \tag{8}$$

In the above $F_{j\alpha\beta}[j=1,2,3;\alpha,\beta=1,2]$ are functions in the basic singular solution near the crack-tip in two-dimensional Modes I and II problems for an anisotropic medium (Sih, and Liebowitz, 1968); while $F_{313}(\theta)$ and $F_{323}(\theta)$ are functions in the basic singular solution near the crack-tip in two-dimensional Mode III (anti-plane shear) problem for an anisotropic medium (Sih, and Liebowitz, 1963). Likewise, S_3^i is a complex number depending on the elastic compliance coefficients of the i^{th} lamina (Sih and Liebowitz, 1968). The notation, $K_{4,3}^i = \partial K_4^i/\partial x_3$ is used.

(*ii*) <u>case of a hole</u>:

$$\sigma_{33} = \left\{ \frac{\alpha_1}{r} + \frac{\alpha_3}{r^2} + \frac{\alpha_5}{r^3} + \frac{\alpha_6}{r^4} \right\} \cos 2\theta + \left\{ \frac{\alpha_2}{r} + \frac{\alpha_4}{r^2} + \frac{\alpha_6}{r^3} + \frac{\alpha_8}{r^4} \right\} \sin 2\theta$$

In the above, $\alpha_1 .. \alpha_8$ are continuous functions in x_3. In the remainder, for want of space, only terms symmetric in θ will be given, while the anti-symmetric terms in θ can be derived by observation.

$$\sigma_{\theta 3} = \left\{ \frac{1}{4}\alpha_{2,3} + \frac{1}{4r}\alpha_{4,3} + \frac{1}{4r^2}\alpha_{6,3} + \frac{1}{4r^3}\alpha_{8,3} \right\} \cos 2\theta$$

$$\sigma_{r3} = \left\{ -\frac{(1-ar^{-1})}{2}\alpha_{1,3} - \frac{r^{-1}}{2}\ln\left(\frac{r}{a}\right)\alpha_{3,3} + \frac{(r^{-2}-a^{-1}r^{-1})}{2}\alpha_{5,3} + \frac{(r^{-3}-a^{-2}r^{-1})}{2}\alpha_{7,3} \right\} \cos 2\theta$$

$$\sigma_{\theta r} = \left\{ -\frac{(r-a^3r^{-2})}{24}\alpha_{2,33} - \frac{(1-a^2r^{-2})}{16}\alpha_{4,33} - \frac{(r^{-1}-ar^{-2})}{8}\alpha_{6,33} - \frac{r^{-2}}{8}\ln\left(\frac{r}{a}\right)\alpha_{8,33} \right\}$$

$$\cos 2\theta + \alpha_9(x_3) F_{r\theta}(r,\theta)$$

$$\sigma_{\theta\theta} = \left\{ -\frac{r}{16}\alpha_{1,33} - \frac{1}{16}\alpha_{3,33} - \frac{r^{-1}}{16}\alpha_{5,33} - \frac{r^{-2}}{16}\alpha_{7,33} \right\} \cos 2\theta + \alpha_9(x_3) F_{\theta\theta}(r,\theta)$$

$$\sigma_{rr} = \left\{ \frac{(-48a+17r+39a^2r^{-1}-8a^3r^{-2})}{96}\alpha_{1,33} + \frac{1}{16}\left(8\ln\frac{r}{a} - 11 + \frac{13a}{r} - \frac{2a^2}{r^2}\right)\alpha_{3,33} \right.$$

$$\left. + \frac{1}{16}\left(\frac{8}{a} - \frac{13}{r}\ln\frac{r}{a} - \frac{4}{r} - \frac{4a}{r^2}\right)\alpha_{5,33} + \frac{1}{16}\left(\frac{4}{r^2}\ln\frac{r}{a} + \frac{4}{a^2} - \frac{13}{ar} + \frac{9}{r^2}\right)\alpha_{7,33} \right\}$$

$$\cos 2\theta + \alpha_9 F_{rr}(r,\theta) \tag{9}$$

It is noted that in Eq. (8) K_3^i, K_4^i, and K_5^i are assumed to be Hermitian polynomials in each lamina such that these functions are continuous at interlayer interfaces; thus the interlayer traction reciprocity is satisfied a priori. However, the functions K_1^i and K_2^i are allowed to be discontinuous at interlayer interfaces, since $\sigma^i_{\alpha\beta}$ $(\alpha,\beta=1,2)$ may be discontinuous at x_3^i. Likewise, in Eq. (9), $\alpha_1 ... \alpha_8$ are continuous functions in x_3. However, α_9 is discontinuous at x_3^i; and F_{rr}, $F_{\theta\theta}$, and $F_{r\theta}$ are asymptotic solutions near a hole in a 2-dimensional anisotropic domain (Savin, 1961).

The details of assuming $\underset{\sim}{\sigma}^{iR}$ such that Eqs. (1), (4, or 5), and (6) are met, a priori, are omitted here and may be found in (Nishioka, and Atluri, 1980a).

Finally, if V_n is a finite element and ∂V_n its boundary (note that in the present procedure, each finite element consists of the entire stack of lamina), the traction reciprocity condition at the interelement boundary, namely,

$$(\underline{n}.\underset{\sim}{\sigma}^i)^+ + (\underline{n}.\underset{\sim}{\sigma}^i)^- = 0 \text{ for each } i=1,2...k \tag{10}$$

where (+) and (-) denote, respectively, the two sides of ∂V_n and $\underline{n}$ is a unit normal to ∂V_n. The above constraint condition, which is impossible to satisfy a priori, in general, is enforced in the present procedure through a Lagrange multiplier technique.

Thus the modified complementary energy principle for the finite element assembly can be written as:

$$\Pi_{mc}(\underset{\sim}{\sigma}^i, \underline{U}^i) = \sum_n \left[\sum_i \left\{ \int_{V_n^i} W_c^i(\underset{\sim}{\sigma}^i)\,dv - \int_{\partial V_n^i} \underline{T}^i.\underline{U}^i\,ds + \int_{S^i_{\sigma n}} \overline{\underline{T}}^i.\underline{U}^i \right\} \right]$$

where W_c^i is the complementary energy density (unit volume) of the i^{th} lamina which depends on the ply elastic-constants and the assumed $\underset{\sim}{\sigma}^i$. The integral on ∂V_n^i is the term which enforces the constraint condition of Eq. (10) through Lagrange multipliers $\underline{U}^i$ chosen at ∂V_n^i. The assumptions for $\underline{U}^i$ are not detailed here, but may be found at (Nishioka, and Atluri, 1980a).

<u>Edge Crack in a (±45°)s Laminate:</u>

The geometry of the 4-ply laminate is shown in Fig. 1, while the finite element mesh is shown in Fig. 2. Only the upper half (in thickness direction) of the laminate is modelled.

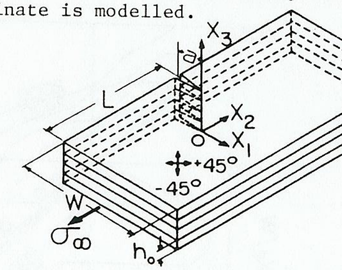

FIGURE 1

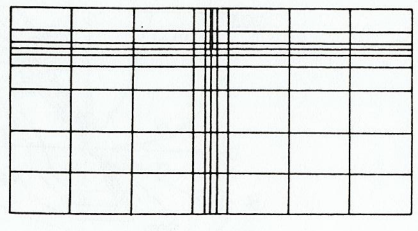

FIGURE 2

The geometric and material property data are: $(L/W=1.0; h=4h_o; a=0.2W; (h_o/W)=(1/100)$. The ply elastic constants, in the ply principal directions, are: $E_{11}=20\times10^6$psi; $E_{22}=2.1\times10^6$psi; $E_{33}=2.1\times10^6$psi; $G_{12}=G_{23}=G_{13}=0.85\times10^6$psi; $\nu_{12}=\nu_{23}=\nu_{31}=0.21$. The problem is studied for the case of uniaxial tension shown in Fig. 1. Note that the present finite element-mesh consists of a total of 2000 d.o.f. The present problem was also studied by Wang, Mandell and McGarry, (1977) using the finite element method development by Mau, Tong, and Pian (1972), who do not account for any crack-front stress/strain singularities. Thus, to extract a meaningful solution, Wang et. al. (1977) use two meshes, the outer mesh with about 1800 d.o.f. and the second inner mesh, near the crack, with about 1950 d.o.f.

Fig. 3 shows the directly computed stress-intensity factors K_1^i and K_2^i (in the in-plane stress $\sigma^i_{\alpha\beta}$ in each lamina, i=1,2) and K_4^i (in the transverse normal or "peel" stress σ^i_{33}). Such results were not presented in Wang et. al. (1977) since the procedures they employed do not permit a convenient extraction of these factors.

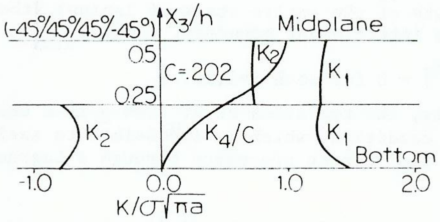

FIGURE 3

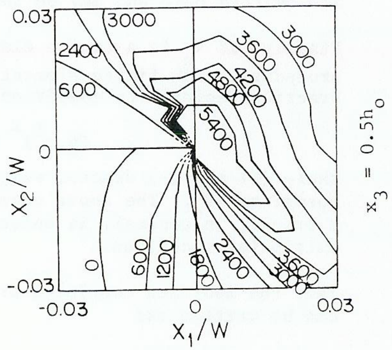

FIGURE 4

From Fig. 3 it is seen that at the point of incipient fracture, if the direction of crack-growth in each lamina is assumed to be determined by the magnitudes and signs of K_1 and K_2, crack-growth will occur in different directions in each of the lamina. This, then, provides a rational model for non-self-similar growth of cracks in multilayered laminates. Figs. 4 and 5 show the contours of the inplane stress σ_{22} (at the mid-section of each lamina) near the crack-tip (x_1,x_2=0). Likewise Figs. 6 and 7 show contours of the inplane stress σ_{11} and σ_{12}. The results in Figs. 4 to 7 correlate well with those in Wang, et. al. (1977), and these point to the relative efficiency of the present method, wherein the solution for detailed stress, as well as the intensity factors, were obtained in a single stage using only 2000 d.o.f.

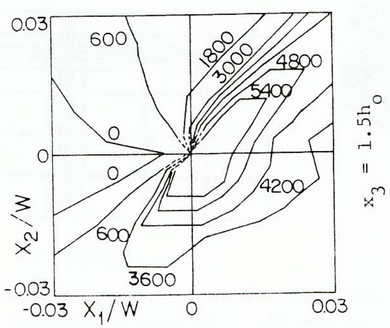

FIGURE 5

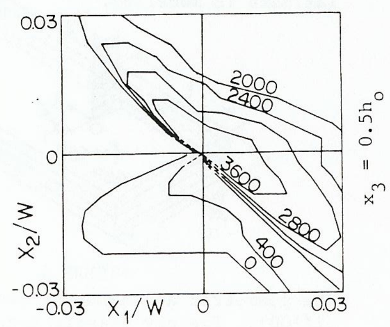

FIGURE 6

The variation of interlaminar shear stress near the crack-front, along with the comparison result of Wang et. al. (1977) is shown in Fig. 8, while the variation of interlaminar normal stress (note that the procedure in Wang et. al. 1968; ignores the effect of σ^i_{33}) is shown in Fig. 9. The results in Fig. 8 and 9 do indicate a peak in the interlaminar shear and normal stresses, thus providing a model for the coupling of the process of delamination with that of fracture, in a laminated composite.

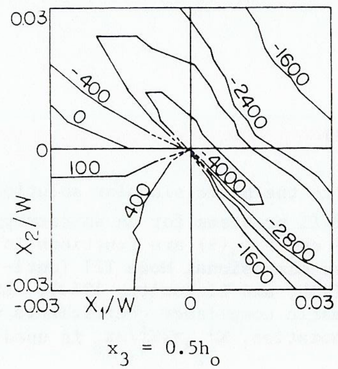

FIGURE 7

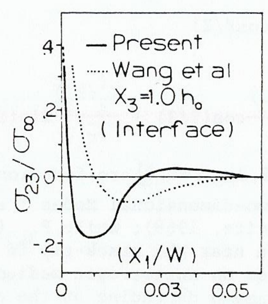

FIGURE 8

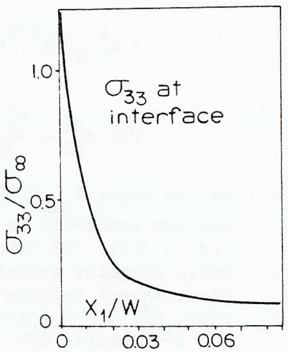

FIGURE 9

Fastener Hole in a (±45°)s Laminate:

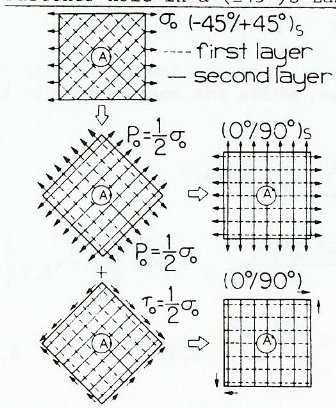

FIGURE 10

The problem is depicted in Fig. 10. As shown in this figure, using the arguments of linear superposition, the problem is decomposed into two simpler problems: (*i*) (0°/90°)s laminate under biaxial tension, and (*ii*) (0°/90°)s laminate under pure shear. Each of these two problems is solved by modeling only a quarter of the laminate as shown in Fig. 11.

The variation of circumferential stress $\sigma_{\theta\theta}$ at the mid-plane of the laminate, along with a comparison result of Rybicki and Hopper (1973) who use a classical Rayleigh-Ritz type 3-dimensional approach based on the virtual complementary energy principle, is shown in Fig. 12. The variation of transverse normal stress at the mid-plane of a lamina, along with the comparison results of Rybicki and Hopper (1973), is shown in Fig. 13. It is seen that while the present results for $\sigma_{\theta\theta}$ correlate well with those of Rybicki and Hopper (1973), the correlation between the two sets of results for σ_{33} is rather poor. However, it should be noted that Rybicki and Hopper (1973) derive a 3-dimensional equilibrated stress field from 3 Maxwell-stress functions, which are assumed to be continuous polynomials in all the 3 coordinates x_i (i=1,2,3). Thus, in Rybicki and Hopper's (1973) formulation the additional artificial constraint that even the inplane stresses $\sigma^i_{\alpha\beta}$ (α,β=1,2) are continuous at the interlaminar interface x_3^i was imposed. This may partly explain the discrepancies shown in Figs. 12 and 13.

The above results for the stress-field near an un-loaded fastener hole in a laminate demonstrate the significant differences between the cases of an anisotropic laminate and isotropic homogeneous media. The problem becomes even more compli-

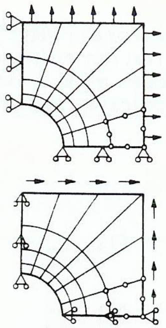

FIGURE 11

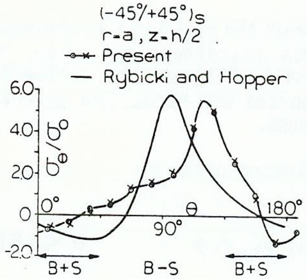

FIGURE 12

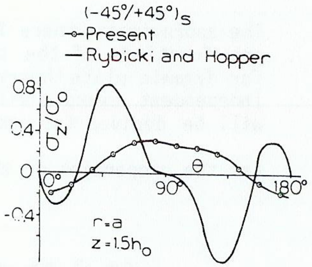

FIGURE 13

cated in the case of the more realistic situation of a pin-loaded fastener hole in a composite laminate. This problem is being currently investigated and the results will be reported.

Closure:

The development of special hybrid "crack" and "hole" elements for fracture analysis of multilayer angle-ply laminates with cracks and holes has been reported. The relative efficiency and accuracy of these new methods have been demonstrated in carefully chosen test cases of 4-ply ($\pm 45^{o}$)s laminates.

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