A J BASED ENGINEERING USAGE OF FRACTURE MECHANICS

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ABSTRACT

The use of J as a one-parameter model of fracture is rationalised in terms of an ideal elastic-plastic-elastic material. Initiation of crack growth, stable crack growth and the final instability are then described in terms of J and dJ/da for ductile cracking mechanisms. Use of a simple design curve including allowance for residual stresses and acceptance of a measure of toughness subsequent to initiation are discussed.

KEYWORDS

Fracture mechanics, stress intensity factor, J contour integral, elastic-plastic design, stable crack growth, unstable crack growth.

INTRODUCTION

Yielding fracture mechanics has developed greatly in the past decade, so that some half-dozen or more concepts have emerged. The intention here is to highlight the degree of commonality that exists between a number of these different theories and to suggest that their common features can best be unified under the umbrella of the J contour integral. It will then be argued that although for fracture of real metals a rigorous interpretation of J is inadequate, the concept can nevertheless be extended to provide a rational pathway through the complex interactions of degree of plasticity, geometric effects and different levels of work hardening for many practical purposes. To change the metaphor, such a rational basis provides a backbone on which a complete skeleton of related theories can be created to accommodate more complex real problems. Without such a backbone, fracture studies are but a collection of unrelated case studies with no central methodology. The use of a simple model to provide the backbone to which a coherent theory can be attached is by no means uncommon: an obvious example is the use of rigid-plastic behaviour as the basis of many plasticity studies in both structures and metal working, whereby a whole range of simple methods are made available for engineering purposes. It is relatively easy to perceive where the offshoots from a simple 'backbone' theory are needed, although the advances themselves may be difficult to achieve. Some of the arguments advanced here are heuristic but few subjects have developed by a succession of logical statements based on rigorous theory and the schematic relations are seen as more important than rigour at this stage.

The first step is to argue that the theory underlying most, if not all, the macroscopic or engineering level fracture models can be related to or even subsumed by J. This has been argued elsewhere and will be repeated below in the barest outline. The second step is to examine the implication of a simple one-parameter model for fracture, here expressed in terms of J. Onset of crack growth and unstable crack growth must be considered separately. The third step is to discuss the development of simple methods for relating fracture data to design or service applications. There then remains the many factors that such a simplified 'backbone' theory does not cover adequately - for example, certain aspects of three-dimensional problems and the interplay of micro-modes of separation and macro-mechanics description thereof. The contention is that these are more readily accommodated in relation to the 'backbone' theory rather than in isolation, although at the present stage of development satisfactory treatment of several such issues remains elusive.

THE INTER-RELATION BETWEEN SEVERAL FRACTURE CONCEPTS

It was argued (Turner, 1979a) that the basic concepts of most engineering fracture theories can be related to J and that any procedure especially advocated can, if deemed important, be translated for use with a J based theory; for example, the use of full thickness rather than small sized test pieces. Several theories offer expressions for the chosen fracture parameter that allow interpolations between lefm and plastic collapse. Either end point may be implicit as in the ln secant term of the original Dugdale (1960) model for Cod, or explicit, as in the Heald, Spink and Worthington (1972) modification to Cod theory and the extension theorof into the Two Criteria method (Dowling & Townley, 1975). Other theories expressing a direct interpolation are the Stress Concentration Theory (Irvine, 1978) and the Two-Parameter Theory (Newman, 1976), although the latter also makes provision for a second parameter to account for the degree of plane stress or plane strain not considered here. For the case of plastic zone correction to lefm it is supposed that use of the Irwin factor for size of plastic zone gives a term, here denoted G_p, that with hindsight, is an estimate of J in contained yield. Although a unique relation between J and Cod seems to exist only in the Dugdale model (Rice, 1968) the two terms appear always to be relatable by

$$J = m\sigma_Y\delta \qquad \text{Eqn.1}$$

where $m \simeq 2 \pm 1$, noting that in computational studies m is a function of degree of plasticity, configuration and work hardening. For this group of theories it is illustrative to express the ratio of plastic to elastic parameter, i.e. a generalised version of the term J/G, as a function of applied stress, Fig. 1. Clearly, any single expression cannot reflect the true complexities of the plasticity behaviour, although most of the interpolations have been shown to be adequate for selected practical purposes.

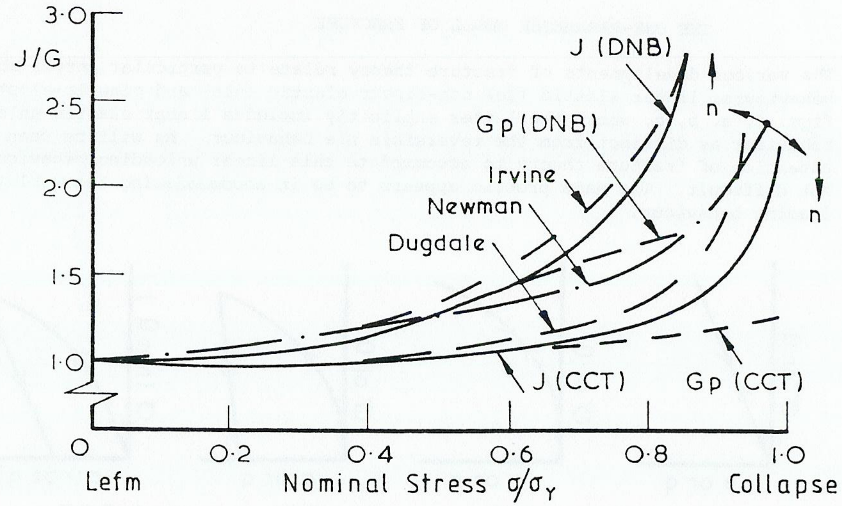

Fig.1. Comparison of several theories expressed as interpolations between lefm and collapse.

For a second group of theories it is simpler to point to particular similarities with J whilst accepting that for some the link is indeed close, whilst for others it is more tenuous. The best example is perhaps Equivalent Energy (Witt, 1971) where a hypothetical linear diagram is constructed, of area w_{el}, that is equated to the overall work, w_o, of the actual plastic diagram. Writing

$$J = \eta_o\, w_o/Bb \qquad \text{Eqn.2}$$

$$G = \eta_{el}\, w_{el}/Bb \qquad \text{Eqn.3}$$

where Bb is the ligament area, then clearly the Equivalent Energy procedure ensures J = G for all cases where $\eta_o = \eta_{el}$, i.e. where η is a function of configuration alone, invariant with deformation. Computational evidence (Turner, 1980a) shows this is more nearly so for a wider range of geometries than at first seemed likely and the term η is discussed more fully later. A close qualitative agreement exists with Soete's concepts (1977) since the four regimes that he identifies experimentally, elastic, contained yield, net section yield and gross section yield are those that emerge as the natural divisions within J theory. The Tangent Modulus or Gross Strain theory (Randall and Merkle, 1971) and the Eftis, Liebowitz and Jones (1975) non-linear $\tilde{G}$ theory also reflect behaviourial features that can be extracted from the J based theory. It is not suggested that all features of these theories are encompassed within a J model. It is suggested that in so far as all these concepts are single parameter models of fracture (for a given degree of plane stress or plane strain) then all can be accepted for some ideal circumstance, but none can be truly representative of real behaviour. In the writer's view the J-based model, extended and applied as described here, provides the best compromise that is currently available between rigour for analysis, flexibility for application to different circumstances, degree of development and reasonable accord with reality.

THE ONE-PARAMETER MODEL OF FRACTURE

The various developments of fracture theory relate to particular stress-strain behaviours, linear elastic (le) non-linear elastic (nle) and plastic-elastic (pe), Figs. 2, a. b, c, where the latter explicitly includes linear elastic unloading behaviour as distinct from the reversible nle behaviour. As will be seen later, extention of fracture theory to accommodate this linear unloading behaviour is not difficult. The main problem appears to be in accommodating the subleties of loading behaviour.

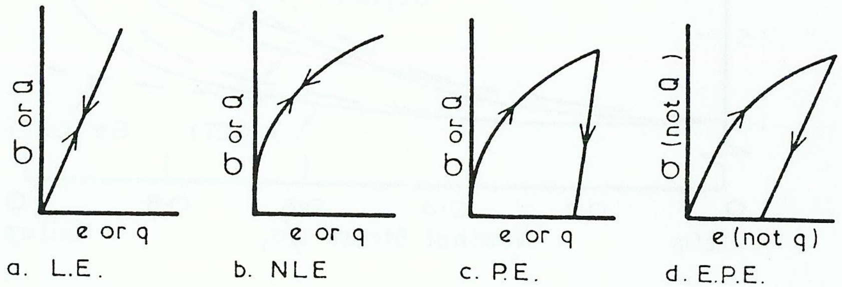

Fig.2. Various stress-strain and load-displacement diagrams.

McClintock (1965) pointed out that there can be no unique crack tip stress state for rigid plastic non-hardening material, since even in plane strain the triaxiality or ratio of stresses $\sigma_x : \sigma_y : \sigma_z$ depends upon the configuration of the component and its loading. Workhardening and perhaps elasticity ameliorate these differences but do not eliminate them for many materials of moderate hardening. The stress state represented by lefm plane strain gives nominal stress ratios 1:1:2ν at the crack tip. The HRR solutions in plane strain are models of the high constraint cases, since for n = 1 the stress ratios on the plane θ = o (at constant volume) are 1:1:1 and even for n → o the ratios are π:2+π : 1+π (approx. 0.6 : 1 : 0.8) as in the Prandtl field. If the differences between the lefm and nle cases of high constraint are ignored, then a stress system exists that varies but little with deformation, and also varies but little with n. Thus, a single parameter representing stress field intensity is adequate to characterise the crack tip field throughout the range of applicability of the lefm and HRR solutions. On the other hand, for certain well-known configurations, notably single edge and centre cracked tension, the stress ratios tend to 1 : 0 : 0.5 for the rigid non-hardening plastic plane strain limit state. If this state is approached in practice, then there must be an appreciable change with degree of deformation from the high constraint of lefm, or perhaps Prandtl field, that is representative of small scale yielding to the low constraint limit state. Such behaviour clearly needs at least two parameters, intensity and triaxiality, to define it. The practical significance of a stress field that is not unique in all its terms and thus requires more than stress field intensity to define it cannot be stated, since in general the consequence will vary with the property under discussion and the relevant micro-mechanisms of deformation. If some modes of separation in some materials are insensitive to triaxiality, then J (or indeed other single parameter) will be an adequate description of onset of fracture, but if some modes or materials are sensitive to triaxiality or the maximum component of stress, then one term, J or other, will be inadequate to relate configurations experiencing

different degrees of in-plane constraint. Several studies (such as Hayes, 1970, Boyle, 1972, Sumpter, 1974) have shown the path independence of J is maintained to within the numerical accuracy of finite element computations even using incremental plasticity, for a variety of configurations including centre cracked tension with no work-hardening. However, for a given value of J the crack tip deformation (Sumpter and Turner, 1976) and stress states (McMeeking and Parks, 1979) are not unique but depend upon configuration once there is extensive plasticity. Thus, the point is not that J is less applicable for low constraint but that there is no available datum or reference against which to assess the physical (fracture) significance of a given value of J, even in plane strain, except for the cases that do indeed experience high constraint, since the universal datum of lefm plane strain is itself of high constraint. It is a matter for conjecture whether a test in the plane strain lefm regime with compressive transverse (σ_x) stresses also applied could simulate the fracture conditions of the low constraint cases that arise with extensive plasticity.

An Ideal Material. Real materials experience a combination of compressible linear elasticity plus plastic deformation according to the incremental flow rules, with linear elastic unloading, and are here denoted "real elastic-plastic elastic" (real epe). For the purposes of the "backbone" engineering fracture theory an "ideal epe material" is postulated, of which the characteristics are elastic (finite modulus) behaviour followed by total theory plasticity loading and linear elastic unloading, Fig.2d. The intention (which may not be rigorously satisfied) is to have a one-parameter stress state loading system followed by linear unloading. As already noted, in the HRR solutions there is a fairly small difference in degree of triaxiality within plane strain, according to the hardening exponent n, but it appears that in reality, with low hardening a larger degree of difference can occur in practice, perhaps for several contributory reasons. In nle (and by implication total plasticity) for constant boundary conditions and proportional loading the stress ratios are invariant with degree of deformation, so even a finite width centre cracked plate with little hardening would acquire and maintain the high constraint of the Prandtl field, as found in the HRR solutions. It appears to have been argued that this constant ratio of stresses is also maintained even for the incremental case (Hutchinson, Needleman & Shih, 1979) albeit with constant volume power law hardening. With non-hardening rigid plastic slip behaviour the triaxiality depends on the boundary conditions of configuration and loading, but not on degree of deformationn. Thus, all these cases appear amenable to a single paramater description of the crack tip stress state, since only the intensity varies with degree of deformation, although the cases of slip with low constraint would again lack a datum to which the physical significance of a given value of J could be related. Yet, for the practical case of compressible elasticity, followed by low hardening incremental plasticity for a finite width centre cracked plate in tension, the triaxiality is a function of degree of deformation.

It is not clear to the writer which particular departure from the various ideal cases, elastic compressibility plus plastic incompressibility, incremental behaviour, finite width or some combination thereof, is the essential feature of this real behaviour. The precise specification of the ideal epe material may therefore be questioned, but its practical attributes are clear: i) the stress ratios are constant with degree of deformation during loading and a one parameter description of the intensity of the field is sufficient, just as in the strict nle (or total theory plasticity) case. J will here be taken as the most appropriate single paramater for ideal epe material. An important practical point seems to be that, although in principle a one parameter scheme cannot describe the stress state for all cases in real epe material, J measures an average intensity which may prove adequate for a wider range of circumstances than a single component parameter such as stress, strain or Cod. Only experiment, or a knowledge of the true criteria of separation on the microscale and its manifestation on the macro-scale, can give this guide. ii) The variables of configuration and degree of deformation are separable (Paris, Ernst & Turner, 1980; Turner, Ernst & Paris, 1980).

Just as in the strict power law hardening case, this means that the work, w, and the work rate $\partial w/\partial a$ are related by a size factor, Bb, and geometric factor, denoted η, that is independent of degree of deformation so that, for a given work hardening, it is a function of configuration alone. Thus,

$$w = \frac{-b}{\eta} \left.\frac{\partial w}{\partial a}\right|_q \qquad \text{Eqn.4}$$

whence, since $BJ = -\partial w/\partial a$, $J = \eta_o w_o/Bb$, as in Eqn.2, where b is the ligament (W-a) B is thickness and the suffix o denotes an overall value based on elastic plus plastic behaviour. For linear elastic behaviour this degenerates to Eqn.3, $G = \eta_{el} w_{el}/Bb$, where suffix el defines the lefm case, so that $w_{el} = Qq/2$ where Q is load and q is displacement, and of course G is the linear elastic value of J. The term η_{el} can be derived from compliance ϕ

$$\eta_{el} = \frac{b}{\phi} \frac{d\phi}{da} \qquad \text{Eqn.5}$$

or from the lefm shape factor, Y, i.e. where K is written

$$K = Y\sigma\sqrt{a} \qquad \text{Eqn.6}$$

then (Turner, 1973)

$$\eta_{el} = bY^2 a / \int Y^2 a \, da \qquad \text{Eqn.7}$$

and the un-notched compliance is used to evaluate the constant of integration in the denominator. In elastic plastic or nle behaviour η remains a function of the exponent n.

iii) In unloading, although a linear load displacement relationship is supposed, it is not assumed a priori that the energy release rate is G because the decrease in load with crack growth at constant displacement, $\partial Q/\partial a|_q$, Fig.3, may itself be a function of the stress strain behaviour. Thus, a term I was defined (Turner, 1979b) as "the elastic energy release rate in the presence of plasticity" (whereas G is here taken as a purely lefm term, the elastic energy release rate in the absence of plasticity). Since the limits are lefm or nle (total recovery)

$$G < I < J \qquad \text{Eqn.8}$$

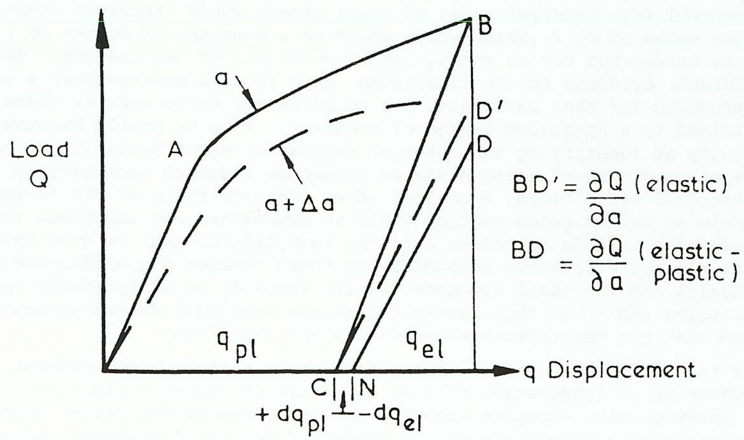

Fig.3. Energy release rate for elastic-plastic-elastic material.

It follows from Eqn. 4 that

$$\left.\frac{\partial Q}{\partial a}\right|_q = \frac{-\eta Q}{b} \qquad \text{Eqn.9}$$

Thus, if η is strictly independent of degree of deformatiom, it must equal the initial value η_{el} irrespective of the value of the hardening exponent n, and in that case $\partial Q/\partial a|_q$ is the same as in lefm, so

I = G for ideal epe behaviour

In brief, the ideal epe material experiences a crack tip stress field that varies in intensity only and can thus be described by a single parameter. J is chosen for this parameter because of its continuity with lefm and because of its relationship with work and work rate through Eqns. 2 and 4. It is argued that for most practical purposes of estimation and usage, even if not with strict rigour, the attributes of a one parameter description, constant constraint (for plane strain), variables separable and hence existence of η that is a function of geometry alone, all stand together and when there is evidence of a loss of these conditions, as for example, when η varies with degree of deformation, the one parameter description of the crack tip is lost.

iv) It may also be convenient to associate with ideal epe material a neutral strain rate behaviour so that there is no change in micro-mode of separation with strain rate or crack speed. Toughness is then taken to vary only slowly with speed, so that there is no need to account for variation of rate in connection with either yield stress or fracture toughness when setting up the mechanics of the problem. This artificially permits examination of the mechanics of the deformation applied to the body without interim complications of the material response other than through its elastic-plastic behaviour on the macro-scale. Once the mechanics are set out, more realism can be introduced into the material response and the mechanics amended in so far as possible.

Real Material and Structural Behaviour. For monotonic proportional loading of uniform (un-notched) test pieces there is no need to distinguish between real and ideal stress strain behaviour. But as already discussed, real material when used in complex stress fields or with complex stress history accommodates itself in a manner that is configuration dependent, and the distinction is then important. Computational studies based on the Prandtl-Reuss equations for elasto-plastic behaviour appear to model actual behaviour with adequate realism, without, of course, incorporating a criterion for separation. The writer is not aware of a treatment in algebraic terms that models the real behaviour, except in so far as it is, explicit or otherwise, simplified by steps such as assumption of constant volume, or infinite plate, or power law (no linear regime) behaviour which features contribute in part to the differences between ideal and real behaviour. For proportional loading the bridge between stress-strain and load-displacement behaviour is often made by taking a power law representation for stress-strain and using Illyushin's Theorem (Illyushin, 1956). The writer confesses to lack of clarity as to whether that step is not warranted for real (compressible elastic plus incremental) materials, since acceptance of it seems to imply the variables separable constant stress ratio behaviour that is the hallmark of the ideal epe case rather than the real epe. For these cases where limit state and elastic predictions of triaxiality differ it has already been argued that a one parameter description is inadequate and an important practical question in which state is the better guide to contained yielding. In so far as near invariance of η with degree of deformation is accepted as a measure of maintenance or loss of a one parameter system, then the computational evidence is that in most cases the elastic estimate remains reasonably valid for deformations up to several (e.g.

3 or 4) times yield in the uncracked body, and that rather extensive deformation is required before the rigid plastic estimate dominates. The near invariance of η for a variety of configurations and its rather slow variation with deformation in others, carrying the implication that the triaxiality is maintained at near lefm levels, is best supported in computations with mild work hardening. For no hardening, or for a plateau of non-hardening at yield representative of mild steel, some uncertainty remains on how well constraint is maintained, and for what degree of deformation.

In summary, it appears that there still exists a large degree of uncertainty over whether a single paramter, in practice mainly δ_i or J_i, is indeed descriptive of the onset of separation in plane strain for a variety of configurations in which the triaxiality alters with degree of deformation. If one parameter is inadequate then triaxiality or stress ratio or the bi-axial (σ_x) component of stress appears the necessary second parameter, just as σ_z has long been accepted for the differences between plane stress and plane strain. The problem is not only finding an acceptable way of specifying it that is applicable to both the non-singular (no work hardening) and singular cases, but also in studying, presumably by experiment, the effect of various constraints on fracture behaviour that is itself material dependent.

These uncertainties carry over into the elastic unloading behaviour because, as already noted, the change in load with crack growth $\partial Q/\partial a|_q$ is a function of the stress-strain law . A general expression for the energy release rate, I, is (Turner, 1980a)

$$I = G - q_{el} \frac{(\eta_o - \eta_{el})}{\eta_o} \frac{\partial \Omega}{B\partial a}\Big|_q \qquad \text{Eqn.10a}$$

$$= G\left(\frac{2\eta_o}{\eta_{el}} - 1\right) \qquad \text{Eqn.10b}$$

where $\eta_o > \eta_{el}$ clearly $I > G$, but if η is not a function of deformation, $I = G$. There appears here to be seeds of contradiction or perhaps the impossibility of strict ideal epe behaviour if deformation starts linearly, since $\eta_o = f(n)$, whereas η_{el}, for $n = 1$, requires η to be a function of geometry alone, i.e. the term $f(n)$ must be a constant for all n, a behaviour substantiated strictly only for configurations describable by one geometric parameter (Rice, Paris & Merkle, 1973). For purposes of estimation there is some uncertainty as to whether to insist on initial compressible linear portion to the stress-strain curve as physically necessary, but implying strict use of power law description is inappropriate, so that Illyushin's Theorem, variables separable and similar attributes such as Eqns. 4 and 9 are not truly applicable, or whether to waive the initial linear elastic behaviour, thereby gaining strict acceptance of the above virtues, but losing a strict reduction to lefm when plasticity is vanishingly small.

The further complexities of fracture behaviour that is both strain rate sensitive and bi-modal on the microscale, as commonly found in the structural steels and bcc metals is recognised as one of the major problems of fracture mechanics not yet adequately explained in the transition region where either mode is possible. The present usage of ideal and real material still excludes this behaviour unless explicitly mentioned.

FRACTURE BEHAVIOUR

Under conditions of strict lefm plane strain, four physical quantities appear to coincide, exactly in the rigorous case of elastic behaviour, no doubt less

exactly so in reality. These four quantities are

i) the attainment of a crack tip field of characteristic severity, K

ii) the attainment of a crack tip strain energy density, G, related by

$$K^2 = E'G \qquad \text{Eqn.11}$$

where E' is the effective tensile modulus

iii) the attainment of an overall work absorption rate, $G = \partial w/Bda|_Q$

iv) the attainment of an energy release rate, $G = -\partial w/Bda|_q$

Even in lefm some of these can cause separate physical effects. As is well-known, a 'remotely' loaded plate has a stress intensity of the form of Eqn.6, where K increases with crack length, whereas for the crack-line loaded wide plate K decreases with crack length. If crack growth commences at a critical value of K, then for a "neutral rate material" unstable growth occurs in the former configuration, whereas in the latter it does not. The reason, of course, is clear; in the latter case where K or G decreases with crack growth, further external work must be done to extend the crack. Thus, despite the mathematical identity between K and G in Eqn.11, even in lefm the physical distinction is invited

a) onset of crack growth occurs at a characteristic severity of crack tip field

b) unstable crack growth occurs under a favourable balance of energy rate, which for a neutral material requires dG/da to be positive.

It is suggested that this physical distinction is crucial for describing fracture in plasticity where the direct link between characteristic severity and energy rates is lost because of the existence of plasticity as an alternative mode of dissipation of energy.

Onset of Crack Growth. It is clear that in nle or ideal epe with the z direction stress or strain and the hardening exponent specified, the whole crack tip stress and strain field is unique in pattern and thus specifiable by the one term J that defines the intensity of the field (i.e. the amplitude of the singularity). For a neutral rate material onset of crack growth would therefore start at a particular value of J, J_i, which value would be a function of degree of plane stress or plane strain but not of configuration or degree of deformation. There is considerable evidence in the literature both for and against such a condition being approached for real materials, but no certainty as to when or where it is attained to a specified degree of accuracy. This is partly because of the difficulty of identifying the onset of separation experimentally, an uncertainty of the minimum size of notch depth or remaining ligament necessary to ensure the characteristic J field, a neglect of an adequate ratio of B/b to maintain plane strain as net or gross section yield is approached and sometimes the use of an inadequate formula to relate J to the load, displacement or work being measured. With all these features accounted for there remains the difference between real material and the ideal epe material for which J_i is a rigorously applicable criterion and it is this latter difference that will demand experimental study even when the theoretical restrictions are satisfied.

For the truly one parameter crack tip stress pattern a measurement of Cod would suffice as an alternative to J as a measure of stress field intensity at onset of crack growth. Despite the more extensive use of Cod rather than J in design and casualty analysis the writer prefers J for the "backbone" model for several reasons. Cod is difficult to define in other than the Dugdale model; it is more difficult to estimate or measure from far field terms than J and it seems likely that as the true one-parameter field is lost the term J, which in essence contains both δ and the somewhat variable term m from Eqn.1 might go some way towards accommodating variations in constraint.

The use of J in the first sense of defining a truly one-parameter crack tip field is, of course, directly analagous to the physical meaning of K in lefm, (i) above. The use of J as a measure of crack intensity averaged in some way when the patterm is in fact different, is analagous to the energy density meaning of G, (ii) above except that in other than nle this measure is of work absorbed, not energy recoverable.

An Increment of Crack Growth.

Initial crack length. By using test pieces with cracks initially cut to longer lengths, the meaning (iii) above of work absorption rate was used by Begley & Landes (1972), in their well known procedure for estimating J from experimental load-displacement records. This procedure is rigorous for nle or ideal epe behaviour. For real material behaviour the relationship between J evaluated by contour integral and by difference in areas under the loading curves for bodies with initial cracks of slightly different lengths has been checked, to within numerical accuracy, in several of the early studies of J by finie element methods, such as Boyle (1972), Sumpter (1974) and others. Nevertheless, the value and meaning of J is open to question for other than the nle or ideal epe loading diagram. For a configuration in which triaxiality is lost as net section yield is approached, the loading diagram will fall below the nle case for which triaxiality is maintained, as sketched, Fig.4. The evidence is that this difference

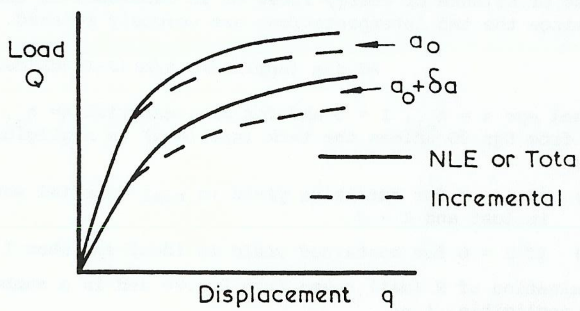

Fig.4. Differences in plastic constraint according to the stress-strain law affect the load-displacement diagram for some configurations.

is but little affected by small differences in initial crack length. Thus, J based on w via Eqn.2 will be different if the value of η used is $\eta_o = \eta_{el}$, as valid for nle. This reflects the fact that a given crack tip stress state in a given material corresponds to a given value of J, exactly for nle or ideal epe, and apparently so for practical purposes in real material. The converse, however, only holds for the former cases. The lack of a stress state unique for a given material and value of J in the real case is associated with the breakdown of the variables separable behaviour. Thus, in the former, values of J derived from either w or $\partial w/Bda$ will ccincide because Eqn.4 is exact, whereas in the latter the two values will differ because Eqn.4 is not exact, although it seems to be reasonably correct for cases where the constraint does not vary with degree of

deformation. When η so varies it is clear evidence that Eqn.4 is not strictly valid, the one parameter system is lost, and the crack tip stress field is altering in pattern (triaxiality) as well as in intensity. It is thus both a weakness and strength of evaluating J from w that the relationship is more sensitive to changes in w (arising from change in triaxiality for a given configuration) than if J were evaluated from $\partial w/Bda$. It is again repeated that these subleties do not imply that J is "better" or "worse" for condition of high or low constraint. It is the datum whereby for a given value of J a meaning can be attached relevant to onset of fracture that may be lost when the constraint does not match the universal datum of lefm plane strain, itself a case of high triaxiality, and the ratio of BJ/w, or$(1/w)(\partial w/\partial a)$, i.e. η/Bb, reflects these changes of constraint because they affect w.

The growing crack. If the increment of crack length occurs under slowly rising load, as distinct from cracks initially cut to longer lengths, then the growth is obviously stable and the rate of absorption of work, dw/Bda, must be greater than the rate of release of energy in the system. Inertia and damping are here neglected so that a simple static balance of energy rate is in question, i.e. for stable crack growth

$$I < \partial w/Bda|_q \qquad \text{Eqn.12}$$

Both terms are functions of length, since w is obviously so and I depends on deflection (i.e. gauge length, as seen Eqn.10a). Thus, compliance of the structure, ϕ_s, can be represented as an effective gauge length d or span S of the component

$$\phi_s = D/BEW \qquad \text{Eqn.13}$$

where D is the length d for tension or S/9, in bending. A rising load is then necessary to drive a stable crack in order to overcome the inequality of Eqn 12. The overall change dw must encompass both external energy, dU, and partial recovery of internal (strain) energy for nle material and J for use after crack growth was defined in this way by Garwood, Robinson & Turner (1975), so that, Fig5,

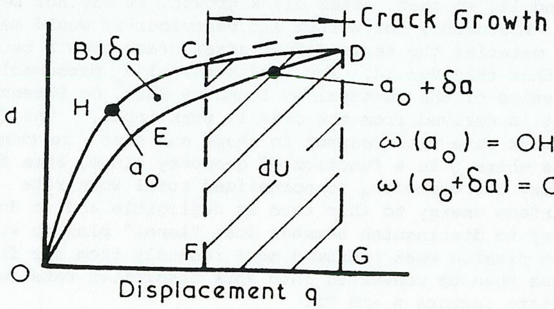

Fig. 5. External, recoverable and net work for nle material.

$$dw = dU - BJda \qquad \text{Eqn.14}$$

This should perhaps be further amended here for epe behaviour to

$$dw = dU - BIda \qquad \text{Eqn.15}$$

so that for ideal epe behaviour where I = G this reduces to

$$dw = dU - BGda \qquad \text{Eqn.16}$$

For a power law relationship between load and displacement

$$Q = Aq^n \qquad \text{Eqn.17}$$

then

$$w = Qq/(n+1) \qquad \text{Eqn.18}$$

and for nle behaviour dw can also be written

$$dw = Qdq/(n+1) \qquad \text{Eqn.19}$$

leading to the well-known lefm case that at constant load Eqn.14 reduces to

$$dU = Qdq; \quad dw = Qdq/2; \quad BG = Qdq/2 \qquad \text{Eqn.20, a,b,c}$$

The resistance to slow crack growth is of course the material and configuration dependent term dw/Bda. After initiation most experimental data show an apparent J value

$$J_{app} = \eta(w+dU)/Bb \qquad \text{Eqn.21}$$

A corrected value based on (w+dw) was shown (Turner,1980b) to be

$$J_{corr} = J_{app}\,[1-\Delta a(\eta-1)/b] \qquad \text{Eqn.22}$$

whence for DNB pieces with η=2 (and approximately so for compact tension)

$$J_{corr} \simeq J_{app}\,[1-\Delta a/b] \qquad \text{Eqn.23}$$

A more rigorous derivation of a resistance curve was given by Garwood, Pratt & Turner (1978). Such resistance curve of apparent toughness R versus crack growth Δa is conceptually independent of configuration. By differentiating Eqn.2

$$dw/Bda = (b/\eta)(dJ_r/da) - (J_r/\eta)f_1(\eta) \qquad \text{Eqn.24}$$

where by definition

$$f_1(\eta) \equiv 1+(b/\eta)(d\eta/da) \qquad \text{Eqn.25}$$

In the foregoing the suffix r denotes that the value of J, i.e. J_r, is derived from Eqns. 2 and 14, so that, after crack growth, it may not be the true contour integral term. Presumably for strict nle behaviour it would be but for real or even ideal epe material the terms differ after crack growth because Eqn. 15 or 16 is followed rather than Eqn.14. For real material J_r presumably corresponds with the far field value of the integral as found by Shih, de Lorenzi and Andrews (1979) since it is derived from the overall work input. The factors η and Bb normalise the work rate with respect to shape and size, so that for the nle and ideal epe cases where η is a function of geometry alone, then the R curve represents a property corresponding to normalised total work rate. The contribution of the true surface energy to this term is negligible and it does not seem feasible or necessary to distinguish between some "local" plastic work ascribed to "toughness" and plastic work consumed more remotely from the fracture surface. This R curve can then be converted into work absorption rate for a component by use of appropriate factors η and Bb.

Unstable Crack Growth.

It follows from Eqn.12 that unstable crack growth will occur (Turner, 1979b) when

$$I > \partial w/B\partial a)|_q \qquad \text{Eqn.26}$$

where $(\partial w/B\partial a)|_q$ is the total dissipation in the system, at fixed displacement overall. As already noted, both terms are functions of length so that a fixed displacement condition can include the compliance of the structure or testing machine plus test piece through use of an effective gauge length. Work may therefore be done on the test piece whilst not being external to the system, so that Eqn.24 is used to express the dissipation rate with the remainder of the system assumed elastic. As shown by Turner (1980a) the relationship between G and $(\partial G/\partial a)|_Q$ is

$$(\partial G/\partial a)|_Q = (G/b)(f_1(\eta)+\eta) \qquad \text{Eqn.27}$$

A similar form of expression relates J to $\partial J/\partial a$ for the nle power law hardening case. Differentiating Eqn.4 also gives a similar form of relationship betwen d^2w/da^2 and dw/da. Thus, at least for the two limits of nle (where I = J) and ideal epe (where as already seen, I = G) with Eqn.4 strictly correct, then Eqn. 26 can be reduced to the Orowan second derivative requirements for instability (Orowan, 1956)

$$\frac{dp^2}{da^2} > \frac{d^2w}{da^2} \qquad \text{Eqn.28}$$

where p is energy available of which G or J is the first derivative. For real material where I falls between G and J it is not clear whether the reduction from Eqn.26 to Eqn.28 will be exact. As discussed (Turner, 1980c) for strict lefm Eqn. 26 is the same as the well known tangency construction, whereby instability occurs when

$$\partial G/\partial a \text{ (applied)} > dR/da \text{ (material)} \qquad \text{Eqn.29}$$

or for strict nle

$$\partial J/\partial a \text{ (applied)} > dR/da \text{ (material)} \qquad \text{Eqn.30}$$

The R curve must of course be evaluated in terms of G or J as appropriate, and the applied term can be at either constant load or displacement, as required. The implication is that for these purely elastic cases instability can be interpreted as either loss of balance of energy rates or an inbalance of the characterising parameters, since the two interpretations are uniquely related. In general Eqn.26 reduces to

$$\partial J_r/\partial a \text{ (applied)} - (\eta/b)(J-I) > dR/da \text{(mat.)} \qquad \text{Eqn.31}$$

where for ideal epe $\eta = \eta_{el}$, I = G and for real material $\eta = \eta_o$, G < I < J. This differs from Eqn.30 unless the term $(\eta/b)(J-I)$ is negligible. This is so in the two cases

a) if $I \simeq J$ for extensive yield in real material where constraint is lost and $I \to J$

b) if $J \simeq G$ for contained yield in ideal epe when I = G

Strictly, evaluation of R (mat) comes from Eqn.24 and in a number of cases the last term is negligible, i.e.

$$\frac{b}{J}\frac{dJ}{da} \gg f_1(\eta) \qquad \text{Eqn.32}$$

As noted (Turner, 1980b) $f_1(\eta)$ is often, but not necessarily, of the order of unity.

This expression thus appears as a generalisation of the Hutchinson & Paris (1979) term $\omega \gg 1$ (where ω was defined as (b/J)(dJ/da) which was introduced to ensure that a J field was maintained by restricting the amount of stable crack growth permissible. Clearly, such an argument implies a unique J field existed prior to crack growth, but as already noted, there is some doubt whether this is so for some configurations in real material. The strict existence of the one parameter J field of known significance (i.e. high constraint) implies either nle or ideal epe behaviour, in which case the energy rate analysis seems rigorous, provided the second term is not neglected in Eqn.24, and the appropriate values are used for η and I. For real materials Eqn.24 is still correct but appropriate values for η and I are not certain. There thus seems no reason to restrict the instability analysis to $\omega \gg f_1(\eta)$ since loss of a high constraint J field, if it occurs, is accommodated on the estimates of η and I, however good or poor they may prove to be, so the existence or otherwise of a particular field characterised by J seems rather less important subsequent to growth if energetic arguments are used rather than prior to growth where characterising severity is used.

PRACTICAL USE OF THE J-BASED THEORY

R-Curves

In the experimental derivation of an R-curve for other than nle or ideal epe there is some difficulty in assigning the correct value to η if it is not independent of degree of deformation for the configuration in question. In comparing R-curves for different circumstances there are several uncertainties, such as the effect of material anisotropy, the effect of configuration on the plastic work absorption, itself usually comprising both a flat fracture and shear lip component, and the effect of degree and deformation. Anisotropy may be of great practical importance (Willoughby, Pratt & Turner, 1978) but is neglected in the present discussion. For the true nle and ideal epe cases where η is a function of configuration alone, then η and Bb are the only terms required for the normalisation of w to a geometry independent R-curve. A single R-curve has been shown to exist for pieces that were initially geometrically similar but of different absolute size (Garwood, Pratt & Turner, 1978). The similarity is of course lost as the crack grows since R is a function of absolute growth, Δa. This evidence relates to a case where η is well known to be independent of configuration, i.e. deep notch bending (where $\eta = 2$ over the range in question) used with deep side grooves to eliminate shear lip. Data also exist suggesting that the R-curve derived from configurations such as centre cracked tension (CCT), or a part through crack in tension (PTCT) differ from the deep notch bend case (DNB) (Garwood, 1978). The writer suggests that apart from possible effects of anisotropy, a major question is whether the values of η used in the analysis are correct. The usual analysis takes elastic and plastic components of both work and η to write

$$BbJ = \eta_{el} w_{el} + \eta_{pl} w_{pl} = \eta_o w_o \qquad \text{Eqn.33}$$

but as discussed (Turner, 1980a) the value of η_o is significantly less than η_{pl} until quite large deformations are reached and it is likely that in deriving an R-curve from deformations only a few (3 or 4) times yield a value for η_o nearer η_{el} is to be preferred. As an example, if the load displacement diagram in tension is substantially linear-non-hardening, then at a deflection $q = Nq_Y$ (say), $w_{pl} = 2(N-1)w_{el}$. If $\eta_{el} = 0.4$ (say) and $\eta_{pl} = 1$, then the first estimate of η_o is

$$\eta_o = (0.4 + 2(N-1))/2(N-1) \qquad \text{Eqn.34}$$

giving N =	1,	2,	3,	4
η_o =	0.4	0.8	0.88	0.91

whereas computed results for a particular tensile plate (Turner, 1979c) shows

η_o =	0.4	0.5	0.6	0.65

so that dJ_r/da derived using Eqn.34 for tension data might be too large by a factor of some 1.5 or two-fold in relation to a case (such as deep notch bend) where η is invariant with degree of deformation. The further problem to which an answer does not yet seem forthcoming is the variation of shear lip and the work consumed therein, with configuration. As seen Fig.6, the R-curve for full thickness, including both flat fracture and shear lip, differs for bend pieces of different proportion B/W, for which the shear lip component changes with both absolute size and B/W ratio. For very ductile behaviour the shear lip shows some tendency to follow the slip line pattern and it might be plausible to expect the value of η to be comparable to that for plastic flow and collapse, itself related

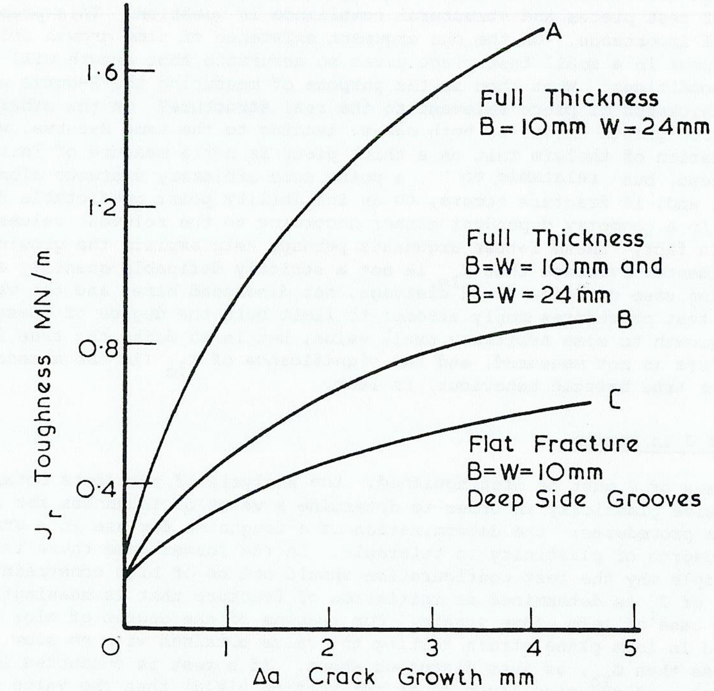

Fig.6. Effect of size and shear lip on J_r curves (DNB pieces: En 32 steel).

to slip line patterns. In that case η might still provide a rough value for normalising work to the J_r curve. For moderate amounts of ductility the shear lip effect seems related to the fracture ligament and some small volume of material adjacent to it and even in bending is more akin to local tensile tearing. In that case there seems no reason why the value of η for plastic bending should be relevant to local tensile tearing, and thus the means of normalising to overall work dissipation is lost. As shear lip becomes negligible and small scale yielding is approached, the elastic value of η dominates and normalisation is again possible to a G or J-R curve.

The relationship of a J_r curve (i.e. an R-curve derived in the plastic regime from overall work dissipation rate) to a G-R curve is not clear. The common usage of G-R curves for thin sheet implies a predominant effect of shear lip and it is the shear lip component in plasticity that is least amenable to normalisation. At the other extreme of full plane strain and rigorous lefm behaviour, the writer believes there is no R-curve, or only a schematic curve with a vertical and horizontal limb, since there is no dissipation other than surface energy. If, however, only plasticity modes of separation are permitted (e.g. micro-void coalescence, not cleavage) then it may be argued that a degree of plasticity and slow crack growth will occur, giving a slight curvature to the load-deflection diagram even in lefm, such that the R-curve is the same as in a small component, although the termination is governed by the instability condition relevant to the size of test pieces and structural compliance in question. This problem seems of crucial importance. On the one argument existence of slow growth and measurement of R-curve in a small test piece gives no assurance that growth will occur under lefm conditions. What then is the purpose of measuring the R-curve unless it is on a thickness of piece relevant to the real structure? On the other argument, some slow growth exists in both cases, leading to the same R-curve, whence the termination of the lefm test on a thick piece is not a measure of initiation toughness, but relatable to a point some arbitrary distance along the R-curve, and, if fracture occurs, to an instability point predictable from the R-curve in a geometry dependent manner according to the relevant values of η and Bb. In fact, these latter arguments perhaps help explain the growing amount of experimental evidence that K_{Ic} is not a strictly definable quantity except in the limiting case of below yield cleavage, not discussed here, and the various practical test procedures simply attempt to limit both the degree of plasticity and slow growth to some arbitrary small value, but in so doing the true initiation toughness is not measured, and the significance of K_{Ic} (in the absence of cleavage or true brittle behaviour) is lost.

Use of J in Design

Two uses of J must be distinguished: the analysis of test data obtained with extensive plasticity in order to determine a value of toughness for use in lefm design procedures; the determination of a toughness for use in a structure where some degree of plasticity is tolerable. In the former case there is no reason in principle why the test configuration should not be of high constraint so that a value of J_i is determined at initiation of fracture that is meaningful for the datum case of lefm plane strain. For reasons of the degree of slow growth permitted in lefm plane strain testing the value obtained with no slow growth will be less than G_{Ic}, as just discussed above. If a test is conducted in a low constraint configuration (such as at net section yield) then the value of J derived might be appreciably greater than found in the lefm plane strain situation with its high constraint.

For use in design where plasticity is tolerated, several more questions arise. How shall the applied severity of J or Cod or other be estimated, particularly if regions of stress concentration and residual or thermal stress are present? How severe a loading condition shall be acceptable and is there need to distinguish between a load or displacement representative of regular use, casual abuse and continued re-use (albeit not fatigue unless explicitly considered) and a more severe condition representative of avoidance of catastrophic failure even if the component is damaged intolerably and must be replaced? In brief, the regular use case implies the restricted degree of yielding associated in non-crack cases with shakedown, whereas the avoidance of catastrophe allows usage up to the limit state provided the structure still "hangs together". The considered distinction between the severity of loading that a structure should withstand, or for which separation is inevitable, is probably the most important aspect of safety. Nevertheless, it will not be discussed here since the arguments are usually peculiar to the object or its condition of usage and cannot be generalised into the treatment of the subject of fracture. Thus, although a "J design curve" has been discussed (Turner, 1979c) in terms similar to that for the well known Cod design curve, it would be more profitable to view the curve only as a procedure for estimating J with no implication on the value of load or deformation at which the curve is entered for usage.

The curve that has been proposed is shown Fig. 7. The equations fitted to the upper edge of related computed cases are

$$J/G_Y = (e/e_Y)^2 \quad \text{for } e/e_Y < 0.85 \text{ (lefm)} \qquad \text{Eqn.35a}$$

$$J/G_Y < 5\,[(e/e_Y) - 0.7] \quad \text{for } 0.85 < e/e_Y < 1.2 \text{ (contained yield)} \qquad \text{Eqn.35b}$$

$$J/G_Y < 2.5\,[(e/e_Y) - 0.2] \quad \text{for } e/e_Y > 1.2 \text{ (uncontained yield)} \qquad \text{Eqn.35c}$$

where $G_Y = Y^2\sigma_Y^2 a/E$

It will be noted that this curve is normalised by use of $JE/\sigma_Y^2 a$ and the dependence on geometry is partly eliminated by use of the lefm shape factor Y, so that the term $JE/Y^2\sigma_Y^2 a$ is necessarily but a single line in the lefm regime. Correction for the size of plastic zone could be applied in this region if desired. If the factor Y were independent of deformation, then of course, a single curve would exist in the plastic regime. That this is not so in terms of applied load was seen, Fig. 1, and that it is not so in terms of applied deformation (in the absence of a crack) is seen, Fig. 7.

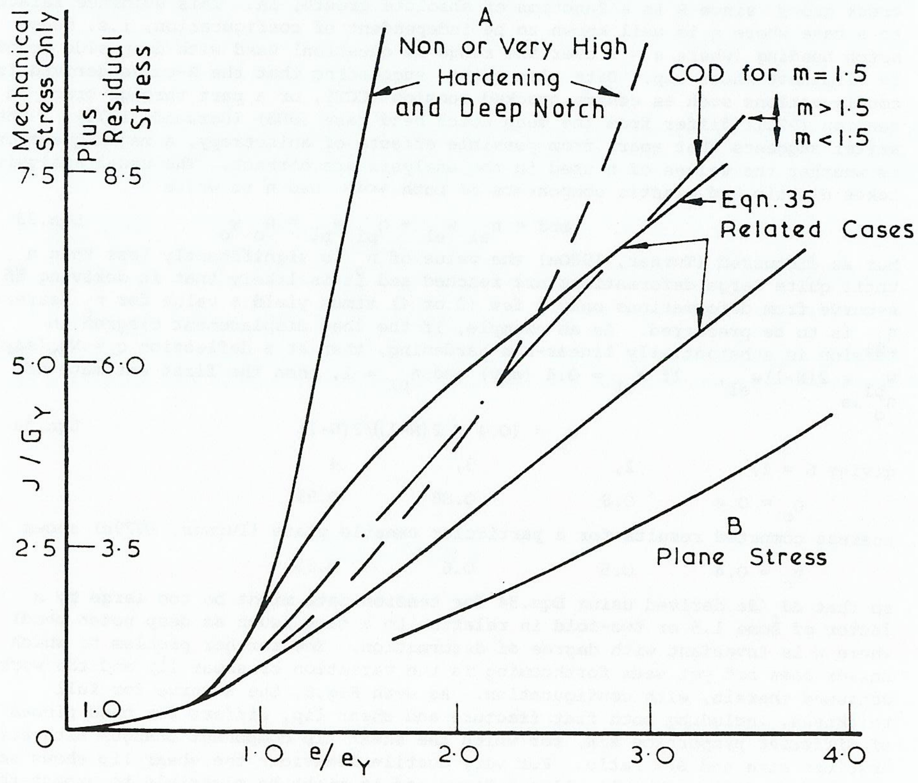

Fig.7. Estimates of J for various groups of configurations: either mechanical stress only or also superposed yield level residual stress.

For ideal epe material the estimation curve can of course be expressed as a unique straight line in terms of work rather than load or displacement, using Eqn.2.

$$J/\eta = w/Bb \qquad \text{Eqn.36}$$

If a power law expression, Eqns. 17 and 18, is used where A = f(a/W) to represent compliance, then Eqn.36 can be written down in terms of either Q or q as required. There remains the estimation of n and A for a given component and material and the selection of η_{el} or η_o according to the assumptions made, but the formal existence of a unique J-based estimation curve is thus demonstrated.

The cases that group together on or just below the given curve (Turner, 1979c) are predominantly shallow notch with mild work hardening behaviour representative of structural steel, where deformation in extensive yield of the gross section is possible because

$$\sigma_Y W < \sigma_{fl} (W-a) \qquad \text{Eqn.37}$$

where σ_{fl} is a work - hardened flow stress. Where this is not possible for reasons of negligible work hardening or of deep notches (i.e. net section yield is possible but not gross section yield) then any plastic deformation is concentrated into the notch and J increases more rapidly, as sketched, Fig.7, Region A. This region also encompasses the opposite extreme of very high hardening, or, in the limit, continuation of lefm, because in such cases the load rises correspondingly with deformation so that J also increases rapidly. Conversely, if the component remote from the crack is in plane stress, then data falls in Region B. Clearly, discussing fracture with yield of the gross section will for some cases be very conservative. Such judgments must be related to the object and service conditions under discussion. Much of the original wide plate data on which the Cod design curve is based fall in that regime; some of Soete's proposals fall in that regime; design against catastrophe (i.e. gross yield at limit state before tearing) would also fall in that regime. However, for many structures in service the two important practical cases are cracks at regions of stress concentration and residual stresses. Both can be accommodated by the above curve, just as in the Cod curve, but are not the basic cases on which either curve is based.

For stress concentrations the somewhat arbitrary rule recommenced for the Cod procedure is also relevant here, on the basis of rather limited data. The procedure is to take the applied strain as ke where k is the elastic stress concentration factor (SCF) in the uncracked body. The two circumstances that the crack extends into a region of lower stress, but that the elastic analysis breaks down at yield, whereafter the strains are concentrated more than the stresses, appear to cancel each other out approximately, although it is not clear whether cases exist where that is not so. An alternative treatment using Neuber's concept of $k^2 = k_\sigma k_e$ where k is the elastic SCF, k_σ and k_e are plastic stress and strain factors was proposed by Begley, Landes and Wilson (1974) but the complications of making the estimate do not seem to justify any extra rigour that there might be.

For residual stresses a three-stage procedure has been proposed (Turner, 1980d) simply overlooking the fact that J is not then strictly relevant. The stages are: allow for any metallurgical effect on toughness; allow for any reaction stress system as an applied load; assume local self-equilibriating residual stresses might be of yield stress magnitude and alter the ordinate of $JE/Y^2\sigma_Y^2 a$ by unity. The original argument was based on stored strain energy and effective values of η for cracks, longitudinal or transverse to a weld, but the procedure is equivalent to adding a yield stress level stress, i.e. a value $G = Y^2\sigma_Y^2 a/E$. The procedure is significantly less demanding than the Cod procedure of moving along the abscissa e/e_Y by unity for yield level residual stress, but in severe cases that difference would be offset in part by the first two steps in the three stage procedure here advocated.

There is little direct evidence in support of a J based design curve, since there are few published case studies that include J data. In broad terms, because of the relationship between J and δ, (Eqn. 1) any data that supports the Cod curve also supports the J curve. In reality, the variation in the factor m does not permit a close comparison. To indicate the general trend of the relationship between the two curves, a line is shown, Fig.7, for m = 1.5 and using $Y^2 = 4$ as typical for many shallow notch cases. A more detailed discussion of the J design curve and a few case studies will be offered elsewhere.

Instability

In all the foregoing it is implied that the critical value of J would be an initiation value, J_i, although in the Cod procedures it has long been accepted that a value of δ beyond initiation, perhaps up to maximum load, can be used by agreement. The possiblity of making a complete instability analysis now exists, in terms of I, as here, or in terms of Paris's tearing modulus T (Paris and others 1979), or dJ/da tangency to the R-curve (Garwood, 1976; Shih, 1980). If, for simplicity only the leading term of Eqn.24 is used, then the present analysis implies unstable growth by Eqn.26 when

$$I > \frac{b}{\eta}\frac{dJ_r}{da} \qquad \text{Eqn.38}$$

where G < I < J and I is estimated from Eqn.10. As already outlined, for nle material, this reduces to the dJ/da tangency usage of Garwood (1976) and Shih (1980).

If Eqn.38 is rewritten

$$(\eta/b)\ (IE/\sigma_Y^2) > (E/\sigma_Y^2)\ (dJ_r/da) \qquad \text{Eqn.39}$$

then the T notation as used by Paris and others (1979) appears, i.e.

$$T_{app} > T_{mat} \qquad \text{Eqn.40}$$

There is the distinction that although in evaluating T the loading terms, such as D/W in tension, correspond in the two analyses, the uncertainty over the multiplicative coefficient is explicit in Eqn.39 (i.e. what value of η shall be used?) whereas it is lost, but nevertheless implicit, in the approximations of the various particular cases of Paris & others (1979). There is also the further distinction of use of J_r from Eqn.38 or use of "J taken beyond crack initiation" by whatever experimental procedures may have been used, such as Eqn.21. These differences are small for ω>>1. Where ω is not large, Hutchinson & Paris (1979) argue the T analysis should not be used because the dominant J field is lost. Thus, within the difference of assumptions and procedures already discussed, the balance of energy rates associated with the ideal epe case encompasses the various other proposals made, so that again there is no conflict other than the practical point of how best to accommodate real material behaviour.

A question of some interest is whether the instability arguments can be used to justify accepting a value of J or δ beyond the initiation value if a full instability analysis is not made, either to avoid the complexities thereof, or perhaps because R-curve data does not exist other than a terminal point of known toughness at fracture after stable crack growth of amount Δ_f. If I is estimated at the end of the test, using a lower bound if need be, then dJ_r/da can be estimated and an R-curve sketched if J_i is known. A yet simpler approximate procedure can be based on only a known end point of value J_f after growth Δ_f, Fig.8, or a lower bound thereto. In the test piece, t, from which J_f is estimated, unstable growth would have occurred when

$$I_t > J_{eff,\,t} \qquad \text{Eqn.41a}$$

$$J_{eff,\,t} = J_{r,f} \qquad \text{Eqn.41b}$$

where J_{eff} is the term dw/Bda defined by Eqn.24. This term is the decreasing R-curve, Fig.8. as discussed by Krafft, Sullivan & Boyle (1961) in connection with earlier literature and represents the effective toughness of a structure or test piece. Its dominant term comprises both the material property dJ_r/da that, of course, reduces with crack growth and also the geometry dependent terms Bb and η, together with a term in J_r and $f_1(\eta)$ that is small, whilst $\omega >> f_1(\eta)$ (Eqn.32). Thus, a structure, s, will sustain stable crack growth, whilst

$$I_s < I_{t,\,f} \qquad \text{Eqn.42a}$$

i.e.

$$I_s < J_f \qquad \text{Eqn.42b}$$

provided also

$$(b/\eta)_s > (b/\eta)_t \qquad \text{Eqn.43a}$$

and

$$(f_1(\eta)/b)_s < (f_1(\eta)/b)_t \qquad \text{Eqn.43b}$$

to ensure case (A) rather than (B), Fig.8.

The first proviso is usually met since normally both the ligament $b_s > b_t$, and if DNB or CT test pieces are used (where $\eta \simeq 2$) then $\eta_s < \eta_t$. The second proviso will certainly be met if the test piece and structure are of the same configuration with $b_s > b_t$ but for different configurations it may well be that $f_1(\eta)_s > f_1(\eta)_t$, so that to satisfy the proviso it would be necessary for $b_s >> b_t$. Thus, recalling $G < I < J$ the crack in the structure will clearly be stable if the provisos are met, and also

$$J_s < J_f \qquad \text{Eqn.44}$$

This may provide a very considerable margin over the crack initiation condition $J_s < J_i$ with yet further margin available beyond J_f if the provisos of Eqn.43 are met by a large margin. A reservation is that in estimating J_s as an upper bound to I_s for cases of epe behaviour with net section yield (such as non-hardening or deep notches, Regime A, Fig.7, as opposed to contained yield or shallow notch gross section yield for which Eqns.35 are relevant) then the effect of compliance must be allowed for since such cases J is length dependent (as in Bucci, Paris & Landes (1972), whereas for strict elastic behaviour (le or nle) this length effect does not arise for remotely applied loading. It must also be recalled that use of J_f must accommodate effects of anisotropy and shear lip on a lower bound basis as already discussed, and that neither the possibility of a change of micro-mode of separation during stable growth nor of time-dependent or environmental effects have been allowed for, except in so far as suitable experiments may have shown such behaviour improbable for the circumstances in question.

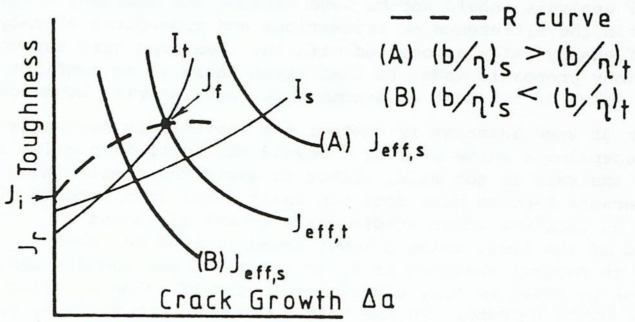

Fig.8. Effective toughness of a structure, J_{eff}, in relation to the final toughness of a test piece, J_f. Note J_i and R-curves are unknown except for the point of fracture, J_f (schematic).

CONCLUSIONS

By postulating an idealised elastic-plastic-elastic behaviour of material it is possible not only to describe the onset of crack growth by a one parameter system for which J is the most suitable candidate, but to extend the scheme to cover slow stable crack growth, R-curves in plasticity, unstable ductile fracture and J estimation procedures, both before and after slow growth in terms of a normalised work dissipation rate expressed in terms of J. The key is the separation of the variables of degree of deformation and configuration permitted by the ideal epe material. The penalty is the uncertain degree of realism to which only experiment can give a sure guide. At the present stage of development of elasto-plastic fracture mechanics the J-based theory, extended as described here, appears to be the most comprehensive basis on which to build a useful engineering tool that, despite its simplifications, gives good insight into fracture behaviour.

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