A CRITERION FOR THRESHOLD STRESS INTENSITY IN FATIGUE CRACK GROWTH

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ABSTRACT

Several models for threshold stress intensity have been proposed in literatures. For instance, the authors pointed out a lower limit stress intensity of ΔK exists for dislocation emission in the dislocation group dynamics theory of fatigue crack growth. After that, they presented a model for threshold stress intensity ΔK_{th} for fatigue crack growth corresponding to this limiting stress intensity. On the other hand, the experimental studies have been made on the effect of ferrite grain size on ΔK_{th}, and an experimental formula was proposed. In this article, another model was proposed based on the condition that it is necessary that lead dislocation after emitted from the source should reach the grain boundary. The mathematical formula for threshold stress intensity thus derived is in good agreement with the experimental formula with respect to ferrite grain dependence.

INTRODUCTION

Several models for threshold stress intensity have been proposed in literatures [1-3]. For instance, the authors pointed out [4,5,6] a lower limit stress intensity of ΔK exists for dislocation emission in the dislocation group dynamics theory of fatigue crack growth. After that, they presented a model for threshold stress intensity ΔK_{th} for fatigue crack growth corresponding to this limiting stress intensity [2,3]. On the other hand, the experimental studies [7,8,9] have been made on the effect of ferrite grain size on ΔK_{th}, and an experimental formula [9,10] was proposed. In this article, another model was proposed based on the condition that it is necessary that lead dislocation after emitted from the source should reach the grain boundary. The mathematical formula for threshold stress intensity thus derived is in good agreement with the experimental formula with respect to ferrite grain dependence.

MODEL

Previously by solving the unstable problem [11] of dislocation emission from the crack tip under applied stress, image stress and ledge stress, the authors derived [4,5,6] the dislocation emission condition and assumed the stress intensity satisfying this condition as threshold intensity ΔK_{th}. In the present article, we assumed another model, that is, the ΔK_{th} corresponds to the stress intensity required for the lead dislocation to reach the grain boundary after dislocation groups have been emitted from the crack tip (Fig.1). When the lead dislocation reaches the grain boundary, it is assumed that the microcrack will initiated by the piled up stress and the main crack extension will occur by joining this. The emission will be controled largely by the dislocation groups near by the source, and the resistance due to the grain boundary itself as an obstacle was neglected as a first approximation in this article. This effect will be a next subject.

ANALYSIS

In the following treatments on dislocation group dynamics with emission [12,13] we use the two fundamental relations. The first one is the power relation between the shear stress τ and velocity υ for each individual dislocation in a linear array:

$$\upsilon = \upsilon_0 \left(\frac{\tau}{\tau_0{}^*} \right)^m \tag{1}$$

where

m = material constant

τ_0^* = a constant representing the stress required to give a dislocation velocity υ = 1 cm/sec (the resistant stress against the dislocation motion),

υ_0 = 1 cm/sec,

The second equation is the equation of motion of the i_{th} dislocation in the array:

$$\frac{dx_i}{dt} = \upsilon_0 \left(\frac{1}{\tau_0{}^*} \right)^m \left[\dot{\tau} t + \frac{Gb}{2\pi(1-\nu)} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{1}{x_i - x_j} \right]^m \tag{2}$$

where

i = the index number of dislocation in order of the emission from the source,

x_i = the distance moved by the i_{th} dislocation in the array,

$\dot{\tau} \equiv d\tau/dt$ = the constantly increasing rate of applied stress τ (Fig.2)

G = the shear modulus,

ν = Poisson's ratio,

b = Burgers vector

From the results of calculations, the position $x_1(t)$ of the lead dislocation at any time emitted from the source near-by the crack tip is given by as follows [12]:

$$x_1(t) = a_1(\eta)\, x_{iso}(t) \tag{3}$$

where

$x_{iso}(t)$ = The position of an isolated dislocation at any time t emitted from the source near-by the crack tip under the constant applied stress rate $\dot{\tau}$.

$\eta \equiv \dot{\tau}^{\frac{m+1}{m+2}} t$,

t = time measured from the instant of stress application

On the other hand, x_{iso} is calculated from Eq.(1) as:

$$x_{iso}(t) = \int_0^t \upsilon_0 \left(\frac{\dot{\tau} t}{\tau_0{}^*} \right)^m dt = \frac{\upsilon_0}{m+1} \left(\frac{\tau}{\tau_0{}^*} \right)^m \frac{\tau}{\dot{\tau}} . \tag{4}$$

According to the calculations, a_1 in Eq.(3) is approximately as [12]:

$$a_1 \simeq 1.5 \qquad \text{for} \quad \eta \gg 1 \tag{5}$$

From Eqs.(3),(4) and (5), we get

$$x_1(t) = \frac{a_1}{m+1} \left(\frac{\tau}{\tau_0{}^*} \right)^{m+1} \frac{\tau_0{}^* \upsilon_0}{\dot{\tau}} . \tag{6}$$

In the present article also let us use the formula by Rice [14] as the local stress σ_ℓ caused by applied stress near-by the crack-tip, but initial yield stress σ_{cy} in cyclic stress strain relation, and cyclic strain hardening exponent β are used [4,5] instead of monotonic yield stress σ_Y and static strain hardening exponent λ, respectively. That is as follows:

$$\sigma_\ell = f(\beta)\, \sigma_{cy} \left(\frac{\Delta K}{\sigma_{cy} \sqrt{x}} \right)^{\frac{2\beta}{1+\beta}} \tag{7}$$

where

ΔK = stress intensity factor

$f(\beta) = [(\beta + \frac{1}{2})(\beta + \frac{3}{2}) \Gamma(\beta + \frac{1}{2}) / \Gamma(1/2) \Gamma(\beta+1)]^{\beta/1+\beta}$.

On the other hand, after dislocation emitted the movement is resisted by image force and ledge force, and, thus, the effective stress $\sigma_{\ell eff}$ exerted on the dislocation will be as follows:

$$\sigma_{\ell,eff} = f(\beta)\, \sigma_{cy} \left(\frac{\Delta K - \Delta K_i}{\sigma_{cy} \sqrt{x}} \right)^{\frac{2\beta}{1+\beta}} , \tag{8}$$

where ΔK_i is the limiting stress intensity for the dislocation emission from the source and corresponds to the larger value of the two critical stress intensities: $\Delta K_{Icr.stress}$ and $K_{Icr.energy}$ [2], and for iron it corresponds to $\Delta K_{Icr.stress}$ [2]. That is,

$$\Delta K_i = Gb/(1-\nu)\sqrt{\pi x} . \tag{9}$$

Substituting Eq.(8) into Eq.(6) using $\tau = \frac{1}{2}\sigma_{\ell,eff}$ and putting x_1 is equal to grain diameter d, then we get ΔK_{th} as follows:

$$\frac{\Delta K_{th} - \Delta K_i}{\sigma_{cy}\sqrt{\varepsilon}} = \left(\frac{2}{f(\beta)}\frac{\tau_0{}^*}{\sigma_{cy}}\right)^{\frac{1+\beta}{2\beta}} \left(d\frac{\dot{\tau}}{\tau_0^* \upsilon_0}\frac{m+1}{a_1}\right)^{\frac{1+\beta}{2\beta(m+1)}} \qquad (10)$$

where

ε = the small distance from the crack tip over which σ_ℓ is averaged.

Assuming the mechanism of initial cyclic yield stress σ_{cy} is similar to that of monotonic yield stress σ_Y in dislocation dynamics aspect, then we can express σ_{cy} as follows [15]:

$$\sigma_{cy} \simeq 2\tau_0{}^* \left(\frac{N^*}{\rho}\frac{1}{\gamma(m)}\right)^{\frac{1}{m}} \left(\frac{Gb\dot{\tau}}{\nu_0 \tau_0^{*2}}\right)^{\frac{1}{m}}, \qquad (11)$$

where

N^* = the specified number of the dislocation emitted

ρ = grown-in dislocation density per unit volume

$\gamma(m) = 1.396m^{-1.45}$

Substituting Eq. (11) into Eq. (10), we get

$$\frac{\Delta K_{th} - \Delta K_i}{\sigma_{cy}\sqrt{\varepsilon}} = \left\{\frac{1}{\{f(\beta)\}^m}\frac{m\gamma(m)}{a_1}\frac{\tau_0{}^*}{G}\frac{\rho}{N^*}\frac{d}{b}\right\}^{\frac{1+\beta}{2m\beta}}. \qquad (12)$$

Let us consider ε as the length corresponding to the region εb near the crack tip (Fig. 3) in which the number of dislocation reaches some critical value N_{crit}, that is, $N_{crit} = \beta N^*$ (β = constant). Denoting mobile dislocation density D_{md} as D^*_{md} corresponding to this, then

$$D^*_{md}\ \varepsilon b = \beta N^*. \qquad (13)$$

On the other hand, D_{md} is given by:

$$D_{md} = \frac{\{k(m)\}^2}{b^2}\left(\frac{b\dot{\tau}\ \tau_0^{*m}}{G^{m+1}\ \upsilon_0}\right)^{2/m+2} \qquad (14)$$

where

$$k(m) = \gamma(m)(m+1)/a_1.$$

Substituting Eq. (14) into Eq. (13), we get

$$\varepsilon = \frac{\beta N^*}{\{k(m)\}^2}\frac{b}{\left(\frac{b\ \dot{\tau}\ \tau_0^*\ m}{G^{m+1}\ \upsilon_0}\right)^{2/(m+2)}}. \qquad (15)$$

Substituting Eqs. (11) and (15) into Eq. (12), we get

$$\Delta K_{th} = \Delta K_i + M_0\left(\frac{\rho}{N^*}\frac{d}{b}\right)^{\frac{1+\beta}{2m\beta}}, \qquad (16)$$

where

$$M = 2G\sqrt{b}\left(\frac{\beta N^*}{k^2(m)}\right)^{\frac{1}{2}}\left(\frac{N^*}{\rho}\frac{1}{\gamma(m)}\right)^{\frac{1}{m}}\left\{\frac{1}{\{f(\beta)\}^m}\frac{m\gamma(m)}{a_1}\frac{\tau_0{}^*}{G}\right\}^{\frac{1+\beta}{2m\beta}}. \qquad (17)$$

Furthermore, it may be more reasonable to use the maximum value ΔK_i^* of ΔK_i as follows:

$$\Delta K^* = Gb/(1-\nu)\sqrt{\pi x_0}. \qquad (18)$$

x_0 = core cut off of dislocation, and Eq. (16) may be written as

$$\Delta K_{th} = \Delta K_i{}^* + M_0\left(\frac{\rho}{N^*}\frac{d}{b}\right)^{\frac{1+\beta}{2m\beta}}. \qquad (19)$$

For iron, let us use $m=10$, $\tau_0^*=1.972 \times 10^2$ MN/m^2, $G=7.943\times10^4$MN/m^2, $b=3\times10^{-10}$m, $\beta=0.11$, $\rho=10^{10}$/m^2, $N^*=10^{15}$/m^2 as reasonable values, respectively, and assume $\varepsilon=1.12\times10^{-4}$m. Then from Eq. (19) we get the following formula for ΔK_{th}:

$$\Delta K_{th} = 1.05 + 1.14 \times 10^3 d^{\frac{1}{2}} \qquad (20)$$

in MN/m$^{3/2}$ unit.

This equation is in good agreement with the following experimental formula [9]:

$$\Delta K_{th} = 3.8 + 1.14 \times 10^3 d^{\frac{1}{2}} \qquad (21)$$

in MN/m$^{3/2}$ unit.

DISCUSSION

Taking ΔK_{T_0} as corresponding to stress ratio, R=0, then $\Delta K_T = K_{max} - K_{min}$ is expressed as: $\Delta K_T = K_{T_0}(1-R)$ [9]. Thus with respect to mean stress, Eq. (19) may be written as: $\Delta K_{th} = (1-R)(\Delta K_i{}^* + Md^{1/2})$.

The first term $\Delta K_i{}^*$ in Eq. (19) may be considered as corresponding to ΔK_{th} proposed based on the model [2] in which rate determning process is for spontaneous emission of a dislocation from the crack tip. The first term $\Delta K_i{}^*$ in Eq.(20) is smaller than that in the experimental formula [9] Eq. (21). This may be due to that in the analysis based on the model [2], source activation stress τ_s at the source is neglected, and, actualy after the exerted effective stress τ_{eff} becomes larger than τ_s, emission of dislocation will occur.

ON THE REASON WHY CYCLIC LOADING IS NECCESSARY FOR START OF CRACK EXTENSION ?

It is very salient feature why cyclic loading is neccessary for starting of crack extension. In other word, it is very important problem to clear whether the value of threshold stress intensity is the same or not for cyclic loading (i.e. fatigue test) and for montonic and single loading (i.e. static tensile test). Next consider this problem.

It is reported that the emission number of dislocations from the source is some finite value [16]. On the other hand under cyclic stress dislocation density increases and sub-grain volume decreases with increase of repeated number and saturates to some value [17]. From these results, it may be reasonable to assume that activation stress τ_s will decrease with increase of the number of repeated cycles, and will probably saturate to some value. Since no attempt has been made to formulate this problem, in a first approximation, let us express this effect as follows:

$$\tau_s = \tau_s{}^* + \frac{\tau_{s0}}{(1+aN)^{\delta}} \qquad (22)$$

where N=the number of repeated cycles. $\tau_s{}^*$= asymptotic value of τ_s. τ_{s0} , a (>0) and δ (>0) are constants.
Thus, as mentioned above, the spontaneous emission of dislocation from the source will not occur until the exerted stress intensity exceed $\Delta K_i + \tau_s\sqrt{\varepsilon}$. That is, in order for the dislocation to be able to emit, the repetition of load cycle is needed for τ_s to decrease according to, for instance, Eq.(22).

In this way, more exactly the criterion for the threshold stress intensity will be:

$$\Delta K_{th} = \Delta K_i + \tau_s{}^* + M_0 d^{\frac{1}{2}} \qquad (23)$$

instead of Eq.(16). The physical significance of $\tau_s{}^*$ may be a future problem needed to study.

CONCLUSIONS

The following conclusions are obtained.

(1) A criterion for threshold stress intensity has been derived as:

$$\Delta K_{th} = \Delta K_i + M_0 \left(\frac{\rho}{N^*} \frac{d}{b} \right)^{\frac{1+\beta}{2m\beta}} ,$$

where d is ferrite grain diameter.

(2) The criterion is in good agreement with the experimental formula with respect to ferrite grain diameter.

(3) Assuming that the source activation stress will decrease with increase of the number of cyclic load repetition, it can be explained why some number of the cyclic load repetition is needed for starting of the crack extension. It is the salient feature for ΔK_{th} **different** from the situation **in** the case of monotonicaly single loading test.

(4) It is needed to study on the effect of τ_s in terms of more detailed model upon the criterion for ΔK_{th}.

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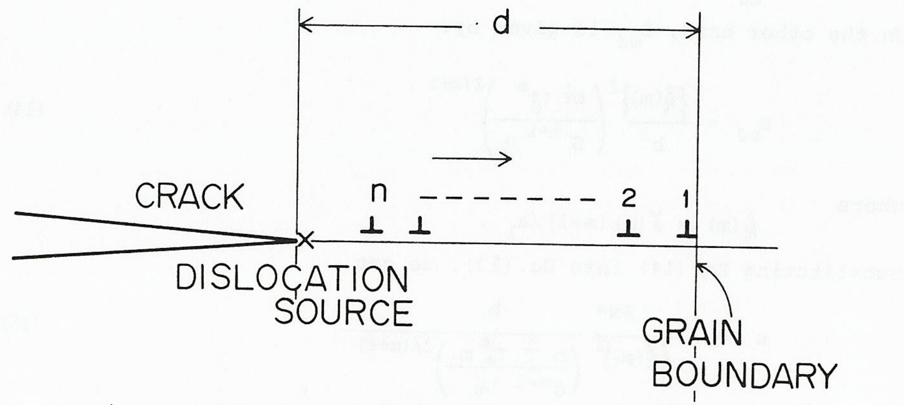

Fig. 1. Emission of dislocation from the source.

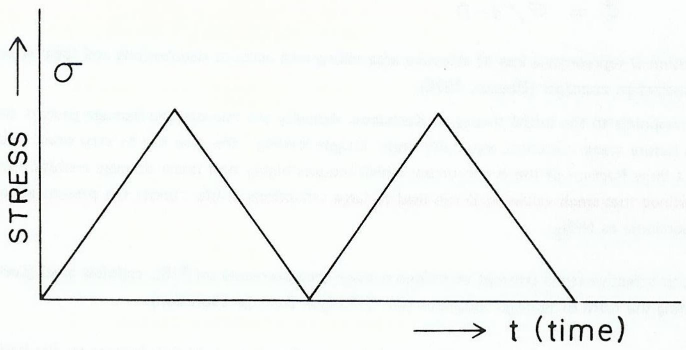

Fig. 2. Cyclic stress by applied load.

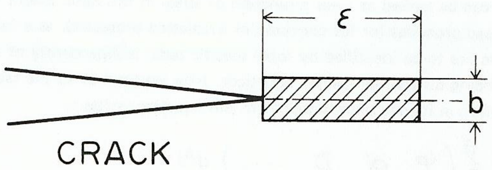

Fig. 3. The high dislocation density area near-by the crack.