THE RELATION BETWEEN MICROSTRUCTURAL FRACTURE PROCESSES AND MACROSCOPIC CRACK TIP CHARACTERIZING PARAMETERS DURING THE STABLE GROWTH OF CRACKS

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ABSTRACT

One of the most rapidly developing areas in the fracture field since the last International Fracture Conference concerns the stability and eventual instability of cracks propagating in ductile materials. The stability of a crack growing under plane strain conditions, has been described in terms of the material's J integral - crack growth (Δc) resistance curve. By comparing these with the system's J-Δc curve for various initial crack sizes and for different loading patterns, it is then possible to assess whether a crack grows stably or unstably, and also when a stably growing crack eventually becomes unstable.

Such a description of stable crack growth is essentially macroscopic, and the present paper focuses on the fracture processes which operate in a process zone in the immediate vicinity of a crack tip, and how this localized behaviour correlates with the macroscopic description. The link between the two scales of description is made via Cottrell's dislocation description of cumulative and non-cumulative fracture processes and Wnuk's use of the Dugdale-Bilby-Cottrell-Swinden (DBCS) model.

The main conclusion arising from the investigation is the clear demonstration that crack growth behaviour is very sensitive to the nature of the localized fracture processes, even in situations where there is extensive plastic deformation. Furthermore, the paper provides a framework for a discussion of the effect of microstructural factors on crack stability, for example the localization of deformation within flow bands, a process that has been observed in many high strength materials, and which encourages unstable crack growth.

KEYWORDS

Microstructural fracture processes; fracture characterizing parameters; stable crack growth; J integral descriptions.

INTRODUCTION

When a perfectly elastic solid deforms under plane strain Mode I conditions, the crack extension condition is $J = 2\gamma$ where γ is the free surface energy, and the J integral is related to the crack tip stress intensification K via the relation $EJ = K^2(1-v^2)$ where E is the Young's modulus and v is the Poisson's ratio. The crack stability-instability condition is then derived by determining J in terms of the crack length c and the applied loads or displacements. The characterization has been extended to crack propagation in thin metal sheets (plane stress deformation) for small scale yielding conditions. The ensuing crack growth analyses are based on the assumption that for each material there is a unique relation between J and the crack growth increment Δc. By calculating J as a function of crack length c for various applied loads or displacements, assuming that the material is perfectly elastic, the resulting relations for the system are matched with the material's J-Δc relation. This enables the applied load (or displacement) to be determined as a function of the crack growth increment for a prescribed initial crack size, thereby allowing the crack instability condition to be readily obtained; this approach is frequently referred to as the R-curve procedure.

More recently, it has been recognized that crack growth can also occur under plane strain conditions and be accompanied by either localized or extensive plastic deformation, a good example of the latter being when ferritic steels are tested in the upper-shelf temperature region. The R-curve approach has therefore been extended to encompass both these situations, the appropriate macroscopic material fracture resistance parameter being J_R, the value of the J-integral as a function of crack extension Δc. J is determined in terms of the crack length and applied loadings, no account being taken of the fracture process, and the ensuing relations are matched against the material's J_R curve, which is assumed to be unique, i.e. independent of geometrical parameters and the extent of yielding. To allow the problem to become non-dimensional, Paris and co-workers (1977) have proposed an instability criterion based upon the normalised gradient $T_{MAT} \equiv (E/Y^2)dJ_R/dc$ of the material's J_R curve, where Y is the yield stress. When $T_{APP} \equiv (E/Y^2)dJ_{APP}/dc$ for the system exceeds T_{MAT}, which is referred to as the tearing modulus, instability should coincide with the onset of crack growth, which should occur when the system's J value exceeds the material's critical J_{Ic} value. Implicit in the use of this approach is the assumption that the material's J versus Δc curve is indeed unique. This curve clearly depends on the operative fracture processes within a process zone in the immediate vicinity of the crack tip. Thus there is a correlation between the macroscopic description of crack growth (i.e. the J_R-Δc curve) and the operative fracture mechanisms.

This paper focuses on this correlation via Cottrell's (1965) description of the fracture events at a crack tip. He highlighted two extreme mechanisms by which a crack can propagate in a solid subject to Mode I plane strain deformation conditions. At one extreme, crack extension can be represented by the injection of edge dislocations into the material from the crack tip along slip lines that are inclined at 45^o to the tensile axis; each dislocation of Burgers Vector b contributes an increment $b/\sqrt{2}$ to both the opening and extension of the crack. A "non-cumulative" situation exists in that each crack growth increment requires the injection of extra dislocations; more importantly, these dislocations push the existing dislocations further away from the crack tip, whereupon the plastic zones spread more rapidly across the section than does the crack. With this mechanism, the material at the crack tip does not actually fracture, but slides-off along specific planes and it is this sliding-off which is responsible for a limited amount of stable crack growth ("stretch zone" formation).

At the other extreme, crack extension can be represented by the climbing of edge dislocations along the crack plane ahead of the crack tip. This process does not

require the injection of extra dislocations into the material, and the same dislocation group is responsible for fracture at all stages of crack propagation. A "cumulative" situation exists, with crack propagation not requiring an increase in the crack tip stress intensification; fracture is therefore unstable for many practical loading systems. The classic example of a cumulative fracture is crack propagation in a perfectly elastic solid; in this case the climbing edge dislocations represent the progressive loss of cohesion as the crack tip is approached.

The Cottrell description focuses on the fracture processes at the crack tip, and by associating these processes with dislocation emission into the surrounding unfractured material, this description provides a basis for relating material behaviour within the fracture process zone with the response of the surrounding material; it is against this background that the present paper extends this description.

THE CONNECTION BETWEEN MATERIAL BEHAVIOUR WITHIN THE FRACTURE PROCESS ZONE AND THE MACROSCOPIC DESCRIPTION OF CRACK GROWTH

As indicated in the Introduction, Cottrell (1965) considered two extreme cases: (a) there is no emission of dislocations from the fracture process zone into the surrounding material and (b) dislocations are emitted such that the associated plastic displacement is equivalent to the crack growth increment. There is, of course, no reason why intermediate situations should not exist. Consider, therefore, the case where a fracture process zone exists at a crack tip, and there is a partial loss of cohesion within this zone; the zone size is Δ which represents the spacing between the inhomogeneities that are responsible for the material losing its cohesion. As the crack tip moves forward a distance Δ, it is supposed that the fracture processes within the zone require the surrounding material to plastically deform; this deformation can be represented by the emission of dislocations into the material. To facilitate the analysis, it will be assumed that these dislocations, which produce a plastic displacement δ, are emitted as edge dislocations which climb along the crack plane. The Dugdale-Bilby-Cottrell-Swinden (DBCS) model (Dugdale, 1960; Bilby, Cottrell and Swinden, 1963) can then be used to describe crack growth. Thus, as the crack tip moves forward a distance Δ, the difference between the displacements at B_2 and A_1 (Fig. 1) is δ.

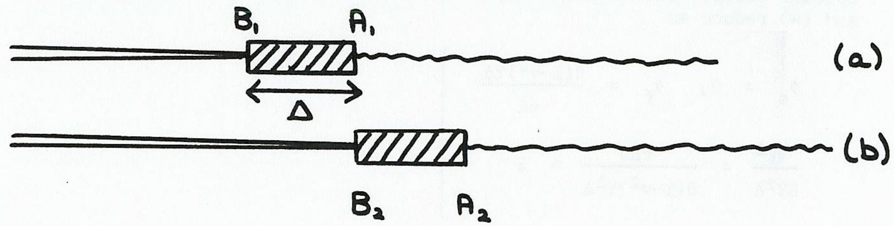

Fig. 1. DBCS model of crack growth: (a) crack length is c, (b) crack length is (c + Δ).

In other words, the tip moves forward a distance Δ if the displacement δ accumulated while a material point is within a distance Δ from the tip attains a critical value δ. This is in fact Wnuk's "final stretch" criterion (Wnuk, 1973) and the extension of Cottrell's fracture description provides a physical basis for this criterion.

The simulated yield region ahead of the process zone is assumed to sustain a stress Y, representative of the material's yield stress, while again to facilitate the analysis, the cohesive stress within the process zone is also assumed to have the value Y. For the small scale yielding case where the combined plastic and process zone size w is small compared with the crack size, superposition of results (Bilby, Cottrell and Swinden, 1963) for the DBCS model gives the displacement Φ within the process zone as

$$\frac{\pi E\Phi}{8(1-\nu^2)Yw} = \sqrt{1-\chi u} - \frac{\chi u}{2}\,\ell n\left\{\frac{1+\sqrt{1-\chi u}}{1-\sqrt{1-\chi u}}\right\} \tag{1}$$

where $u = \Delta/w$ and $\chi = r/\Delta$, r being the distance measured from the crack tip. Relation (7) gives the displacements Φ_s and Φ_f at the extremeties, $\chi = 1$ and $\chi = 0$, of the fracture process zone as

$$\frac{\pi E\Phi_s}{8(1-\nu^2)Yw} = \sqrt{1-u} - \frac{u}{2}\,\ell n\left\{\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}\right\} \tag{2}$$

$$\frac{\pi E\Phi_f}{8(1-\nu^2)Yw} = 1 \tag{3}$$

the crack tip stress intensification K and the J-integral being given by the relations

$$\frac{\pi K^2}{8Y^2w} = \frac{\pi EJ}{8(1-\nu^2)Y^2w} = 1 \tag{4}$$

Wnuk (1979) has recognised that this general description degenerates into two special cases. With a perfectly elastic material, $\Delta = w$, and equations (2), (3) and (4) reduce to

$$\left.\begin{aligned}\Phi_s = 0, \quad \Phi_f &= \frac{8(1-\nu^2)Y\Delta}{\pi E}\\ \frac{\pi K^2}{8Y^2\Delta} = \frac{\pi EJ}{8(1-\nu^2)Y^2\Delta} &= 1\end{aligned}\right\} \tag{5}$$

The crack growth criterion accordingly becomes

$$\frac{8(1-\nu^2)Y\Delta}{\pi E} = \delta \tag{6}$$

the crack tip stress intensification or J integral required for growth being given

by relations (5) with $\Phi_f = \delta$; i.e. $J = Y\delta$. In this case a constant displacement δ is maintained at the crack tip during crack propagation.

On the other hand, if the fracture process zone is small compared with the yield zone (i.e. $\Delta \ll w$), relations (2), (3), and (4)) simplify to

$$\left.\begin{aligned}\frac{\pi E\Phi_s}{8(1-\nu^2)Yw} &= 1 - \frac{\Delta}{2w}\,\ell n\left\{\frac{4ew}{\Delta}\right\}\\ \frac{\pi E\Phi_f}{8(1-\nu^2)Yw} &= 1\\ \frac{\pi K^2}{8Y^2w} = \frac{\pi EJ}{8(1-\nu^2)Y^2w} &= 1\end{aligned}\right\} \tag{7}$$

The displacement $\Phi(\Delta,c)$ at the process zone front when the crack tip is at c, is given by the first of relations (7), while the displacement $\Phi(o,c+\Delta)$ at the rear of the process zone, i.e. at the crack tip, when the crack tip is at $(c+\Delta)$ is given by the second of relations (7) with w replaced by $(w+\delta w)$. Since $\delta w = (dw/dc)\,\Delta$, the crack growth condition becomes

$$\frac{dw}{dc} = \frac{1}{2}\,\ell n\left[\frac{\Delta}{4ew}\exp\left\{\frac{\pi E\delta}{4(1-\nu^2)Y\Delta}\right\}\right] \tag{8}$$

This relation describes how the deformation pattern changes near an advancing crack tip, in that it shows how the plastic zone size w increases as the crack advances. Furthermore, the relation shows how this deformation pattern depends on the fracture processes operative within the process zone; these processes are defined by the parameters δ and Δ. Relations (5) allow the growth condition to be expressed in the form

$$\frac{dJ}{dc} = \frac{Y\delta}{\Delta} - \frac{4(1-\nu^2)Y^2}{\pi E}\,\ell n\left\{\frac{\pi EJ}{2(1-\nu^2)Y^2\Delta}\right\} \tag{9}$$

and the J versus Δc relation for crack growth is therefore unique for a particular material.

The preceding results for crack growth under small scale yielding conditions ($\Delta \ll w \ll c$) are of the same mathematical form as those obtained from some recent theoretical and numerical calculations by Rice and Sorensen (1978). Their analysis of the deformation field, consistent with a Prandtl stress distribution at the tip of an advancing plane strain crack in a plastic-elastic solid, gives the functional form of the crack tip profile: the crack opening is of the form r ℓn (const/r) where r is the distance from the tip. This observation, coupled with data generated from finite element studies of cracks growing under small-scale yielding conditions, allows the derivation of a relation characterizing the deformation at an advancing crack tip. By assuming that a crack extends when a critical opening is obtained at a small distance from the tip, i.e. by assuming a crack tip opening angle criterion, Rice and Sorensen obtained a growth condition essentially equivalent to relation (9). At first sight, it is surprising that there should be this accord, since the Wnuk criterion is concerned with the situation ahead of the crack tip, whereas the Rice-Sorensen criterion is concerned with the situation behind the crack tip. However it is easily shown that one obtains the same crack growth condition (relation (9)) with the DBCS model, if the displacement at a material point coincident with the tip of a crack of length c is assumed to increase by an amount δ as the tip moves forward as distance Δ. This latter criterion is infact the DBCS analogue of Rice and Sorensen's criterion; thus the Wnuk criterion may be regarded as a crack tip opening angle criterion with a constant angle $\theta \equiv \delta/\Delta$ being maintained during growth. (It is emphasized that θ must be defined in terms of a finite Δ; it cannot be defined when $\Delta = 0$).

This paper's considerations have so far been with reference to plane strain crack growth under small scale yielding conditions. To discuss the situation where crack growth is associated with extensive plastic deformation, consider the Mode I plane strain model (Fig. 2) of a crack of length 2c within an infinite solid which is

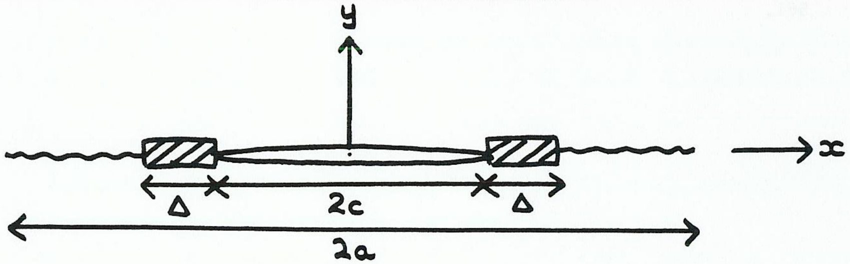

Fig. 2. DBCS model of a crack of length 2c in an infinite solid. The applied tensile stress is σ and thin plastic zones extend to the points $x = \pm a$. Fracture process zones are as shown.

subject to an externally applied tensile stress σ; the yield zones, which sustain the yield stress Y, are assumed to extend to the points $x = \pm\, a$. The relative displacement Φ across the yield zone at a distance s ahead of the crack tip is

$$\Phi = \frac{8Y(1-\nu^2)c}{\pi E}\left[\ln\left(\frac{a}{c}\right) + \frac{s}{2c}\ln\left\{\frac{a^2 s}{2ec(a^2-c^2)}\right\}\right] \qquad (10)$$

assuming that the DBCS model is valid for the extensive plasticity case. Remembering that both a and c change during crack growth, application of the Wnuk crack growth criterion shows that J satisfies the differential equation (Wnuk, 1979; Smith, 1980)

$$\frac{dJ}{dc} = \frac{Y\delta}{\Delta} - \frac{4Y^2(1-\nu^2)}{\pi E}\ln\left\{\frac{2e(a^2-c^2)c}{a^2\Delta}\right\} \qquad (11)$$

with a being given in terms of J by the equation

$$J = \frac{8Y^2(1-\nu^2)}{\pi E}\ln\left(\frac{a}{c}\right) \qquad (12)$$

Relation (11) is valid, irrespective of the extent of yielding. However, when crack growth proceeds under small scale yielding conditions, i.e. $w = (a-c) >> c$,

relation (11) reduces to (9). At the other extreme, where crack growth proceeds under large-scale yielding conditions, and the plastic zone size $w = (a-c) >> c$, relation (11) reduces to

$$\frac{dJ}{dc} = \frac{Y\delta}{\Delta} - \frac{4Y^2(1-\nu^2)}{\pi E}\ln\left(\frac{2ec}{\Delta}\right) \qquad (13)$$

simplifying to

$$\frac{dJ}{dc} = \frac{Y\delta}{\Delta} - \frac{4Y^2(1-\nu^2)}{\pi E}\ln\left(\frac{2ec_o}{\Delta}\right) \qquad (14)$$

for small amounts of crack growth, with c_o being the initial crack size. The J versus Δc relation will be unique, i.e. independent of the initial crack size, with equation (14) integrating to give

$$J = J_{Ic} + \frac{Y\delta}{\Delta}(\Delta c) \qquad (15)$$

provided the second term on the right hand side of relation (14) is small compared with the first term; J_{Ic} is the J value appropriate to the onset of crack growth. The ratio R of these two terms is

$$R = \frac{4(1-\nu^2)}{\pi}\cdot\frac{(Y/E)}{(\delta/\Delta)}\ln\left(\frac{2ec_o}{\Delta}\right) \qquad (16)$$

and it is reasonable to proceed on the basis that the J versus Δc relation is unique if $R \lesssim 0.2$. The magnitude of R is not particularly sensitive to the logarithmic term, and with a typical value $\Delta \equiv \delta_{Ic} \equiv J_{Ic}/Y \sim 10^{-1}$ mm and c_o in the range 1-10 cm, R is approximately equal to 10 $(Y/E)/(\delta/\Delta)$; the J versus Δc relation is therefore unique provided

$$(Y/E)/(\delta/\Delta) \lesssim 2 \times 10^{-2} \qquad (17)$$

Thus a low yield stress, i.e. low Y/E, and a high crack growth resistance, i.e. high δ/Δ, favour uniqueness. For example, with a typical value $Y/E = 3 \times 10^{-3}$ for a pressure vessel steel, there is uniqueness if the crack tip opening angle δ/Δ exceeds 0.15. Several experimental results (Marston, 1978) for A 533 B, suggest that δ/Δ exceeds this value, and uniqueness is expected with such a material.

The large scale yielding results (small initial crack size) can be compared with the small-scale yielding results (large initial crack size). For the latter, since $J_{Ic} \equiv Y\delta_{Ic} \sim Y\Delta$, the governing equation (9) becomes

$$\frac{dJ}{dc} = \frac{Y\delta}{\Delta} - \frac{4Y^2(1-\nu^2)}{\pi E}\ln\left\{\frac{\pi e E}{2(1-\nu^2)Y}\right\} \qquad (18)$$

for small amounts of crack growth. Thus when condition (17) is satisfied, the second term is small compared with the first term. Thus the overall conclusion is that the J versus Δc curve is independent of both the initial crack size and the extent of yielding, for small amounts of crack growth, provided the yield stress is sufficiently low and the material's crack growth resistance is sufficiently high; the appropriate J - Δc relation is

$$\frac{dJ}{dc} \sim \frac{Y\delta}{\Delta} \qquad (19)$$

This linear and unique dependence of J on Δc, irrespective of geometrical parameters, and for both small scale and large scale yielding situations, forms the basis of Paris and co-workers (1977) use of the tearing modulus parameter T_{MAT}. With T_{MAT} defined by $T_{MAT} \equiv (E/Y^2)\ dJ/dc$, it can be expressed via relation (19) as $T_{MAT} \equiv (E/Y).(\delta/\Delta)$, and it follows from relation (17) that the J versus Δc curve should be unique provided $T_{MAT} \gtrsim 50$. The materials considered by Paris and co-workers have T_{MAT} values in excess of 100; the present comments therefore provide valuable support for the tearing modulus approach with such materials.

DISCUSSION

The main feature of the preceding considerations is the link developed between the mechanics of fracture within the immediate crack tip region, and the macroscopic description of crack growth. The local fracture process controls the magnitude of the crack tip opening angle $\theta \equiv \delta/\Delta$, which in turn governs a material's macroscopic crack growth (J - Δc) curve. Furthermore, each material has its own characteristic curve, at least for small crack growth increments; thus, for example, relation (19) is independent of both loading and geometrical parameters, and the extent of plastic yielding, provided the yield stress is sufficiently low and the material's crack growth resistance is sufficiently high. Crack growth can therefore be predicted by determining J for various loadings and crack lengths, while ignoring the detailed fracture processes, and then matching the results against the material's J versus Δc curve.

In the light of these comments, metallurgical factors which affect the magnitude of the parameter δ/Δ should also influence the material's macroscopic fracture behaviour. For example, if a material is hardened by, for example, the introduction of a distribution of second phase particles by an appropriate heat-treatment procedure, or as a result of neutron irradiation in a nuclear reactor, plastic deformation will tend to be localized within a few slip bands rather than be homogeneously distributed within the component crystals, (Smith, Cook and Rau, 1977). Such flow localization facilitates the processes by which voids coalesce ahead of a crack tip, thereby allowing crack growth to proceed more readily, i.e. δ/Δ is lowered. In the Paris terminology, this produces a reduction in the tearing modulus, and should increase the likelihood of occurrence of an unstable fracture, which is a well-known characteristic of some high strength materials.

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