THE GROWTH OF MACROSCOPIC CRACKS IN CREEPING MATERIALS

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ABSTRACT

The stress field at the tip of a growing crack in an elastic-nonlinear viscous material is derived, and is combined with a crack growth criterion based on critical strain. There results an (integral) equation of motion for the crack tip which is solved numerically to yield the growth rate history. Special attention is paid to instability effects which arise during crack growth as a consequence of the properties of the asymptotic near-tip field.

KEYWORDS

Creep, fracture mechanics, stress analysis.

INTRODUCTION

As a special aspect of the highly complicated processes that accompany and determine the growth of a macroscopic crack in a creeping material, the stress analysis of a growing crack is presented, and conclusions are drawn with respect to fracture mechanics and to crack growth behavior. The stress analysis is based on the usual (small strain) compatibility and equilibrium equations of continuum mechanics and on the material law of an elastic-nonlinear viscous solid. The nonlinear viscous term describes Norton-type, power-law creep. The total strain rate is given by

$$\dot{\varepsilon}_{ij} = \frac{1+\nu}{E}\,\dot{\sigma}'_{ij} + \frac{1-2\nu}{3E}\,\dot{\sigma}_{kk}\,\delta_{ij} + \frac{3}{2}\,B\,\sigma_e^{\,n-1}\,\sigma'_{ij}. \quad (1)$$

The equivalent stress is defined as $\sigma_e = (3\sigma'_{ij}\,\sigma'_{ij}/2)^{1/2}$, the primes denote the deviatoric part of the stress tensor σ, the dot denotes the total time derivative at a material point, δ_{ij} is the unit tensor, and the summation convention is applied. Material parameters are Young's modulus, E, Poisson's ratio, ν, the creep exponent, n, and the creep coefficient, B.

We consider two-dimensional problems with plane cracks in plane-strain and plane-stress tension (Mode I). The crack extends at a constant or variable rate $\dot{a}$ in the positive x-direction. The current crack tip position is denoted by a, with a = o at the beginning of crack growth. The absolute value of the initial crack length need not be specified. It is implicitly contained in the load parameters K and C* introduced in the next section. Attached to the moving crack tip is a polar coordinate system (r,θ) with $\theta = o$ lying directly ahead of the crack.

The asymptotic stress and creep strain fields near the tip of a growing crack in such a material, according to Hui and Riedel (1980), have the form

$$\sigma_{ij} = \alpha_n \left[\frac{\dot{a}(t)}{BEr}\right]^{1/(n-1)} \hat{\sigma}_{ij}(\theta), \qquad \varepsilon^{cr}_{ij} = \frac{\alpha_n}{E}\left[\frac{\dot{a}(t)}{BEr}\right]^{1/(n-1)} \hat{\varepsilon}^{cr}_{ij}(\theta) \tag{2}$$

for $r \to o$ and if $n > 3$. The dimensionless angular functions, $\hat{\sigma}_{ij}$, $\hat{\varepsilon}^{cr}_{ij}$, and the dimensionless factor α_n have been given numerically by Hui and Riedel (1980) for plane strain and plane stress. The asymptotic fields, eq. (2), are valid for both, steady-state and unsteady crack growth. They further have the remarkable property to be independent of the load that is applied to the specimen and of the prior growth history. Besides material parameters, only the current crack growth rate appears in the asymptotic fields.

Another surprising feature of the asymptotic field will be addressed in the present paper: According to eq. (2) the asymptotic creep strain increases the faster the crack grows since $\varepsilon^{cr} \propto \dot{a}^{1/(n-1)}$. Therefore crack growth is inherently unstable if the near-tip field dominates over a length scale which is physically relevant for the microscopic failure mechanism, and if a critical strain criterion, or a similar criterion involving strain besides other quantities, controls crack growth. In the following section, the characteristic lengths for the near-tip field to dominate are derived for the cases where the near-tip field is embedded in a predominantly elastic far field or in an extensively creeping specimen. Finally the stress fields are combined with a critical strain criterion. This leads to an equation of motion for the crack tip from which the particular effects of the singular field, eq. (2), on crack growth behavior are derived.

DERIVATION OF THE STRESS FIELDS

Small-scale yielding. First we consider the case where the crack grows while the bulk of the specimen still behaves predominantly elastic except in a small zone near the crack tip where the singular field, eq. (2), dominates the stress (short-time behavior or 'small-scale yielding'; Riedel and Rice, 1980). Within the region where the stress intensity factor K dominates the linear elastic stress field, the development of stress and strain can then be analyzed as if the crack were of semi-infinite length, and the stress can be calculated subject to the remote boundary condition of asymptotic approach to the elastic singular field,

$$\sigma_{ij} = K f_{ij}(\theta)/\sqrt{2\pi r} \tag{3}$$

for $r \to \infty$, where $f_{ij}(\theta)$ describes the well-known angular distribution of stress around the crack in linear elasticity.

In the present paper, only steady-state solutions to this boundary layer problem are sought. Here steady-state means constant growth rate or, less stringently, that the stress field as seen from the moving crack tip varies slowly enough in time so that total time rates can be approximated by $\dot{\sigma} \approx -\dot{a}\partial\sigma/\partial x$. This implies that K and $\dot{a}$ must vary slowly enough. For the steady-state problem so defined, the general form of the stress field has been derived by Hui and Riedel (1980) employing dimensional considerations. There results

$$\sigma_{ij}(r,\theta) = \left[\frac{\dot{a}}{K^2 EB}\right]^{1/(n-3)} \Sigma_{ij}(R,\theta) \tag{4}$$

with

$$R = r/r_1 \quad ; \quad r_1 = \left[\frac{EBK^{n-1}}{\dot{a}}\right]^{2/(n-3)} \tag{5}$$

where r_1 represents the characteristic length over which the near-tip stress field, eq. (2), dominates; r_1 must be small enough compared to the crack length and ligament width to ensure the validity of the small-scale yielding solution, eq. (4). The dimensionless shape functions Σ_{ij} are not yet known for Mode I. In the following theoretical development, the interpolation formula between the known near-tip and far-field solutions

$$[\Sigma_e(R,o)]^{1-n} = (\alpha_n \hat{\sigma}_e)^{1-n} R + f_e^{1-n} (2\pi R)^{(n-1)/2} \tag{6}$$

is employed for the equivalent stress ahead of the crack tip, which, if applied to Mode III, approximates the finite element results of Hui (1980) for Mode III within 10 per cent. The detailed form of Σ_{ij} between the asymptotic limits is irrelevant for the general conclusions of the present paper; f_e and $\hat{\sigma}_e$ are the angular parts at $\theta = o$, of the equivalent elastic and near-tip stress, respectively.

The steady-state stress field, eq. (4), is approximately valid if the variations of K and $\dot{a}$ are sufficiently small compared to K and $\dot{a}$ themselves while the crack traverses the zone, r_1, where the near-tip field dominates. For continuous velocity or load variations this leads to the requirements

$$|\ddot{a}| \lesssim [\dot{a}^{n-2}/(EBK^{n-1})]^{2/(n-3)}, \qquad |\dot{K}| \lesssim [\dot{a}^{n-1}/(E^2B^2K^{n+1})]^{1/(n-3)} . \tag{7}$$

Crack growth accompanied by extensive creep of the whole specimen. If the test duration is long enough so that the whole specimen creeps extensively, elastic straining can be neglected except in the zone where the singular field of the growing crack, eq. (2), dominates. If the size of this zone, r_2, is small enough compared to the crack length and uncracked ligament width, the bulk of the specimen can be considered as nonlinear viscous. Then the C*-integral introduced by Landes and Begley (1976) is path-independent in the bulk of the specimen except in the near-tip zone, and the associated HRR-field (named after Hutchinson, 1968, and Rice and Rosengren, 1968)

$$\sigma_{ij} = \left[\frac{C^*}{BI_n r}\right]^{1/(n+1)} \tilde{\sigma}_{ij}(\theta) \tag{8}$$

sets the remote boundary conditions $(r \to \infty)$ for the near-tip zone. The dimensionless factor I_n and angular functions $\tilde{\sigma}_{ij}(\theta)$ are given numerically by Hutchinson (1968), and Rice and Rosengren (1968).

As in the small-scale yielding case the general form of the steady-state stress field follows from dimensional considerations to be

$$\sigma_{ij}(r,\theta) = \left(\frac{EC^*}{\dot{a}}\right)^{1/2} \Sigma_{ij}(R,\theta) \tag{9}$$

with

$$R = r/r_2 \quad \text{and} \quad r_2 = \left(\frac{\dot a}{EB}\right)^{(n+1)/2} \left(\frac{B}{C^*}\right)^{(n-1)/2} \quad . \tag{10}$$

Here, r_2 is the characteristic length over which the near-tip singular field dominates. It must be small compared to the crack length if the boundary layer formulation, eq. (8), is to work properly. The dimensionless shape functions Σ_{ij} are unknown as yet, but the asymptotic behavior for $R \rightarrow o$ and $R \rightarrow \infty$ is known, and a reasonable interpolation ahead of the crack is expected to be

$$[\Sigma_e(R,o)]^{n-1} = (I_n R)^{(1-n)/(n+1)} \tilde\sigma_e^{\,n-1} + R^{-1}(\alpha_n \hat\sigma_e)^{n-1} \quad . \tag{11}$$

The steady-state stress field, eq. (9), is approximately valid if $\dot a$ and C^* vary to a small extent only while the crack grows through the near-tip zone whose size is r_2. For continous variations this implies

$$|\ddot a| \lesssim B\,(E^{n+1}\, C^{*\,n-1}/\dot a^{\,n-3})^{1/2} \quad , \quad |\dot C^*| \lesssim B[(EC^*)^{n+1}/\dot a^{\,n-1}]^{1/2} \quad . \tag{12}$$

THE CRACK GROWTH RATE AS A FUNCTION OF THE LOAD AND OF THE PRIOR CRACK GROWTH HISTORY

The crack growth rate-vs.-crack length relations presented in the sequel are based on a critical strain criterion, i.e., the crack is to grow in such a manner that the creep strain has a critical value ε_c at a distance x_c ahead of the propagating crack tip. In order to demonstrate the effect of the near-tip singular field, eq. (2), on crack growth behavior, it is convenient to write the critical strain criterion in the differentiated form

$$\frac{B\sigma^n}{\dot a}\,\frac{x_c}{\varepsilon_c} = -\,\frac{\partial \varepsilon^{cr}}{\partial x}\,\frac{x_c}{\varepsilon_c} \quad , \tag{13}$$

where σ and $\partial\varepsilon^{cr}/\partial x$ are understood at the distance x_c ahead of the current crack tip, and where the material law $\dot\varepsilon^{cr} = B\sigma^n$ has been inserted. The ratio x_c/ε_c has been included for later convenience. The creep strain, ε^{cr}, consists of a part, ε_o which is accumulated while the crack is still at rest, and of a contribution due to the growing crack. For small-scale yielding it suffices for the present purpose to approximate the creep strain of the stationary crack by the creep strain that would develop in a purely elastic stress field, i.e., $\varepsilon_o \propto r^{-n/2}$. Correspondingly is in the extensive creep limit $\varepsilon_o \propto r^{-n/(n+1)}$. The creep strain added to ε_o during the growth period is given by an integral over all prior crack tip positions, viz. $B\int_o^a \sigma^n(a+x_c-a')/\dot a(a')da'$. Now, as an important approximation, the stress is expressed in this integral and in eq. (13) by the steady-state stress field described in eqs. (4 to 6) or eqs. (9-11). With the (interpolated) steady-state stress field inserted, eq. (13) is an equation of motion that is ideally suited for a step-wise numerical solution. Its right-hand-side (rhs) depends only on the prior crack growth history (via the integration for ε^{cr}), whereas the steady-state stress on the left-hand-side (lhs) depends on the current growth rate only. Thus, at each crack tip position, calculate the rhs by integration over the prior history, and then solve eq. (13) for $\dot a$ numerically. With this new $\dot a$, the integration on the rhs can be carried out one step further, the next $\dot a$ can be calculated and so on. Figure 1 illustrates a peculiarity involved in the solution of eq. (13) for $\dot a$ at each step. It shows the lhs of eq. (13) with the steady-state stress fields inserted. As a function of the normalized growth rate $\dot A$, the lhs must exhibit a maximum for small-scale yielding, and a minimum in the extensive creep limit, whereas the rhs, being a function of the prior history only, is independent of the current growth rate (horizontal lines). It is obvious that eq. (13) either has two solutions at a given crack tip position or it has none, depending on the load parameter (K or C^*) and on the rhs. The results in Figs. 1 and 2 are universal for all model parameters not specified in the figures if the following normalizations are employed; for small-scale yielding:

$$\dot A = \frac{\dot a \varepsilon_c}{Bx_c}\left[\frac{\sqrt{2\pi x_c}}{Kf_e}\right]^n \quad , \quad k = \frac{Kf_e(\alpha_n\hat\sigma_e)^{n-1}}{\sqrt{2\pi x_c}\,E\,\varepsilon_c} \quad ,$$

and in the extensive creep limit:

$$\dot A = \frac{\dot a \varepsilon_c/\tilde\sigma_e^{\,n}}{(Bx_c)^{1/(n+1)}(C^*/I_n)^{n/(n+1)}} \quad , \quad c^* = C^*\,\frac{[(\alpha_n\hat\sigma_e)^{n-1}\,\tilde\sigma_e]^{n+1}}{(E\varepsilon_c)^{n+1}\,BI_n\,x_c}$$

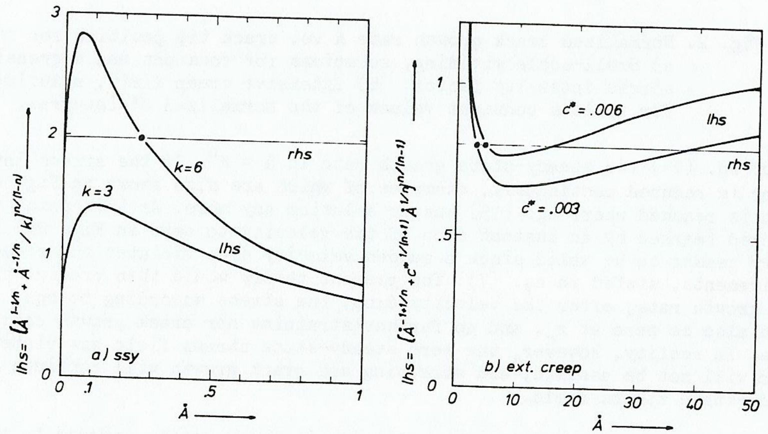

Fig. 1. Graphical representation of lhs and rhs of eq. (13), vs. normalized velocity $\dot A$, for n = 4. a) Small-scale yielding, with k = normalized stress intensity factor. b) Extensive creep limit, with c^* = normalized C^*-integral. The circles indicate stable solutions.

In the small-scale yielding limit, two solutions to eq. (13) exist if the initial stress intensity factor (at a = o) satisfies the inequality

$$K > \frac{n}{2}\,\frac{n^{n/(n-1)}}{(n-1)\,(\alpha_n\hat\sigma_e)^{n-1}\,f_e}\,\sqrt{2\pi x_c}\;E\varepsilon_c \quad . \tag{14}$$

In the dimensionless notation of Fig. 1 this means k = 4.23 if n = 4. The solution with the lower $\dot a$ is not physically meaningful since it is unstable in the sense that the strain at x_c would increase for increasing $\dot a$. As a consequence, the crack would further accelerate until it reaches the solution with the greater $\dot a$ which is stable. Figure 2a shows results of the step-wise solution of eq. (13) for small-scale yielding. If the stress intensity factor is kept constant the growth rate increases until it reaches a steady state which has already been considered by Hui and Riedel (1980). For stress intensity factors that are large compared to the

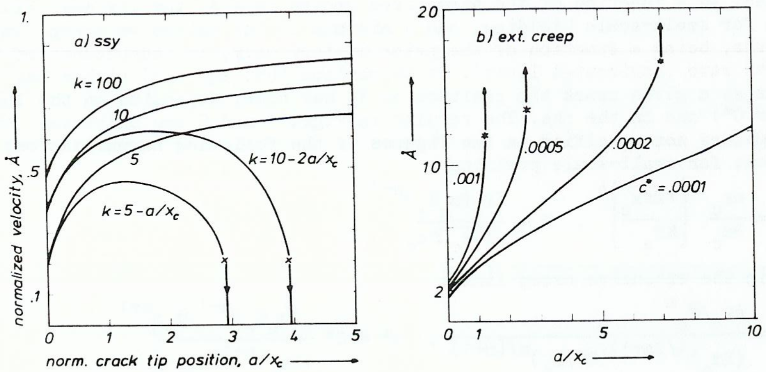

Fig. 2. Normalized crack growth rate $\dot{A}$ vs. crack tip position for n = 4. a) Small-scale yielding, solutions for constant and decreasing stress intensity factor. b) Extensive creep limit, solutions for various constant values of the normalized C*-integral.

rhs of eq. (14) the steady-state growth rate is $\dot{a} \propto K^n$. If the stress intensity factor is reduced continously, examples of which are also shown in Fig. 2a, the point is reached where eq. (13) has no solution any more. As this point is approached (marked by an instant drop of the velocity to zero in Fig. 2a) the present theory ceases to be valid since a sudden velocity drop violates the steady-state requirements, stated in eq. (7). The present theory would then predict that for zero growth rate, after the velocity drop, the stress according to eqs. (4 to 6) would also be zero at x_c, and no further straining nor crack growth could take place. In reality, however, the zero steady-state stress field associated with $\dot{a}$ = o will not be assumed, and straining and crack growth will continue in a non-steady-state stress field.

In the extensive creep limit, the situation is, in a sense, inverse to the small-scale yielding situation. Solutions exist if initially (at a = o) the load parameter C* is small enough to satisfy

$$C^* < \left[\frac{(n-1)n^{1/(1-n)}}{(n+1)(\alpha_n \hat{\sigma}_e)^{n-1} \tilde{\sigma}_e}\right]^{n+1} I_n B x_c (E\varepsilon_c)^{n+1} , \qquad (15)$$

and the solution with the smaller $\dot{a}$ is stable. As it is shown in Fig. 2b, the crack accelerates under constant-C* conditions and never reaches a constant growth rate. Rather, if a critical growth rate $\dot{A}_{max} = (n-1)/c^{*\,1/(n+1)}$, or, in physical coordinates

$$\dot{a}_{max} = (n-1)\left[\frac{\tilde{\sigma}_e/I_n}{\alpha_n \hat{\sigma}_e}^{1/(n+1)}\right]^{n-1} E\,(Bx_c)^{2/(n+1)}\; C^{*\,(n-1)/(n+1)} \qquad (16)$$

is exceeded, no solutions to eq. (13) exist any more, and crack growth becomes unstable. This is because the near-tip field gains an increasing influence on the strain at x_c as $\dot{a}$, and thereby r_2, increase. It has already been remarked in the introduction that the near-tip field inherently tends to cause instabilities. Near the instability, large crack acceleration occurs, which invalidates the present theory.

Kubo and co-workers (1979) have also studied crack growth under the condition of extensive creep, however, ignoring the near-tip field, eq. (2). Their results correspond to the low-C* limit of the extensive creep case of the present study. In this limiting case, also analytical solutions of the integral eq. (13) are possible based in the Laplace transformation method (unpublished results of Riedel and Hui, 1978).

DISCUSSION

The results shown in Fig. 2 have been derived using steady-state stress fields and a crack growth criterion based on critical strain. The assumption of the steady-state stress fields is seriously violated at the onset of crack growth, where the velocity changes abruptly. Here for a certain period of time the stress field of the stationary crack rather than the steady-state field of the growing crack persists to dominate at x_c. The field of a stationary crack leads to an initial velocity of $\dot{A}$ = 2/n (=.5 in Fig. 2a) for small-scale yielding and $\dot{A}$ = (n+1)/n (=1.25 in Fig. 2b) in the extensive creep case. The deviations from these numbers in Fig. 2 illustrate the error which arises from the employment of steady-state stresses despite of the velocity jump at a = o. The smooth velocity variations later on in the growth history admit the employment of steady-state stresses, and the present theory is a good approximation in this respect.

The critical creep strain criterion for crack growth may be too simple in many cases. However, the instability effects due to the near-tip field are not principally removed if the diffusional mechanism of microvoid growth is included in the local failure criterion in addition to the creep strain mechanism. This conclusion may have to be changed if corrosion plays a major role for crack growth.

For larger amounts of crack growth (crack growth comparable to the initial crack length or ligament width), differently shaped specimens can no longer be unified in terms of K or C* as in the preceding sections, and the crack growth history will depend on the detailed specimen geometry.

CONCLUSIONS

1) The near-tip asymptotic field at a crack growing in an elastic-nonlinear viscous material, with a stress exponent n > 3, can have striking effects on crack growth behavior. Crack growth that is accompanied by extensive creep of the whole specimen is unstable if the growth rate exceeds a critical value. Under small-scale yielding conditions crack growth at low growth rates occurs in a non-steady, possibly intermittent way which is not a cosequence of a possible inhomogeneity of the material. Inserting typical numbers from the experimental literature for the material parameters in eqs. (14 to 16) indicates that the instability effects due to the near-tip field might occur within the practical range of $\dot{a}$-, K-, and C*-values in which fracture mechanics tests can be done.

2) Whether or not the near-tip singular field is important, the crack growth rate is history-dependent even for a time-independent load parameter, K or C*. In the examples considered in Fig. 2, the history dependence is less pronounced in small-scale yielding than it is under conditions of extensive creep of the whole specimen.

3) One can expect crack growth to be dominated by K or C* only if the respective lengths, r_1 or r_2, as well as the amount of crack growth are small enough compared to the crack length and ligament width. This condition for K or C*-controlled crack growth must be fullfilled in addition to, and is independent of, the conditions derived for stationary cracks by Riedel and Rice (1980).

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