STRESS BIAXIALITY EFFECTS ON SLOW CRACK GROWTH IN POLYMETHYLMETHACRYLATE

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ABSTRACT

The theories of linear elastic fracture mechanics (LEFM) have been investigated in detail for uniaxial loads, but the effects of biaxial loads have often been ignored because they are assumed to cause no stress singularity at the crack tip and do not appear to cause any relative displacement of crack surface. However, recent work has revealed that the effect of biaxial load on fatigue crack behaviour may in fact be significant. Tests were conducted on PMMA to investigate the transition from uniaxial to biaxial load on the crack growth rate in a slow growth regime. This transitional behaviour is described in detail. As anticipated from the previous fatigue work, the results obtained in the slow growth tests confirmed a decreasing growth rate of the crack with increasing biaxiality. Conversely, the tests conducted with decreasing biaxiality produced a distinctive increase of the crack growth rate. The retardation and acceleration periods thus obtained can be compared with the respective reduction and increase in the effective stress intensity factor range, ΔK. Further evidence from low cycle fatigue tests with increasing biaxiality ratio is also discussed.

KEYWORDS

Slow crack growth; retardation; acceleration; arrest; biaxiality.

INTRODUCTION

The concepts and techniques of fracture mechanics (FM) are now widely used in the study of the strength and stability of structures. Flaws and defects are always present in a structure, however careful the production process and the prediction of the service life of a structure by the FM approach assumes that such flaws will be present. The minimum flaw considered is usually that detectable by non-destructive testing techniques. Detailed knowledge of the behaviour of cracks is essential to determine the safe, yet efficient, life of a structure.

The mathematical and physical models used in the FM approach generally concentrate, however, on uniaxial stress conditions, and in particular stresses which cause the relative displacement of crack surfaces at the crack tip. Recent work has demonstrated that stresses parallel to a crack may also have an effect on crack behaviour, even though such transverse stresses do not appear to cause any displacement of the crack surfaces. This also contradicts the basic assumption of LEFM that only loads which cause a stress singularity at the tip of the crack can affect crack behaviour. Thus, transverse stress has often been ignored as a crack growth factor.

Stress Biaxiality

As few real situations involve simple uniaxial stresses, it is important to investigate the effects of biaxial stresses on crack behaviour and hence on the performance of structures. In the following, biaxial stress is defined as the ratio of the stress parallel to the crack to the stress normal to it.

The stress biaxiality factor, B_t, is defined by:

$$B_t = \frac{P_t (\pi a)^{\frac{1}{2}}}{K_1} \quad (1)$$

where a is half crack length, K_1 is the stress intensity factor, defined later, and P_t is the transverse applied stress (in Williams expansion).

The inherent stress biaxiality factor is:

$$B_o = \frac{P_{to} (\pi a)^{\frac{1}{2}}}{K_1} \quad (2)$$

where P_{to} is the transverse applied stress under uniaxial P_y (normal stress). Therefore:

$$B_t = B_o + \frac{P_x (\pi a)^{\frac{1}{2}}}{K_1} \quad (3)$$

where P_x is the stress in the x-direction. Therefore, as B_o and F_T are functions of $\lambda = (2a/W)$:

$$B_t(\lambda) = B_o(\lambda) + \frac{B}{F_T(\lambda)}$$

$$= \frac{B}{(1 + 0.018\,\lambda + 1.306\,\lambda^2)} - 1 - 0.085\,\lambda \quad (4)$$

In practical work, the load biaxiality ratio $B = 2$ implies that the load in the x-direction parallel to the crack is double that in the y-direction, and similarly $B = 0$ implies a uniaxial load.

Basic Theory

The stress analysis of cracks may be represented by the Williams stress field solution of the crack distribution pattern. The Williams solution yields stresses as:

$$P_{ij} = \sum_{n=1}^{\infty} (a_n\, r^{(n/2-1)} f_n(\theta) + b_n\, r^{(n/2-1)} g_n(\theta))$$

$$= a_1 r^{-\frac{1}{2}} f_1(\theta) + b_1 r^{-\frac{1}{2}} g_1(\theta) + a_2 f_2(\theta) + b_2 g_2(\theta) \quad (5)$$

where a_n are coefficients determining the symmetrical part of the stress field, and b_n are those fixing the anti-symmetrical part.

At $r = 0$, only a_1 and a_2 contribute to the stress field; a_1 is directly related to K, the stress intensity factor, and a_2 to $B\,P_y$. The former, a_1, dominates due to the $r^{-\frac{1}{2}}$ singularity. Hence, as r tends to 0, the stress normal to the crack tip tends to infinity.

The stress intensity factor, K, is defined by:

$$K_1 = F_T\, P_y\, (\pi a)^{\frac{1}{2}} \quad (6)$$

where F_T is the correction factor reflecting geometrical parameters, and P_y is the applied stress normal to the crack. Hence, the stress intensity factor is the only factor which distinguishes one crack problem from another as the $r^{-\frac{1}{2}}$ pattern and θ functions will be similar. Two specimens will exhibit the same crack growth characteristics if they are loaded with the same K history.

The correction factor, F_T, is a function of the normalised crack length, λ, and the relationship between F_T, K and half crack length, a, is included above. The effect of F_T on load and stress biaxiality factors is demonstrated in equation (4).

When the stress intensity factor, K, reaches its critical value for fracture, this value is termed the fracture toughness, K_c. When K_c is measured under plane strain conditions, which represent the worst situation, it is a material constant.

In real materials, the Williams solution is modified by plastic flow during loading which blunts the crack and smoothes out the stress singularity. However, if the crack is long enough and the specimen is large enough, plastic flow should not affect the solution. The effects of plastic flow on crack growth will be discussed in more detail in the next section.

In glassy polymers, such as polymethylmethacrylate (PMMA), polyvinyl chloride (PVC) and polystyrene (PS), the predominant failure micromechanism is craze growth, a phenomenon not observed in metals. A craze is a zone of voided and fibrillated material, and growth is induced by plastic flow processes occurring at the crack tip. The initial rate of development of micro-cavities is determined by the locally concentrated deviatoric stress level and the subsequent expansion of these voids into a fibrillated craze structure depends on the hydrostatic stress. It has been shown by Argon et al (1977) that the rate of craze growth is virtually unaffected by the stress biaxiality, as such a structure could not support transverse stress. Hence, although it may be possible to explain biaxiality effects in metals in terms of plastic flow, such a solution in polymers may not be possible without more understanding of craze growth mechanisms.

PREVIOUS WORK

Effect of Stress Biaxiality on Plastic Zone Size

FM was initially developed for linearly elastic brittle materials such as glass, in which fracture occurs with little or no local plastic deformation. It was mentioned in the previous paragraph that plastic flow may affect the situation by blunting the crack and smoothing out the local stress singularity, so acting as an energy barrier. This reduces the effective work available for interior separation processes.

A generally accepted estimate of the plastic zone boundary is that calculated by substituting the singular stress terms into the Von Mises yield criterion. This yields the extent of the zone ahead of the crack tip for plane stress conditions:

$$r_p = \frac{1}{2\pi}\left(\frac{K_1}{P_o}\right)^2 \qquad (7)$$

where P_o is the uniaxial yield stress. In plane strain, r_p is $(1-2\nu)^2$ times this size, where ν is Poisson's ratio. Increasing the plastic zone size improves toughness since the plastic flow both absorbs energy and reduces the stress concentration by blunting.

Detailed solutions for the plastic zone shape under biaxial loading have been obtained from elastic-plastic finite element analyses by, for example, Miller & Kfouri (1974) and Hilton (1973). Miller & Kfouri varied B between -1 and +1 and observed that the plastic zone reduced with increasing biaxiality. A simultaneous effect was that the plane stress plastic zone "wings" rotated back from the crack extension line as their size reduced.

By investigating the deformation field inside the plastic zone, Hilton (1973) demonstrated that increasing the load biaxiality of a centre-notched plate led to a reduction in the plastic zone size which, he proposed, corresponded to a reduction in the "plastic strain intensity factor". By separating the stress and strain singularity strengths, this is a parameter which extends the stress intensity factor beyond the small scale yielding regime while maintaining the $1/r$ dependence in local energy density. Hilton concluded that if fracture is associated with a critical value of this parameter, then positive biaxiality will increase the fracture initiation stress.

Liebowitz (1978) discussed a similar effect to that described by Hilton and calculated the energy release rate, G, showing that a positive biaxiality would lead to substantial strengthening of a centre-notched plate as predicted by a G_c fracture criterion.

As originally suggested by Dugdale (1960), a relationship between the length of the plastic zone, r_z, and the applied yield stress, P_o, is:

$$r_z = \frac{\pi}{8}\left(\frac{K_1}{P_o}\right)^2 \qquad (8)$$

so that analysis of the total elastic strain field then yields the crack opening displacement (COD), δ, at the crack/line zone interface as:

$$\delta = \frac{K_1^{\,2}}{E\,P_o} \qquad (9)$$

Hence, the fracture criterion is the attainment of a critical COD. This model is particularly useful in that no specific voiding or yielding criterion is involved so that it is applicable for glassy thermoplastics in which a crack tip craze zone is formed. However, as the elastic effects of transverse stress or δ are slight, some sort of voiding criterion is necessary if the model is to be extended to biaxial loading.

Adams (1973) used the Von Mises criterion to modify the yield stress according to the level of transverse stress and as positive load biaxiality ratios raise the yield stress, they would lower r_z and δ. However, the presence of a craze rather than of a line plastic zone suggests that this model does not apply, as the fibrillated craze structure cannot support a transverse stress. Hence, Von Mises' yield criterion will not apply, so craze length, craze stress and COD can be presumed to be independent of the stress state. This criterion is useful, however, for the plastic flow phase preceding craze initiation and two conflicting arguments can be proposed for the effect of transverse stress (Leevers, 1979):

(1) It may inhibit crazing by suppressing the ductile processes necessary for micro-void initiation; or

(2) It may enhance crazing by elevating the hydrostatic stress which controls the expansion of the voids into a fibrillated craze structure.

Previous theoretical work has suggested that the effect of positive biaxiality ratio is to reduce the plastic zone size relative to that due to uniaxial load. The effect of a reduction in plastic zone size is to raise the critical fracture initiation stress. The effect of stress biaxiality on crazing has not so far been clearly determined.

Effect of Biaxiality on Static Fracture Toughness

Conflicting arguments have been put forward as to the effect of stress biaxiality on fracture toughness. Adams (1973) suggested that fracture is controlled by a critical COD, a plastic strain parameter related in some way to the plastic zone size which is reduced as B is increased. Hence, the crack opening load for increasing B must be increased to achieve the same level of plastic strain. Hence, the fracture toughness is increased.

Another, and substantially different, argument developed by Kfouri & Miller (1977) is that a reduced plastic zone size implies a lower plastic work contribution to crack separation energy absorption per unit area of exposed surface. If the cohesive term in this quantity can be assumed constant, then G_c at fracture must be reduced. Hence, the fracture toughness is reduced.

Experimental work on the effect of stress biaxiality in fracture toughness has been inconclusive but does suggest that the effect is negligible. Kibler & Roberts (1970) measured a 25% increase in fracture toughness on increasing B from 0 to 2 using PMMA cruciform specimens. However, these results were attributed to plane stress conditions. They also suggested that the effect on fracture toughness could be either an increase or a decrease, depending on the fracture controlling parameters used. If the plastic zone or COD must attain a critical dimension to cause fracture, then fracture would occur at lower levels of σ_y under biaxial as opposed to uniaxial conditions. However, if the stresses required to maintain equilibrium in the plastic zone control fracture, then larger values of stress would be required to cause fracture under biaxial loading. A similar experimental study to this by Ueda et al (1977) showed no effect of B on K_{Ic}, as did the experiments conducted by Leevers (1979) using the same apparatus and specimen design as in this project.

Effect of Biaxial Stress on Fatigue Crack Propagation in PMMA

In FM, the cyclic growth rate of a fatigue crack is related to the stress intensity factor by the Paris law:

$$\frac{da}{dN} = C\,\Delta K^{m} \tag{10}$$

Paris' law only applies to the "linear" portion (central region II) of the graph, exemplified by Fig. 1. It does not apply at very low growth rates near to the threshold stress nor at high growth rates approaching fracture. The Paris law assumes that K is the factor determining crack behaviour in fatigue.

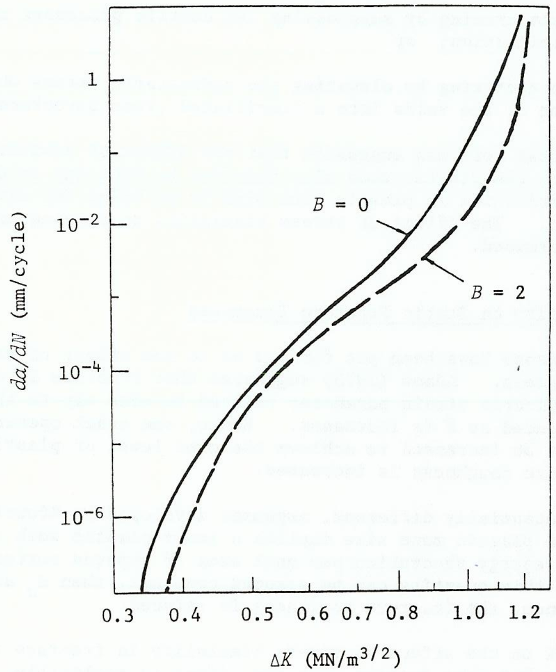

Fig. 1. ΔK versus da/dN. B = 0 and 2. dK/dt = 0.2 MPa.m$^{\frac{1}{2}}$.

The possible biaxiality effects on fatigue crack growth are complicated by the fact that no single crack tip separating micromechanism is likely to operate over the entire range of da/dN. It is well known that there are three clearly

distinguishable regions and, for example, in the intermediate region (II), growth rates may be determined by ductile blunting and/or cyclic craze extension mechanisms. Fatigue crack growth results available so far for biaxiality ratios B equals 0 and 2 are presented in Fig. 1. It will be seen that increasing B may reduce fatigue growth rates by a factor of 5 approximately (Leevers et al, 1979). However, this effect is irregular and appears to be larger in the slow crack growth dominated regime.

The mechanics of fatigue crack propagation in PMMA were also investigated and it was suggested that the crack advances at a critical crack opening displacement. Transverse stress is unlikely to affect this process as the craze is unable to support it, although it could affect the process converting polymer to craze material at the craze tip.

Slow Crack Growth (SCG)

PMMA exhibits slow stable crack growth at sub-critical K values, but above a certain threshold value (about 0.8 MPa.m$^{\frac{1}{2}}$). The crack surface of PMMA clearly reveals the SCG region which is distinguished from the relatively featureless fast fracture surface by "river" markings. Johnson & Radon (1973) suggested that these markings are due to ribbon-like fibres torn out from the material, having been formed by two connecting fractures on the two partial crack planes. Two observations by Leevers (1979) support the contention that slow crack growth appears as an independent mechanism of fatigue crack growth at high ΔK levels:

Firstly, low ΔK fatigue (less than 0.75 MPa.m$^{\frac{1}{2}}$) produced a mirror-like surface, marked only by microscopic striations. Above this level, the threshold for SCG, river markings appear with increasing density.

Secondly, direct observations of large cyclic crack extension (greater than 1 mm) showed that:

(1) The crack remained sharp, i.e. the opening displacement was at least an order of magnitude smaller than the cyclic extension.

(2) The rate of crack extension attained a maximum at peak load and continued during unloading.

Johnson & Radon (1972) discussed the molecular relaxations for a range of polymers and established the dependence of the slow crack growth rate, da/dt, on the applied K value for several specimen geometries and loading speeds. Tests conducted on PMMA at a constant temperature (Fig. 2) showed a rapid increase in K_{Ic} (cf. point A) over a limited range of speeds compared with the range covered. Two basic types of fractures were recorded: stable slow growth of a semi-brittle nature occurring below a critical point dependent on testing speed (i.e. fracture mode transition temperature) and a fast brittle fracture, occasionally preceded by a negligible amount of slow growth (Johnson & Radon, 1973); only the former fracture mode is considered in the following. The plot of log (da/dt) versus K (Fig. 2) may be conveniently described by a simplified relationship:

$$\frac{da}{dt} = D\,K^{l} \tag{11}$$

where $D = 0$ for $K < 0.8$
$D = 0.56$, $l = 60$ for $0.8 < K < 0.94$
$D = 0.032$, $l = 13$ for $K > 0.94$

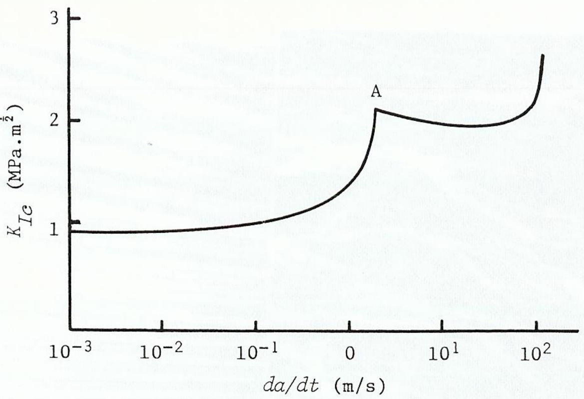

Fig. 2. K_{Ic} versus crack speed at 21°C

Previous Experimental Work in Fatigue

Previous experimental work on the effect of stress biaxiality on fatigue crack growth has again shown conflicting results. Joshi & Shewchuck (1970) realised that their experiments were not really applicable to the analysis discussed so far as they made their assessments using the maximum strain energy theory as opposed to Von Mises' theory. They discovered, by bulge bending tests on round and elliptic surface notched plates of aluminium alloy, that increasing biaxiality led to an increase in the fatigue crack growth rate.

Kibler & Roberts (1970), by load cycling centre-notched plates of aluminium alloy and restraining lateral extension to produce a B value equal to Poisson's ratio, showed slight reductions in the variables C and m of the Paris law equation as B was increased. However, the value of these results is reduced somewhat by the fact that the specimen geometry they used did not ensure that the stress intensity factor was independent of transverse stress. This led to large increases in fracture toughness as B increased, which contradicts the general consensus that no effect of B on K_{Ic} should be observed.

Pook & Holmes (1976), using nickel alloy centre-notched specimens, fully tested for stress field uniformity and load independence, performed path deviation tests for B ratios of up to 2. They noticed that crack growth rates appeared to have a negligible dependence on B. Hopper & Miller (1977) held the transverse load on centre-notched specimens constant during axial cycling so that B varied cyclically to peak values of -1, 0 and +1. They discovered that negative B increased and positive B decreased the crack growth rate, although the effect was small. Although he was unable to present sufficient experimental evidence, Adams (1973) concluded that if fracture is based on either plastic zone size or COD, which are reduced by biaxial loading, then tensile B will lead to a decrease in fatigue crack propagation rates.

Experimental work by Leevers (1979) revealed a retardation effect on increasing B from 0 to 2 at constant ΔK. The crack arrested for about 1500 cycles before acquiring a new, considerably reduced, steady-state growth rate. On decreasing B from 2 to 0, the growth rate was observed to increase but without any noticeable acceleration or retardation. Comparison of the effects of increasing biaxiality

and decreasing stress intensity on the growth behaviour showed a remarkable similarity. A change in B from 0 to 2 had a similar effect on the growth rate as a reduction of about 15% in ΔK. However, the retardation period was not repeatable and an increase in ΔK often caused an acceleration period before settling into a stable growth rate (higher than the previous rate). This acceleration was not observed in decreasing biaxiality tests.

A possible explanation of these effects is in terms of crack closure effects. Fatigue crack growth rates may be a function of ΔK_{eff}, the effective stress intensity range given by:

$$\Delta K_{eff} = K_{max} - K_{cl} \tag{12}$$

where K_{cl} is the K level at which the crack faces close at the tip during unloading, eliminating the stress singularity. Biaxial load is expected to increase the value of K_{cl}, thereby reducing ΔK_{eff}, hence reducing the crack growth rate by blunting the crack (increasing the COD). However, the actual effect of B on ΔK_{eff} has been shown to be small. Ogura et al (1977) discovered that although an increase in B decreased ΔK_{eff} for tension/compression loading, the effect on zero to tension loading was minimal. Hence, closure effects do not appear to provide a suitable explanation for the effects of B on fatigue crack growth rates.

The most likely explanation would appear to be in terms of craze formation, although very little is known about the mechanics of a craze at a crack tip, under fatigue loading. As craze fibril extension is an irreversible process, it can only bear its share of the load if it is being extended or has only been slightly relaxed.

The rate determining process in both true fatigue and slow crack growth is likely to be the conversion of polymer into craze matter. Hence, if the rate of craze matter formation is reduced with increasing B, a corresponding reduction in SCG rates would be observed. This has not yet been demonstrated for statically applied K but FCG rates in the cyclic SCG region have been accounted for completely by this argument. Hence, the results of Leevers can be explained in terms of a reduction in the rate of formation of crack tip craze matter by increasing transverse stress.

While complementary tests in true fatigue were already well advanced and the results will be reported at an early date, it was considered appropriate to investigate the effect of stress biaxiality on slow crack growth. PMMA is an ideal material for this purpose and the results available are discussed in the following paragraphs.

EXPERIMENTAL

The material used in the present investigation was polymethylmethacrylate (PMMA, ICI, UK) 4 mm thick cast plate. The cruciform specimens were provided with a central notch sharpened by a razor blade. The uniformity of the stress field of the central region, 200 mm × 200 mm, has been checked *in situ* by a photoelastic analysis (Wachnicki & Radon, 1979). The test rig applied in-phase monotonic or cyclic loading to both axes at various biaxiality ratios, B. The hydraulic system, tensile loading equipment and crack growth recording method have been described elsewhere (Radon et al, 1977).

In order to enable an immediate change of the biaxiality ratio, a new modification of the loading mechanism was incorporated by the following means. The actual load applied was measured on an x-y plotter from the output of two strain gauge load cells. The point of application of the actuator on the cross-head determined the load biaxiality applied. Previous tests had required the removal of the load so that the actuator could be moved across to a different pair of linkage points in order to change the biaxiality ratio. However, this method was not entirely satisfactory as the effect of instantaneous biaxiality transition was required in order to ensure that crack propagation occurred under identical test conditions for each ratio. By designing a restraining force, it was possible to apply uniaxial load to the specimen with the actuator and cross-head set up for biaxial load. Hence, on removal of the restraining force, the biaxial load could be applied immediately. This restraining force was applied by a specially designed mechanism.

In order to monitor the growth of the crack, a strip of acetate sheet provided with a printed scale was attached to one side of the specimen and the crack measured using a travelling microscope.

A scanning electron microscope was used in the investigation of the fracture surface. In the slow growth region, surface features included highly irregular river patterns, similar in appearance to those observed on metal surfaces or on glass. A typical and well developed river pattern is shown in Fig. 3a. The increasing smoothness of the surface leads then to the development of a regular "herring bone" structure (Fig. 3b). On reading the critical length, the crack arrests momentarily, propagating subsequently with an increasing speed (Fig. 3c). Well known cusp formations may be observed in the fast crack initiation zone, followed by a featureless, mirror-like region. A distinct ribbon-type boundary between a slowly moving and a fast crack may be observed in Figs. 3c and 3d. In Fig. 3d, a part of a lance-like facet is also included. This phenomenon of a torn out fibre is a typical feature of slow growth fractures formed on two partial crack planes and has also been noted in fractures of other materials.

In some of the tests conducted, a few cycles of fatigue were applied in order to start the crack propagation, as occasionally the crack was initially reluctant to move. However, in most tests, this was not necessary. Results were taken for about 4 mm to 5 mm growth under each biaxiality ratio as it was observed that this crack growth could reasonably be assumed to be stable. Occasionally, it was found that more than one test could be conducted on a single specimen, but care had to be taken to reduce the load and thus to ensure that the value of the intensity factor between each test was constant.

Basically, the test programme discussed in the following paragraphs consisted of two series of tests. The first series concerned the investigation of static slow growth during increasing biaxiality, while the second series dealt with the tests of decreasing biaxiality. During all the tests performed, the conditions, including the loading rate and temperature, remained constant, apart from an increase in the ΔK ratio, the value of which was adjusted in order to cover completely the whole section of regime II, together with the adjoining part of the third regime.

DISCUSSION

The purpose of the first few experiments was to investigate the functioning of the rig at high biaxiality ratios and also to test the new modification whereby a sudden change of the load cycling process could be obtained. The subsequent results were obtained from a series of twenty static tests applying an increasing biaxiality from $B = 0$ to $B = 2$. Two additional tests were performed in which the increase in biaxiality was from 0 to 1, the results of which were similar but not

Fig. 3a. River pattern (×150)

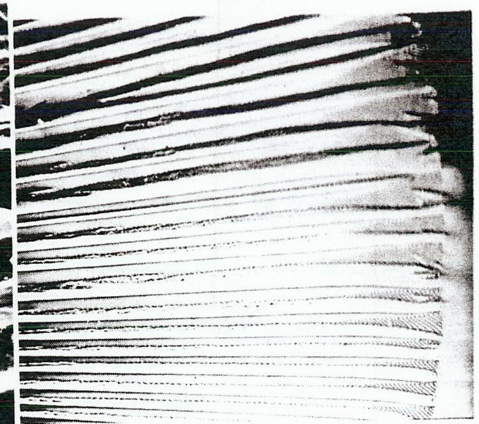

Fig. 3b. Herring bone structure (×20)

Fig. 3c. Boundary between slow crack growth and fast crack (×34)

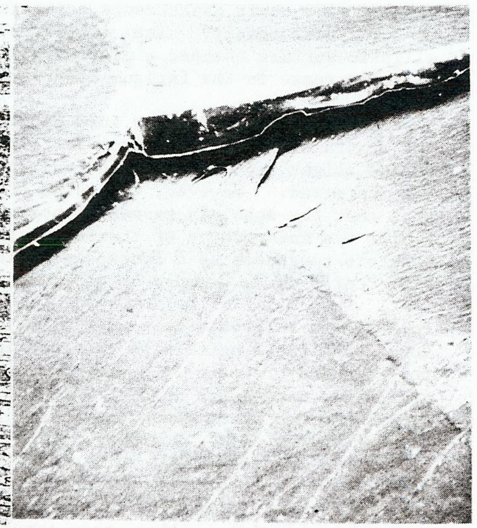

Fig. 3d. Boundary between two lances. Fast crack leading edge towards the right hand side (×130)

In all micrographs, the crack propagates from left to right.

so pronounced. The majority of the tests showed a distinct crack arrest (Fig. 4), but two tests at a very high value of applied K_I were inconclusive. The retardation period increased with lower K values and the results are shown in Fig. 5. It was noted that at very high K values, the crack arrested only for a

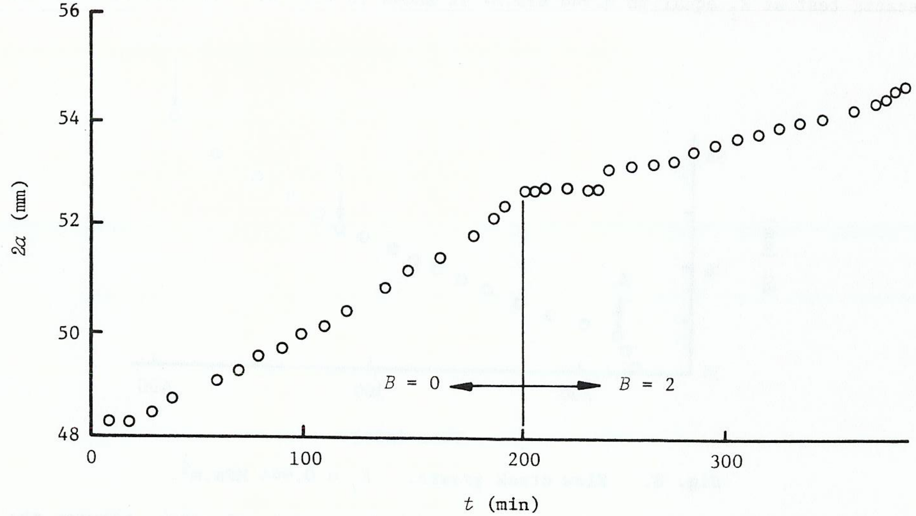

Fig. 4. Increasing load biaxiality. K_I = 0.923 MPa.m$^{\frac{1}{2}}$.

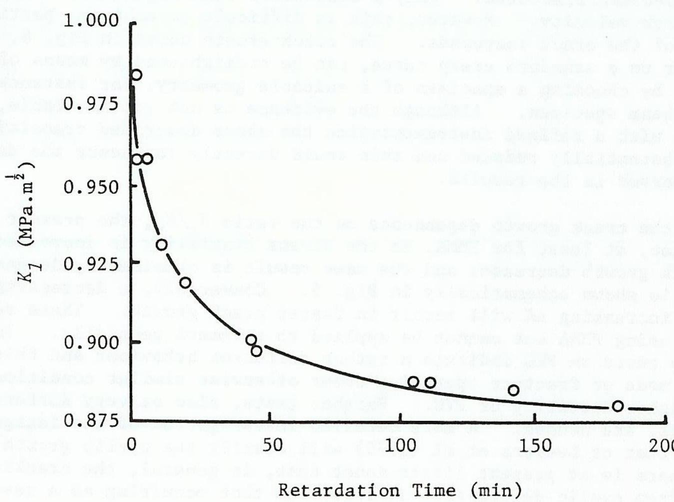

Fig. 5. Retardation time versus K_I

few moments, if at all. The longest retardation time recorded here was 180 minutes and this reduced to close to zero at K values above 0.970 MPa.m$^{\frac{1}{2}}$. Subsequently, the crack growth rate slowed down noticeably (Fig. 4). This reduction in the crack growth rate was recorded in all the tests, but the

difference in the growth rates decreased at higher stress intensities and it was apparent that this process may be connected with the changes in the retardation time. Thus, in the tests at high values of K, where the transient period decreased to zero, the recorded crack growth rate appeared to follow one continuous straight line.

Some discrepancies in this otherwise very regular behaviour were also noted. Two tests showed an unexpected decrease in the crack growth rate; here, the fractographic study of the crack front showed that the crack was growing only on one side with the front progressing at an angle. While these tests were not included in the analysis, it was possible to straighten the crack front by uniaxial fatiguing. It was also found that the crack growth in the tests performed at very high K values was no longer stable but was now in a regime where da/dt increased sharply, leading to fast fracture. Thus, in the test performed at K = 0.982 MPa.m$^{\frac{1}{2}}$, the fracture followed within one minute after the increase of the biaxiality ratio from 0 to 2.

The tests described above support the low cycle fatigue results reported recently by Leevers et al (1979). There, the fatigue crack growth rates had been measured on the same material and, as in the case of the static tests described above, transient deceleration as well as a permanent reduction in growth rate was recorded. In the present investigation, additional fatigue tests were performed and the propagation of the crack was consistent with that observed in the earlier tests referred to.

The purpose of the second group of static tests was to investigate the effects of decreasing the load biaxiality by reducing B from 2 to 0 (see Fig. 6). An

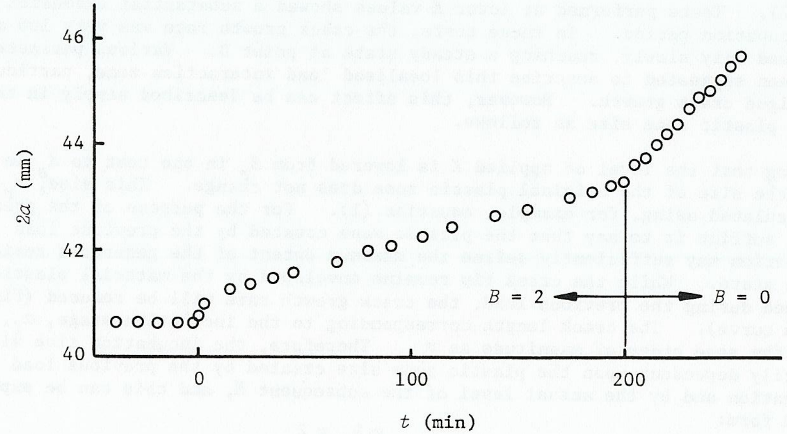

Fig. 6. Decreasing load biaxiality. K_I = 0.892 MPa.m$^{\frac{1}{2}}$.

increase in crack growth was recorded in all tests, and this was dependent on the ΔK applied. The growth change occurred immediately upon the reduction of the biaxiality ratio. However, at the beginning of a new block loading or on changing the biaxiality ratio, an incubation period was observed. The present results indicate two types of incubation process (Fig. 7). At high ΔK values, the incubation time was comparatively short and the crack growth rate fast. The crack velocity gradually decreased and a steady-state was reached at point A

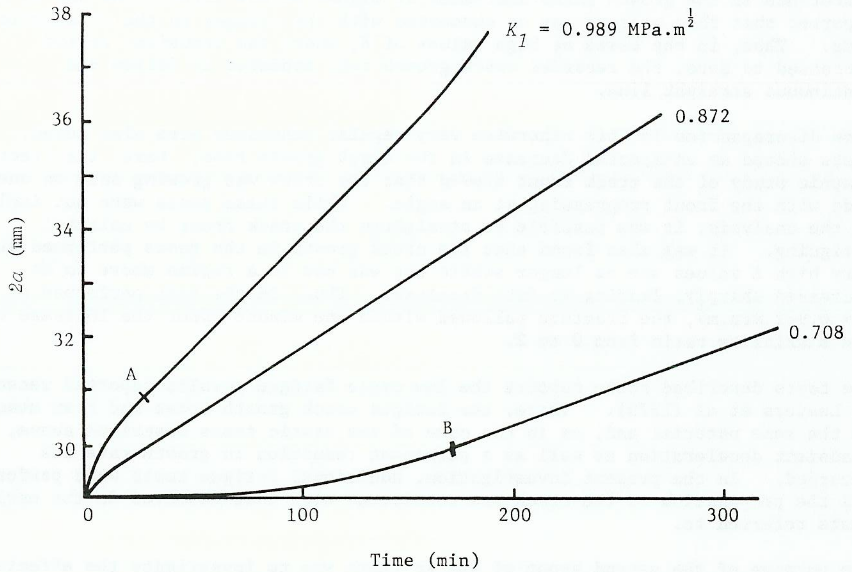

Fig. 7. Crack growth versus time

(Fig. 7). Tests performed at lower K values showed a substantial extension of the incubation period. In these tests, the crack growth rate was very low and increased only slowly, reaching a steady state at point B. Various parameters have been suggested to describe this localised load interaction zone, particularly in fatigue crack growth. However, this effect can be described simply in terms of the plastic zone size as follows.

Assuming that the level of applied K is lowered from K_r in one test to K_s in the next, the size of the original plastic zone does not change. This size, r_r, can be calculated using, for example, equation (1). For the purpose of the present study, suffice it to say that the plastic zone created by the previous load application may sufficiently define the maximum extent of the generated residual stress state. While the crack tip remains enveloped by the material plastically deformed during the previous load, the crack growth rate will be reduced (Fig. 7, bottom curve). The crack length corresponding to the incubation stage, a_i, would be of the same order of magnitude as r_r. Therefore, the incubation time will be primarily dependent upon the plastic zone size created by the previous load application and by the actual level of the subsequent K, and this can be expressed in the form:

$$t_i = C \left(\frac{K_r^2 - K_s^2}{K_s^2}\right)^n \tag{13}$$

where C and n are constants dependent on the material and, in particular, on its strain hardening behaviour.

With an increasing value of K_s, the influence of the loading history diminished, and when the ratio K_s/K_r equalled 1, a steady state of the crack growth was reached and the incubation period disappeared. This limiting case is shown in Fig. 7 as a straight line.

The crack velocity appeared to increase as the K_s was enlarged and then stabilised at a somewhat lower speed. It was concluded that at a constant ratio of K_s/K_r, the crack growth rate would also be constant. The crack growth recorded in a static test at K_1 equal to 0.944 MPa.m$^{\frac{1}{2}}$ is shown in Fig. 8. It will be observed

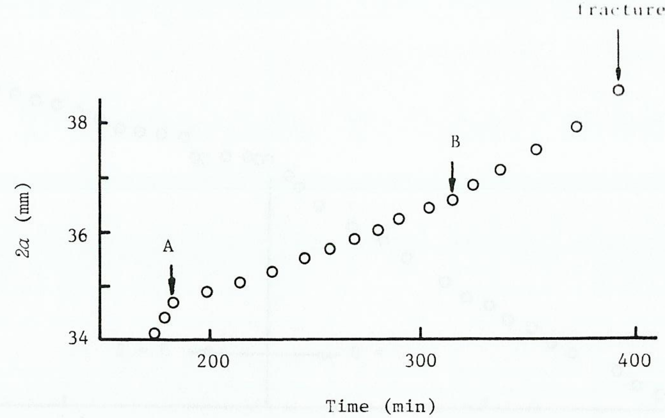

Fig. 8. Slow crack growth. $K_1 = 0.944$ MPa.m$^{\frac{1}{2}}$.

here that, at first, the crack propagated with decreasing velocity; between the points A and B, its speed was constant, after which it accelerated to the point at which the specimen fractured. Only a continuous load adjustment could ensure a constant crack velocity. However, this is difficult to achieve, particularly as the length of the crack increases. The crack growth curve in Fig. 8, which is very similar to a standard creep curve, can be straightened by means of a fine load control, or by choosing a specimen of a suitable geometry, for instance a double cantilever beam specimen. Although the evidence is not yet available, it is likely that with a refined instrumentation the above described transition effects could be substantially reduced and this would directly influence the degree of scatter observed in the results.

Apart from the crack growth dependence on the ratio K_s/K_r, the present static tests indicate that, at least for PMMA, as the stress biaxiality is increased the speed of the crack growth decreases and the same result is obtained by decreasing the ΔK values, as is shown schematically in Fig. 9. Conversely, a decreasing biaxiality as well as increasing ΔK will result in faster crack growth. These results were consistent using PMMA but cannot be applied to polymers generally. In fact, preliminary tests on PVC indicate a rather different behaviour and this may be due to another mode of fracture operating under otherwise similar conditions and/or to the much higher ductility of PVC. Further tests, also at very different temperatures, are needed. A more detailed investigation of the fatigue process, similar to that of Leevers et al (1979) will clarify the cyclic growth conditions, although there is at present little doubt that, in general, the cracking process resulting from cyclic deformation differs from that occurring as a result of monotonic deformation.

CONCLUSIONS

A stable crack growth in PMMA was investigated under plane strain conditions and at 21°C. It was found that:

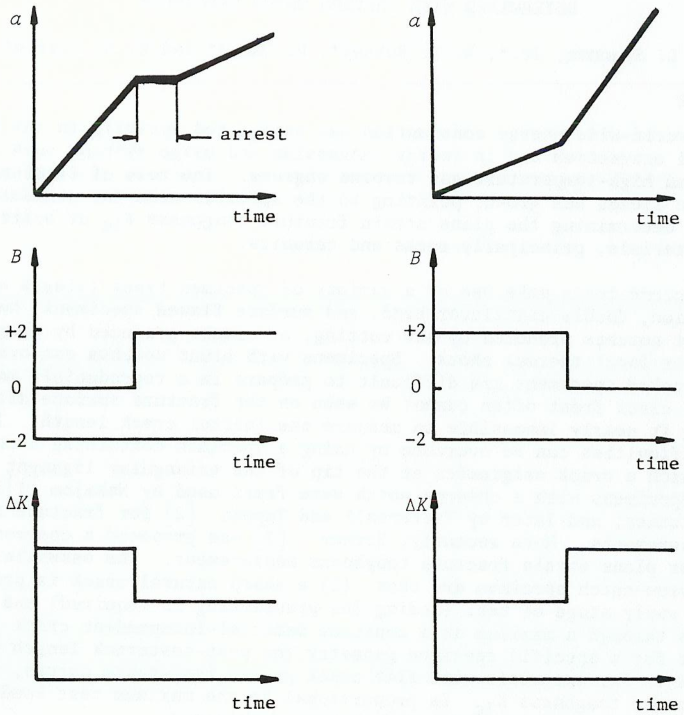

Fig. 9. Transient effects of load biaxiality and of ΔK (schematic)

1. The crack growth will substantially decelerate as the biaxiality is increased or the ΔK is decreased.

2. Conversely, the crack growth will accelerate as the ΔK is increased or the biaxiality is decreased.

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