ON CRACK PROPAGATION IN ROCK FOUNDATIONS OF MASSIVE CONCRETE DAMS

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ABSTRACT

The study is aimed at the investigation of the transition of concrete dams and their foundations to the limit equilibrium state on the basis of the theory of brittle fracture. Probable development of a crack along an experimentally found trajectory is considered. The stress intensity factors are obtained by the finite element method with the use of a special singular element which enables the stress concentration in the region of a crack tip to be taken into account. A calculation example presents the evaluation of the stress intensity factors along an experimental trajectory of a primary crack formed in a model structure on rock foundation. The conclusion drawn emphasizes the necessity of accumulating both experimental and calculation data to establish more reliable safety factors of dams.

KEYWORDS

Concrete dam; rock foundation; limit state; plate shear test; experimental crack trajectory; stress intensity factor; singular finite element.

The governing criterion in the safety analysis of a concrete dam on rock foundation is the shearing resistance along the structure-foundation interface

$$f \cdot N + c \cdot S \geqslant k \cdot F. \qquad (1)$$

Here f and c are the parameters of friction and bond between concrete and rock, S is the dam base area, N and F are the vertical and horizontal forces exerted on a dam, k is the safety factor (Fig. 1a). The f and c values are generally established by shear tests of concrete plates (Fig. 1b).

From a physical point of view the criterion (1) means that the dam under shearing and frictional forces applied horizontally is to retain an equilibrium state.

However, according to the experimental results the actual fracture mechanism is of far greater complexity than that assumed for (1). With increasing a horizontal load the shear crack on the concrete-rock interface (Fig. 2a) may develop only in the presence of a contact layer with considerably lower strength than a plate and a foundation. When such a layer is not available and concrete and rock strengths are not much different the primary crack (Fig. 2b) sets up in the foundation (Fishman, 1979).

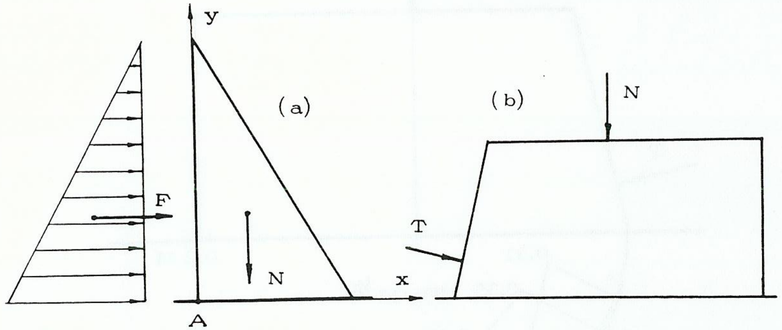

Fig. 1. Forces applied to a dam and a test plate.

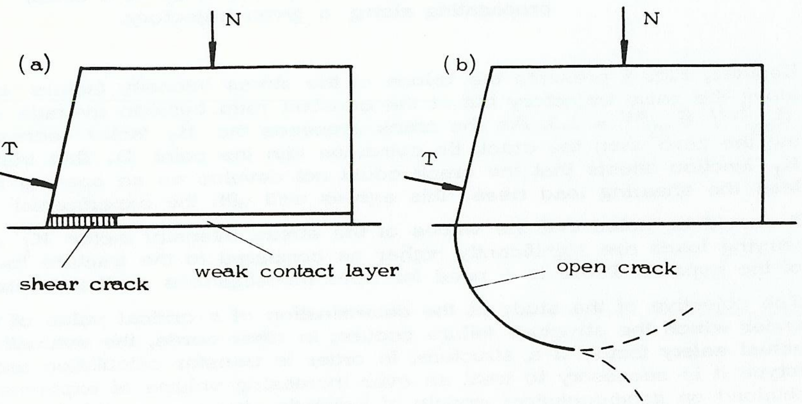

Fig. 2. The mode of failure in shear tests of a plate: the development of a shear crack in a weak contact layer (a); the development of an opening mode crack in foundation (b).

As a rule, a high concrete dam is built of high quality concrete on sound rock. The dam foundation surface is adequately treated, the blanket grouting being used for consolidation of a weak top layer, if necessary. So most probable pattern of fracture is the development of a primary crack (initiating at point A, Fig. 1a) under the upstream face of a dam.

A dam is designed so that normal and tangential stresses, σ_{yy} and σ_{xy}, respectively, in rock under the upstream face close to the structure-foundation interface are very low while the normal stresses σ_{xx} are high. Under such a distribution of stresses a primary crack starts as an opening mode crack oriented vertically and propagates into the foundation. When the orientation of principal stresses varies due to cracking the above mentioned crack extends along a trajectory similar to an arc with the centre in the middle of the dam base and the radius equal to a half-width of the base.

Having reached the vertical section crossing the middle point of the base the crack turns either downwards into the foundation, or upwards, to a dam toe, depending on an external load combination. When the crack crosses the middle section, the dam is assumed to reach its limiting state. The risk of the dam failure arises due to an appreciable increase in the seepage rate and uplift pressures in rock.

Further development of a crack once app-eared can be analyzed by the methods of the linear brittle fracture mechanics. For this the character of rock fracture along a crack trajectory should be evaluated on the basis of experimental data. First of all the normal and tangential stress intensity factors K_I and K_{II} near the crack tip are to be established. They are defined as

$$K_I = \lim_{r \to 0} \sqrt{2\pi r}\, \sigma_{nn}, \qquad K_{II} = \lim_{r \to 0} \sqrt{2\pi r}\, \sigma_{n\tau},$$

where σ_{nn}, $\sigma_{n\tau}$ are the stresses along a crack extension, r is the distance to a crack tip (Fig. 3a).

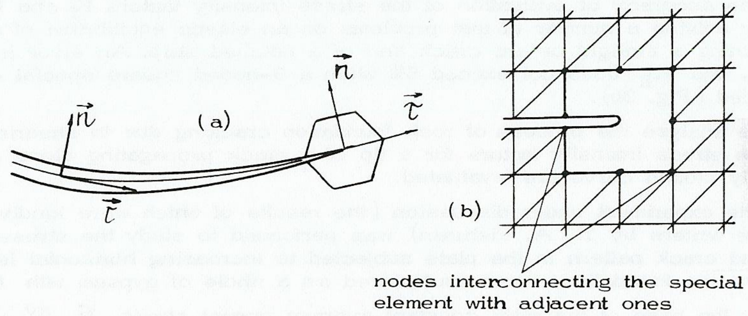

Fig. 3. A region of a crack tip and the location of a special element (a); a 9-noded square as a special element (b).

Stresses and strains in a solid containing a crack as well as the factors of stress intensity at a crack tip are obtained by the finite element method. The crack is simulated by a cut with perfect unilateral constraints imposed on its surfaces

$$\sigma_{n\tau} = 0; \quad \sigma_{nn} - P \leqslant 0;$$

$$\Delta U_n + \delta \geqslant 0; \quad (\sigma_{nn} - P)(\Delta U_n + \delta) = 0.$$

Here σ_{nn}, $\sigma_{n\tau}$ are the stresses at points of the crack surface; P is the prescribed distributed load; ΔU_n is the normal displacement of points at one of crack surfaces relative to those at the other (crack opening); δ is the initial opening. In the case under consideration the initial opening is zero; P may be either taken as zero or used for simulating the seepage water pressure.

To take account of the stress concentration around a crack tip a special singular element is used in addition to ordinary finite elements. It is shaped as a symmetrical polygonal with an edge notch along the axis of symmetry (Seiliger, Khrapkov, 1980). Its stiffness matrix and influence vectors interrelating stress intensity factors for a notch root and nodal displacements are determined through an analytical solution to the problem of an elastic equilibrium of a polygonal solid with given boundary displacements, ensuring the inter-element compatibility. The solution of the elasticity problem for a symmetrical polygonal is developed by means of Kolosov-Muskhelishvili's complex potentials (Muskhelishvili, 1954). The special element covers a region of a crack tip (Fig. 3a), replacing a number of conventional elements and not interfering with the mesh over the rest area. The number of nodes interconnecting the special and the adjacent elements is arbitrary. Crack propagation is simulated by transferring the special element, the mesh being not changed except closely to the crack tip.

The use of the above element in combination with perfect unilateral constraints imposed on crack surfaces may permit to investigate both local processes (the stress concentration) in the region of the crack tip and the crack pattern on the entire length (the extent of crack opening, the presence of closed stretches, etc.).

The procedure described is involved into the BESM-6 computer program arranged for the two-dimensional elasticity problem formulated with perfect unilateral constraints (Vovkushevsky, 1978). Simple triangular elements with the piecewise linear approximation of displacements are used in the program.

The accuracy of evaluation of the stress intensity factors K_I and K_{II} is verified by solving a number of test problems on an elastic equilibrium of a plane containing a straight or arc crack and of a notched strip. An error in calculating K_I and K_{II} does not exceed 5% when a 9-noded square special element is applied (Fig. 3b).

To analyze the process of rock foundation cracking due to shearing a test plate the stress intensity factors for a tip of a crack propagating along an experimentally plotted curve are evaluated.

The experiment under discussion (the results of which were kindly suggested to the writers by Yu. A. Fishman) was performed to study the stress distribution and crack pattern in the plate subjected to increasing horizontal load. The plate and the foundation were manufactured as a whole of gypsum with E = 4000 MPa.

At the base of the plate constant average normal stress $\sigma_{yy}^{av} = 0.4$ MPa was created and maintained by rejecting portions of the force N with the increase in the force T (Fig. 4). With increasing a shear load a crack set up and opened under the face to which the force T is applied. On the crack trajectory (Fig. 4) the points A, B and C can be seen; they illustrate the location of the crack tip at the ratio $\sigma_{xy}^{av}/\sigma_{yy}^{av}$ equal to 1.5, 2.0 and 2.5, respectively, σ_{xy}^{av} being the average tangential stress over the plate base. For these points the stress intensity factor values under corresponding loads applied are found. The K_I values are given in Fig. 4, the K_{II} values appear to be an order less than K_I. This shows that the trajectory under consideration is pretty close to that of the opening mode crack.

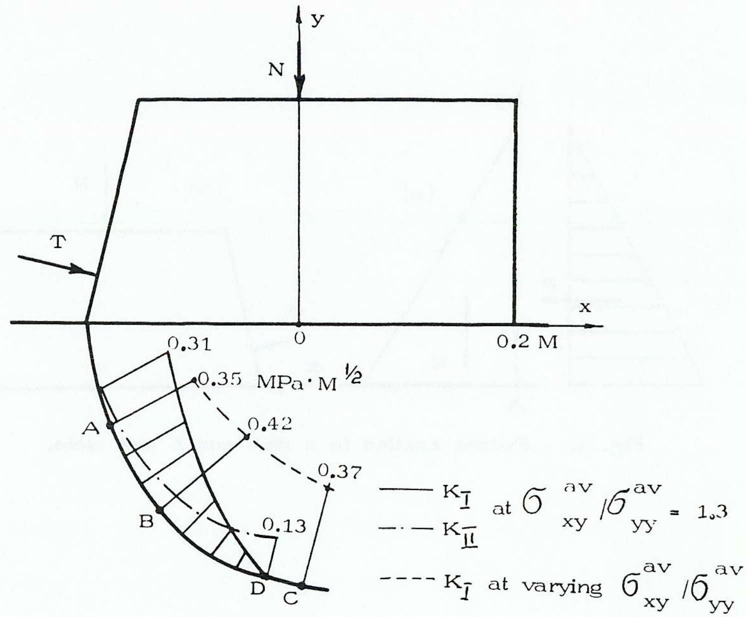

Fig. 4. Stress intensity factors near a tip of a crack propagating along a given trajectory.

Besides, Fig. 4 presents the values of the stress intensity factors K_I and K_{II} along the same trajectory but at the constant ratio between average stresses $\sigma_{xy}^{av}/\sigma_{yy}^{av} = 1.3$. As the crack proceeds the K_I factor decreases reaching the zero when the crack tip coincides with the point D. This behaviour of the K_I function attests that the crack could not develop as an opening mode one unless the shearing load rises. This agrees well with the experimental results.

It should be noted that the values of the stress intensity factor K_I obtained at varying loads are significantly higher as compared to the fracture toughness K_{Ic} of the gypsum. There is a need for more investigations on the problem.

The objective of the study is the determination of a critical value of the load under which the structure failure occurs; in other words, the evaluation of the actual safety factor of a structure. In order to transfer calculation data to a prototype it is necessary to treat an ever increasing volume of experimental results obtained on geomechanical models of materials whose properties approach those of rock. Accumulation of experimental data enables the onset of the limit equilibrium state of a dam-foundation system to be more exactly predicted.

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