K-DETERMINATION IN MIXED-MODE
CRACK PROBLEMS BY INTERFEROMETRY

H.P. Rossmanith* and D.C. Holloway**

* Inst. f. Mechanics, Technical University Vienna, Austria
** Dept. of Mechanical Engng., University of Maryland
College Park, MD., USA

ABSTRACT

Analytically generated isopachic fringe patterns around crack tips subjected to mixed-mode stress conditions are compared with experimentally recorded fringe patterns obtained by interferometric techniques. A Westergaard type stress function technique is utilized for fringe pattern construction. Multi-parameter and multiple-point data reduction methods recently developed were employed for optimal determination of stress intensity factors.

KEY WORDS

Biaxial crack problem, interferometry, isopachics, mixed-mode stress intensity factors.

INTRODUCTION

The vicinity of a crack or flaw in a solid body is a region of high stress concentration if the material is subjected to certain loading situations. By utilizing fracture mechanics concepts it is possible to characterize the stress singularity at a static crack tip by means of stress intensity factors K_1, K_2, and K_3. In plane elasticity problems the stress intensity factors K_1 and K_2 are associated with opening mode (mode-1) and shearing mode (mode-2) crack deformation.

Considerable effort has been devoted to both experimental and theoretical determination of K-factors for a variety of specimen configurations and various loading situations. Among the optical techniques which have proven very powerful for K-determination, interferometry has been employed for mode-1 crack problems (Irwin and co-workers, 1980). This study involves a comparison of analytically generated static isopachic crack tip fringe patterns with experimentally recorded interferometric patterns. The effect of higher order terms in the stress function series expansion on the shape of the isopachics is investigated.

ANALYSIS

Consider a semi-infinite straight crack in an isotropic, homogeneous, elastic plate specimen subjected to in-plane loading as shown in Fig. 1. Employing a generalized Westergaard type stress function approach (Sanford, 1979; Rossmanith, 1979) the stress components σ_x, σ_y, τ_{xy} for a crack tip subjected to mixed-mode stress loading are obtained by superposition of the stress fields $\sigma_{x(j)}$, $\sigma_{y(j)}$, and $\tau_{xy(j)}$ $(j = 1,2)$ of the individual modes:

$$\text{mode 1:}\quad \sigma_{x(1)} = \mathrm{Re}(Z_1 + 2Y) - y\,\mathrm{Im}(Z_1' + Y')$$
$$\sigma_{y(1)} = \mathrm{Re}\,Z_1 + y\,\mathrm{Im}(Z_1' + Y') \qquad (1)$$
$$\tau_{xy(1)} = -y\,\mathrm{Re}(Z_1' + Y') - \mathrm{Im}\,Y$$

$$\text{mode 2:}\quad \sigma_{x(2)} = 2\,\mathrm{Im}\,Z_2 + y\,\mathrm{Re}\,Z_2'$$
$$\sigma_{y(2)} = -y\,\mathrm{Re}\,Z_2' \qquad (2)$$
$$\tau_{xy(2)} = \mathrm{Re}\,Z_2 - y\,\mathrm{Im}\,Z_2'$$

where Re and Im denote real and imaginary parts of a complex function.

The stress functions Z_j and Y are suitably chosen in the form

$$Z_j(z) = \sum_{n=0}^{N} B_{nj}\, z^{n-\frac{1}{2}} \qquad (j = 1,2) \qquad (3)$$

and

$$Y(z) = \sum_{m=0}^{M} A_m z^m \qquad (4)$$

where $z = x + iy = re^{i\theta}$, the coefficients A_m and B_{nj} are real constants, and $K_j = B_{0j}\sqrt{2\pi}$. If, in general, the higher order terms B_{nj} $(n > 1)$ are unequal for different modes (i.e., $B_{n1} \neq B_{n2}$) the mixed-mode situation is called non-isomorphic (Rossmanith, 1979). If $B_{n1} = B_{n2}$, the mixed-mode situation is called isomorphic.

Classical interferometry is sensitive to thickness and index of refraction changes in plane models usually employed in experiments. These changes can be related to the first stress invariant $I = \sigma_x + \sigma_y$, and by eqs. (1)-(4), I becomes

$$\tfrac{1}{2}I = \mathrm{Re}(Z_1 + Y) + \mathrm{Im}\,Z_2$$
$$= \sum_{n=0}^{N} r^{n-\frac{1}{2}}[B_{n1}\cos(n-\tfrac{1}{2})\theta + B_{n2}\sin(n-\tfrac{1}{2})\theta] + \sum_{m=0}^{M} A_m r^m \cos m\theta. \qquad (5)$$

For many applications the two lowest higher order terms $(N = 1,\ M = 0)$ deserve retention only. Then, the solution of the equation

$$\tfrac{1}{2}I = \frac{K_1}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2}(1 + r\beta_1) - m\sin\frac{\theta}{2}(1 - r\beta_2)\right] + A_0, \qquad (6)$$

where $\beta_j = B_{1j}/B_{0j}$ and $m = K_2/K_1$, gives the governing formula for the shape of the contours of constant $\sigma_x + \sigma_y$. Such lines have traditionally been known as isopachic lines or lines of constant specimen thickness. When I is determined optically, fringes result from the superposition of two interference patterns. The first when the specimen is in its unloaded state and the second in its loaded state.

The fringe order of the isopachic pattern, N is related to the stress invariant by

$$I = N f_p / t \qquad (7)$$

where t is the specimen thickness, and f_p is an isopachic fringe constant that is most easily determined by calibration with a known stress state, i.e., a disk under diametrial load.

EXPERIMENTAL PROCEDURE

There are several methods of obtaining isopachic patterns, and in this study a series interferometer (Post, 1955, 1956) along with a HeNe laser was used. The experimental arrangement is shown in Fig. 2. The inset shows the light paths through the specimen. The two emerging rays form an interference pattern from light rays that have traversed the model once and twice respectively. By adjusting the first mirror a grid of interference lines can be produced and recorded by the camera. A stop is placed in the focal plane of decollimating lens so as to prevent other rays from entering the camera. The interference grid is an indication of the taper of the specimen as well as the orientation of the mirrors. For the purpose at hand the mirrors were aligned to produce a fine pitch. A single exposure was made with the specimen in a slight preload. The specimen was then loaded and the second exposure taken. The resulting doubly exposed film is thus a moire of the two interference patterns, and the moire fringes are the isopachic contours. Polaroid PN 55 film was used for the experiment.

RESULTS AND DISCUSSION

Several special cases of eqn. (6) will be considered in detail.

(a) Singular mixed-mode isopachic $(\beta_1 = \beta_2 = 0)$:

The governing formula takes the simple form

$$r_0 \equiv r(\theta) = K_0^{\,2}\left(\cos\frac{\theta}{2} - m\sin\frac{\theta}{2}\right)^2 \qquad (8)$$

with

$$K_0 = K_1(2\pi)^{-\frac{1}{2}}(I/2 - A_0)^{-1}. \qquad (9)$$

A series of selected isopachic crack tip fringe patterns pertaining to various values of the mixed-mode ratio $m = K_2/K_1$ are shown in Figs. 3a-f, where Figs. 3a and 3b are associated with pure mode-1 loading situation. Notice the close similarity between the analytically generated and experimentally recorded fringe patterns.

The integer higher order term of lowest order, A_0, does not affect the shape of the isopachic, only its size. The parameter A_0 may be determined by considering the intersection points $P_i(r_i,0)$ of two different isopachics of known invariants I_i and I_j with the positive branch of the x-axis. Employing eqn. (9) one obtains (Der, 1978; Rossmanith, 1979)

$$A_0 = \frac{1}{2}\frac{I_i\sqrt{r_i} - I_j\sqrt{r_j}}{\sqrt{r_i} - \sqrt{r_j}} \tag{10}$$

(b) Higher order term mixed-mode isopachic ($\beta_1 \neq \beta_2 \neq 0$):

Equation (6) represents a quadratic equation for $R = \sqrt{r}$ and hence can be solved exactly:

$$r = r(\theta) = \frac{1}{(2F_1K_0)^2}[1 \pm \sqrt{1 - 4F_0K_1K_0^2}]^2 \tag{11}$$

where

$$F_0 = \cos\frac{\theta}{2} - m\sin\frac{\theta}{2}$$
$$F_1 = \beta_1\cos\frac{\theta}{2} + m\,\beta_2\sin\frac{\theta}{2}\,. \tag{12}$$

If the higher order parameters, however, are very small, a perturbation analysis may be applied where the smallest of the β-parameters is considered a small perturbation. This leads to the representation:

$$r = r(\theta) = r_0[1 + 2\Omega + 5\Omega^2 + O(\Omega^3)] \tag{13}$$

with $\Omega = K_0^2F_1F_0$.

Figures 4a-d show the influence of various combinations of the higher order parameters β_1 and β_2 on the shape of an isopachic fringe loop.

Stress loading situations with predominant mode-2 contribution and the limiting case of pure mode-2 deserve special attention. Equation (6) then becomes

$$\tfrac{1}{2}I = \frac{K_2}{\sqrt{2\pi r}}[\frac{1}{m}\cos\frac{\theta}{2}(1 + r\beta_1) - \sin\frac{\theta}{2}(1 - r\beta_2)] + A_0 \tag{14}$$

which, for pure mode-2, reduces to

$$\tfrac{1}{2}I = \frac{|K_2|}{\sqrt{2\pi r}}\sin\frac{\theta}{2}(1 - r\beta_2) + A_0 \tag{15}$$

Analytically generated and experimentally recorded isopachic fringe patterns are compared in Figs. 3e and 3f, whereas the influence of the higher term B_{12} on the shape of the mode-2 isopachic is shown in Fig. 4e. The mode-2 isopachic pattern follows from a mode-1 pattern upon replacement of K_1 by K_2 and the transformation $\theta \to \theta - \pi$.

Advanced techniques based on multiple-point data acquisition of the overall isopachic fringe pattern may advantageously be employed for effective determination of K_1, K_2, and the higher order parameters. The overdeterministic data reduction method (Sanford and Dally, 1979), may easily be adopted for isopachic fringe patterns (Barker, Fourney and Holloway, 1980).

Generalization of the isopachic method to dynamic mode-1 crack propagation problems is possible under certain restrictions from both the theoretical and experimental point of view. The analysis of a crack traveling along a curved crack path under mixed-mode stress conditions is subject of intense research and offers considerable mathematical complexities.

ACKNOWLEDGEMENT

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REFERENCES

Barker, D.B., W.L. Fourney, and D.C. Holloway (1980). Comparison of stress-intensity factors determined from isochromatics, isopachics and caustics. SESA-Spring Meeting, Boston.

Der, V. (1972). Techniques for Dynamic Fracture Toughness Measurements in Opaque Materials. Ph.D. Thesis, University of Maryland.

Irwin, G.R., and Co-workers (1980). Photoelastic Studies of Damping, Crack Propagation, and Crack Arrest in Polymers and 4340 Steels. NRC-Report No. NUREG/CR-1455, University of Maryland.

Post, D. (1955). A new photoelastic interferometer suitable for static and dynamic measurement. Proc. SESA, Vol. XII, No. 1, 191-201.

Post, D. (1956). Photoelastic evaluation of individual principle stresses by large field absolute retardation measurements. Proc. SESA, Vol. XIII, No. 2, 119-132.

Rossmanith, H.P. (1979). Analysis of mixed-mode isochromatic crack-tip fringe patterns. Acta Mechanica, 34, 1-38.

Rossmanith, H.P. (1979). Dynamic stress-intensity-factor determination from isopachics. Exp. Mech., 19, 281-285.

Sanford, R.J. (1979). A critical re-examination of the Westergaard method for solving opening mode crack problems. Mech. Res. Com., 6, 289-294.

Sanford, R.J., and J.W. Dally (1979). A general method for determining mixed-mode stress intensity factors from isochromatic fringe patterns. Eng. Fract. Mech., 11, 621-633.

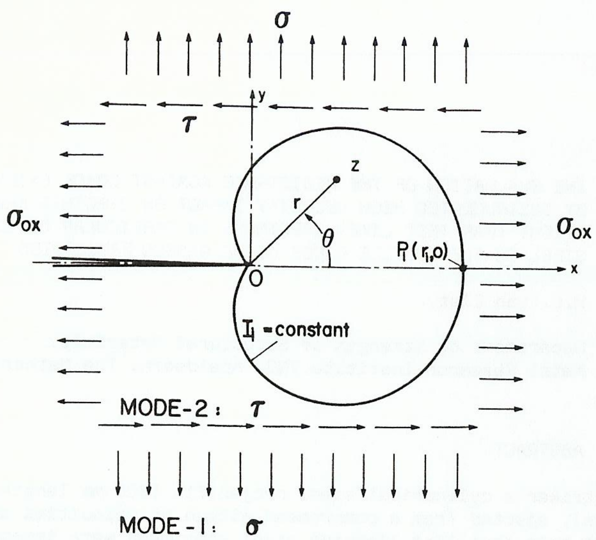


Fig. 1 Cracked plate specimen subjected to mixed-mode in-plane stress loading

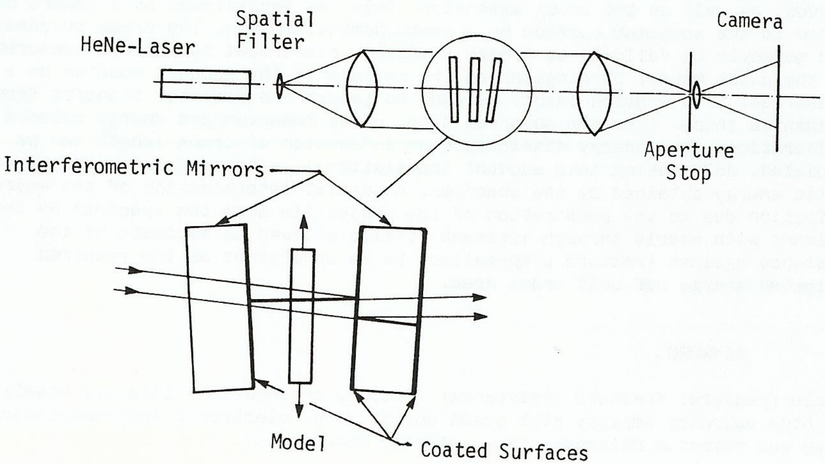
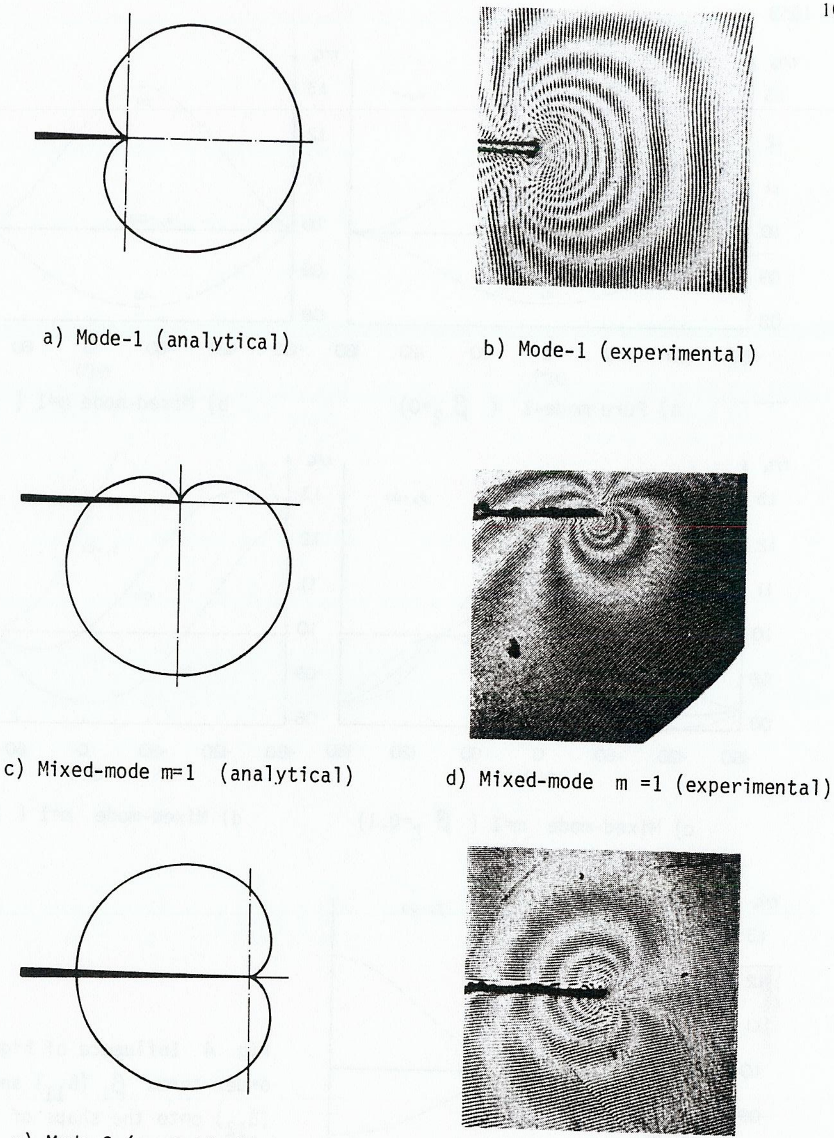


Fig. 2 Optical arrangement for interferometric recording of isopachics about the tip of a static crack



a) Mode-1 (analytical)

b) Mode-1 (experimental)

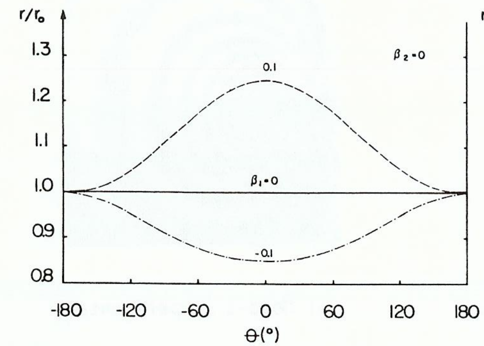
c) Mixed-mode m=1 (analytical)

d) Mixed-mode m =1 (experimental)

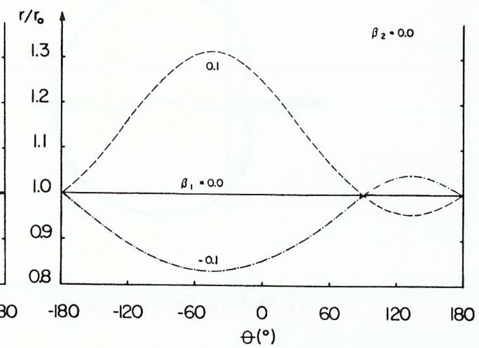
e) Mode-2 (analytical)

f) Mode-2 (experimental)

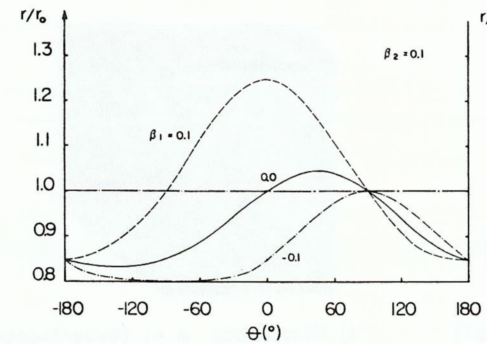
Fig. 3 Analytically generated and experimentally recorded isopachic fringe patterns around a static crack tip subjected to mixed-mode stress loading (A_0=0)



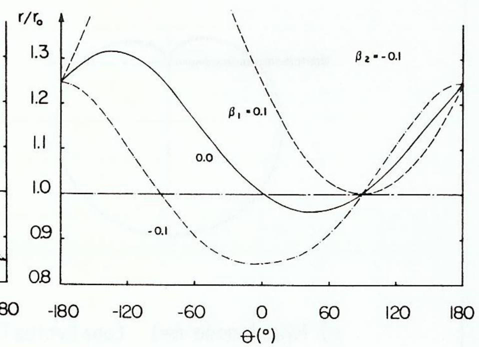
a) Pure mode-1 (β_2=0)



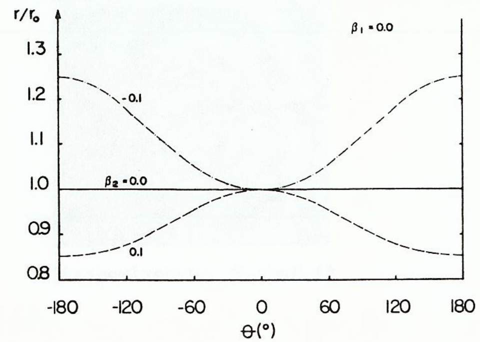
b) Mixed-mode m=1 (β_2=0)



c) Mixed-mode m=1 (β_2=0.1)



d) Mixed-mode m=1 (β_2=-0.1)



e) Pure mode-2 (β_1 =0)

Fig. 4 Influence of higher order order terms β_1 (B_{11}) and β_2 (B_{12}) onto the shape of an isopachic crack tip fringe pattern