GEOMETRY AND SIZE REQUIREMENTS FOR FATIGUE LIFE SIMILITUDE AMONG NOTCHED MEMBERS

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ABSTRACT

The question is posed: Under what conditions of geometry, size, and loading is the same fatigue life expected for two different notched members, such as laboratory specimens versus actual components in service? Equivalent loading in terms of the elastically calculated notch stress is expected to result in similar notch strains, hence in similar crack initiation lives. In addition, the laboratory specimen geometry and size can be tailored based on fracture mechanics so that similar crack propagation lives occur, resulting in similar total fatigue lives.

KEYWORDS

Fatigue, notch, cycling, specimens, components, simulation, strain, stresses, fracture mechanics

INTRODUCTION

In dealing with fatigue problems in notched components under service loading, it is sometimes desirable to design and test notched laboratory specimens which are expected to have similar fatigue lives as a particular type of component. In other situations, a judgment must be made concerning the suitability of existing notched specimen data for analysis of a particular component. As laboratory specimens versus actual components often differ considerably as to geometry, loading mode, or size, two questions are raised: 1) What aspects of geometry, loading mode, and size must be related between laboratory specimen and component? 2) How should the magnitudes of loading for specimen and component be matched?

Answers to the above two questions are proposed and subjected to experimental test. Fatigue crack initiation and propagation are both considered. The discussion is simplified by considering only two dimensional situations with a single applied load. Some important aspects of laboratory simulation of components are beyond the scope of this paper: determination of load histories, material condition and surface finish, and time-dependent effects of hostile chemical or thermal environments.

FATIGUE CRACK INITIATION

The widely employed hypothesis is adopted that initiation of fatigue cracks is controlled by the notch surface strain. Thus, fatigue crack initiation is considered from the viewpoint of designing and loading a notched laboratory specimen to produce the same notch strain as in a given component.

Local Strains at Notches

For three of the notched specimens of Fig. 1, Fig. 2 gives notch strain from elastic-plastic finite element analysis. Notch surface strain from analysis is plotted versus k_tS, the notch stress calculated assuming elastic behavior, where k_t and S are, respectively, elastic stress concentration factor and nominal stress.

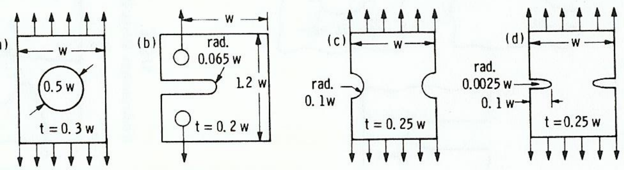

Fig. 1. Notched specimens tested: (a) center hole, (b) compact, (c) blunt double edge notch (DEN), and (d) sharp DEN. In each, w = 25.4 mm, t = thickness.

TABLE 1 Test Specimen Parameters

Specimen	k_t	k_p	ℓ'/r	ℓ', mm	k_t'
Center Hole	4.24[1]	2.12	0.071	0.451	2.37[1]
Compact	2.62[2]	4.31	0.180	0.297	2.41[2]
Blunt DEN	3.03[1]	2.42	0.099	0.252	2.03[1]
Sharp DEN	13.4[1]	10.7	0.185	0.012	12.9[1]

[1]Value based on axial load averaged over gross area.
[2]Value based on tension and elastic bending on net section.

These analyses assume plane stress, and they employ the stable, or half-life, cyclic stress-strain curve for an AISI 4340 steel, which curve is also shown in Fig. 2. The analyses were performed just as for monotonic loading, except for the use of a cyclic stress-strain curve. Similar analysis is described in more detail in a paper by Dowling and Wilson (1979), where it is indicated that such analysis is valid for cyclic loading, provided that creep/relaxation effects of both the time dependent and cycle dependent types can be neglected.

Concerning Fig. 2, note that values of k_tS for each member are related to applied load by a constant obtained from elastic analysis. Although the value of k_t for a given member depends on the choice of a definition for S, such as the use of net section versus gross section area, values of the product k_tS are independent of this choice. The plotting of k_tS versus notch strain in Fig. 2 causes all of the analytical results to coincide where $k_tS < \sigma_y$, the cyclic yield stress.

For each member in Fig. 2, there are three regions of behavior: 1) simple elastic behavior, 2) local notch yielding under gross deformations that are elastic, and 3) general yielding. The transition between the first and second regions occurs at the same point for all geometries, specifically at $k_tS = \sigma_y$. At the transition marking the beginning of general yielding, the strain begins to increase rapidly with k_tS, with the k_tS value where this occurs being geometry dependent. The transition to general yielding is evident in Fig. 2 for the center hole and blunt DEN specimens, but for the compact specimen the analysis was terminated prior to general yielding.

In Fig. 2, where $k_tS > \sigma_y$, the strain is in all cases larger than the strain from the dashed line representing the elastic solution. However, if only k_tS values not involving general yielding are considered, an approximate correlation exists among the three specimens between k_tS and notch strain. Additional analytical results in support of this trend have been presented by Wilson (1974). This correlation, along with the hypothesis that notch strain controls fatigue crack initiation, suggests that fatigue crack initiation lives for various notched members should be approximately the same on a plot of k_tS versus life.

Fatigue data confirming this trend are plotted in Fig. 3 for the same material and specimens as in Fig. 2. The data in Fig. 3 are for completely reversed constant amplitude loading, and the lives plotted correspond to initiation of a specific small crack size. Note that a reasonable correlation prevails except at short lives. The trends observed at short lives are consistent with the k_tS versus strain curves of Fig. 2. In particular, wherever k_tS is increased into general yielding the life decreases rapidly due to the large strains which occur.

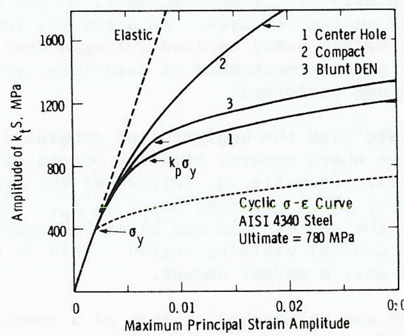

Fig. 2. Notch surface strains from finite element analysis.

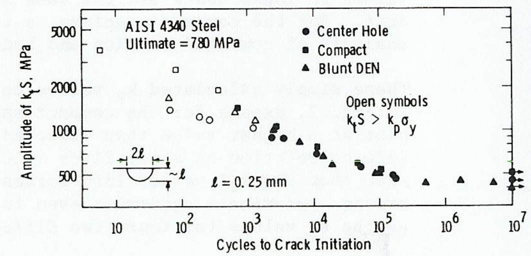

Fig. 3. Fatigue crack initiation life data.

General Yielding Behavior

General yielding can in many cases be predicted with sufficient accuracy using simple mechanics of materials type analysis. In other cases, notably where effective elevation of the yield strength due to biaxial or triaxial constraint occurs, more sophisticated analysis may be required, as by finite elements or slip-line fields. For example, the yield strength is considerably elevated in axially loaded round bars containing deep, sharp, circumferential grooves.

Following Seeger (1977), it is useful to define the quantity:

$$k_p = \frac{\text{S at fully plastic limit load}}{\text{S at first notch yielding}} = \frac{S_p}{S_y}$$

in which S_p is calculated for an ideal elastic, perfectly plastic material of yield strength σ_y. Since $k_t S_y = \sigma_y$,

$$k_t S_p = k_p \sigma_y \quad (1)$$

For a material with strain hardening, Eq. 1 can be used to estimate the $k_t S$ value where general yielding begins.

From Eq. 1, two notched members of the same material, hence with the same σ_y, are expected to begin general yielding at the same $k_t S$ value if their k_p values are the same. Matching of k_p values is also expected to result in similar $k_t S$ versus strain curves in the general yielding region. This further consequence of matching k_p values is due to the notch strain, ε, during general yielding being related to applied load by the strain hardening exponent of the material, n, so that

$$k_t S = C\varepsilon^n \quad (2)$$

with the coefficient C being approximately the same for two members having the same k_p values.

Values of k_p which ignore any constraint effects are given in Table 1 for the four notched members of Fig. 1. For the three axially-loaded specimens, k_p was calculated by simply assuming fully plastic yielding on the net area. As a result, the k_p values in these cases are the same as k_t values based on load averaged over the net area. For the compact specimen, a fully plastic mechanics of materials type analysis of combined tension and bending was performed.

These simply calculated k_p values correlate with the beginning of general yielding in Fig. 2, except for the compact specimen where general yielding occurs off the plot at a higher value than expected. Also, from Fig. 3, failure of the $k_t S$ versus life correlation at short lives occurs only where expected based on k_p. However, note that the $k_t S$ versus life curves for the center hole and blunt DEN specimens are in approximate agreement even in the general yielding region. This is expected as the k_p values for these two differ by only a modest amount.

The value of k_p for a laboratory specimen may be matched to that of a component by trial and error design. If constraint effects are ignored, k_p increases with any geometry change that increases k_t, and also generally with increased degree of bending, that is, increased eccentricity of the load. Note that ignoring constraint effects gives lower bound values of k_p. Hence, if such values are used, general yielding effects are never expected for $k_t S < k_p \sigma_y$.

FATIGUE CRACK PROPAGATION

In notched members, a significant or even dominant fraction of the fatigue life may be spent in crack propagation (Dowling, 1979). Therefore, fatigue life similitude is also considered from the viewpoint of fracture mechanics, with emphasis on the linear elastic stress intensity, K. Since equivalent $k_t S$ loading is required for simulation of the crack initiation life, the load magnitude is fixed. Hence, the geometry of the specimen must be adjusted so that the K versus crack length relationship is similar to that of the component, which is expected to result in similar crack growth lives.

Crack Growth Within the Local Notch Field

For cracks growing from notches, as in Fig. 4, detailed stress intensity solutions, such as those of Newman (1971), are available for a number of particular cases. These are all qualitatively similar. A typical analytical result is given in Fig. 5, specifically for the circular hole in an infinite plate case of Fig. 4b, where a uniaxial stress, S, is remotely applied normal to the crack plane. In Fig. 5, the dashed line is the actual numerical solution, and the two solid lines are limiting cases for short and long cracks. Note that the numerical solution first follows the short crack limiting case, then there is a transition in behavior followed by approximate agreement with the long crack limiting case.

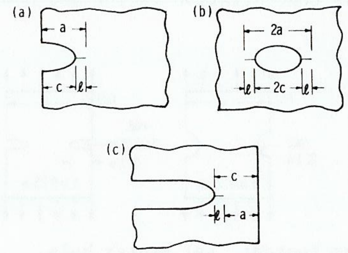

Fig. 4. Nomenclature for notches: (a) edge, (b) interior, and (c) deep.

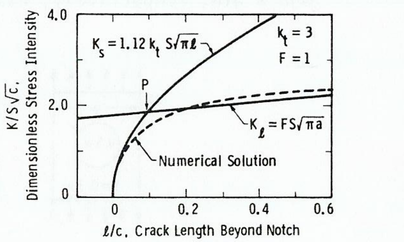

Fig. 5. Stress intensity solution for cracks growing from a circular hole in an intinite plate.

All short crack limiting cases are given by

$$K_s = 1.12\, k_t S\sqrt{\pi \ell} \quad (3)$$

where the crack length, ℓ, is measured from the end of the notch, and the controlling stress is $k_t S$. For long cracks, and for cases similar to Fig. 4a or b, the limiting case stress intensity is obtained by idealizing the geometry so that the notch becomes part of a long crack of dimension $a = c + \ell$.

$$K_\ell = F\, S\sqrt{\pi a} \quad (4)$$

where F is a dimensionless function of geometry, for example, a finite width correction factor.

Specific values of F may be obtained from a handbook of stress intensity factors, such as that of Tada, Paris and Irwin (1973). Nominal stress, S, is defined in any convenient manner allowing the stress intensity expression to fit the form of Eq. 4, with the same definition being used for Eq. 3 and in defining k_t. It may be convenient to consider the member as being deeply notched. In this case, $a = c - \ell$ from Fig. 4c, and Eq. 4 still applies.

Consider the crack length corresponding to a point such as P in Fig. 5, the intersection of the short and long crack limiting cases. This length, here called ℓ', locates the transition between behavior controlled by the local notch stress and behavior controlled by the bulk stress. For crack propagation, ℓ' may be interpreted as the extent of the local notch field.

Combining Eqs. 3 and 4 gives:

$$\ell' = \frac{c}{(1.12\ k_t/F)^2 \mp 1} \tag{5}$$

where the negative sign in the denominator is used for cases corresponding to Fig. 4a or b, and the positive sign for Fig. 4c. Values of ℓ' are generally a small fraction of the notch radius, r, and for moderate and sharp notches usually fall in the range r/20 to r/4, approaching approximately r/5 for all very sharp notches.

As long as the crack length in neither specimen nor component exceeds its respective ℓ' value, loading at equivalent k_tS automatically causes the K versus ℓ relationships to be approximately matched. Thus, the life to any specific crack length less than either ℓ' value is expected to be similar in specimen and component. For the initiation life data in Fig. 3, note that this condition is met, the crack length chosen as initiation being less than any of the three ℓ' values, with these values being given in Table 1.

Values of k_tS only 20 to 30% above σ_y are sufficient (Dowling, 1979) to generate a notch tip plastic zone which engulfs the entire small crack region, ℓ'. Thus, the use of linear elastic fracture mechanics is often suspect for small cracks at notches. However, since the notch strain is approximately the same for two members under equivalent k_tS loading, and since this loading produces similar elastic K solutions for $\ell < \ell'$, K values modified to account for plasticity effects would be similarly modified in the two members, so that fatigue life similitude is not expected to be seriously compromised. Beyond ℓ' the plastic zone size may be compared to the dimension a rather than ℓ, so that little difficulty with linear elastic fracture mechanics is expected, except under general yielding.

Crack Growth Beyond the Local Notch Field

If two members have the same ℓ', the long crack limiting case K solutions must have the same value at $\ell = \ell'$. As a result, the actual K solutions, as from numerical analysis, will be similiar at this point and for some distance beyond, diverging more or less rapidly depending on the particular members involved. The crack growth life beyond ℓ' will generally be more sensitive to the K values at the beginning of this portion of the life than to those later. This is of course due to higher K values causing higher rates of growth at longer crack lengths, so that the number of cycles spent at long crack lengths may be relatively small, often not of major importance in determining the life. Hence, the length ℓ' for specimen and component can be matched to obtain roughly equivalent crack growth lives.

Note that k_t and F in Eq. 5, and also k_p from the earlier discussion, are determined by the geometric shape and mode of loading, but they are unaffected by size. Hence, ℓ' in Eq. 5 is proportional to c, which represents the size of the member, so that matching of ℓ' can be accomplished by simply adjusting the size of the test specimen.

If it is desired to obtain better matching of the crack growth lives than is provided by equal ℓ' values, a plot of K/k_tS versus ℓ as in Fig. 6 should be made. In the absence of detailed analysis, it is useful to approximate the K solutions by using Eq. 3 for $\ell < \ell'$, and Eq. 4 for $\ell > \ell'$. The geometry of the test specimen can be varied, and trial and error used with the F functions provided in a handbook of stress intensity factors, to obtain curves which approximately match on this type of plot. Crack growth lives are expected to be similar to whatever crack length this matching is successful.

In Fig. 6, for the compact and blunt DEN specimens, the approximate K/k_tS versus ℓ curves match quite closely for a considerable range of crack lengths. Since the

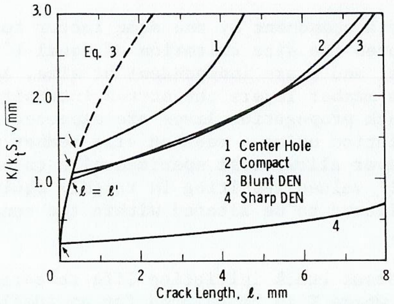

Fig. 6. Stress intensities approximated by short and long crack limiting cases.

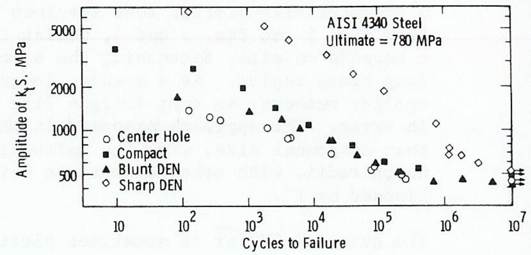

Fig. 7. Total fatigue life data.

initiation lives have already been shown to correlate by Fig. 3, the total fatigue lives should also agree. Figure 7 shows that this is indeed the case, except that the data diverge at short lives due to general yielding effects.

From Fig. 3, excluding general yielding effects, the initiation lives for the center hole specimen are in reasonable agreement with those for the compact and blunt DEN specimens. However, the total lives in Fig. 7 are somewhat shorter due to shorter crack growth lives. Figure 6 suggests that this is due to the higher and rapidly increasing K/k_tS curve for this specimen.

For the sharp DEN specimen, with reference to Table 1 and Fig. 6, the ℓ' value is much smaller than for the other specimen types, resulting in drastically lower K values under equivalent k_tS loading. As a result, much longer crack growth lives occur when the comparison is made in terms of k_tS, causing the total fatigue lives to fail to correlate in Fig. 7.

CONCLUDING REMARKS

Four areas have been identified that should be considered in designing and loading notched laboratory specimens to simulate components in service. Two of these are of primary importance, and two are of lesser importance. Of primary importance is the loading of specimen and component at the same k_tS, which is expected to produce similar notch strains and, therefore, similar crack initiation lives. Also of primary importance is the sizing of the notched specimen to obtain the same ℓ' as for the component, which results in similar K values during the early phases of crack growth.

If loads approaching general yielding are of concern, then the specimen can be designed to have the same k_p value as the component. However, most engineering designs avoid such loads, so that matching of k_p values will probably seldom be required. If the crack propagation life is thought to be important, then the test specimen can be designed so that K/k_tS is matched to as large a crack length as possible. It will often be difficult to match both the k_p values and the K/k_tS curves, as conflicting requirements may be placed on the specimen design.

It is noteworthy that values of the elastic stress concentration factor, k_t, depend on the manner of defining nominal stress, S, so that a particular numerical value of k_t has no significance. Therefore, matching of k_t values has no significance.

Note that reducing all of the dimensions of a component by the same factor to make a geometrically similar test specimen violates the size criterion of equal ℓ' values. From Fig. 5 and Eqs. 3 and 4, noting that k_t and F are independent of size, and that c depends on size, decreasing the size of a member lowers the stress intensity in the long crack region. As a result, longer crack propagation lives are expected in smaller members, so that fatigue life simulation using a reduced size member may be in error. The approach proposed in this paper allows test specimen size to be smaller than component size, with the matching of ℓ' values resulting in roughly equivalent notch radii, with other dimensions being allowed to be altered within the constraints imposed by ℓ'.

The quantity $2K/\sqrt{\pi r}$ is sometimes plotted versus crack initiation life to correlate the behavior of different notched members, where K is calculated for an idealized geometry in which the notch is reduced to a crack. This is equivalent to plotting k_tS using an estimated value for k_t of

$$k_t' = 2K/S\sqrt{\pi r} = 2F\sqrt{c/r} \qquad (6)$$

The value of k_t' is close to the actual k_t only for notches that are crack-like in form. Table 1 gives k_t' values for the members of Fig. 1, with these differing significantly from k_t in two cases. As a result, a plot of $k_t'S$ versus initiation life similar to Fig. 3 for the same data does not produce a satisfactory correlation. Equation 6 is useful in estimating k_t values for crack-like notches, but indiscriminate use of $2K/\sqrt{\pi r}$ can lead to serious error.

The simulation approach proposed has the advantage that it is not necessary to assume that the component life is controlled by either crack initiation or propagation. Also, size and stress gradient effects are to a large extent automatically included. For irregular k_tS versus time histories, all significant aspects of component behavior are expected to be reproduced in the laboratory specimen, so that no serious limitations appear to exist in this area. A similar approach may be applicable to some situations of biaxial notch surface stresses or multiple external loads, but caution and further development in this area is required. The simulation approach proposed may be used with tests in a hostile chemical or thermal environment, but differences between test time and service time may affect the results.

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PREDICTION OF CRACK FORMATION LIFE IN NOTCHED SPECIMENS

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ABSTRACT

The present study recognises that a finite volume of material must be involved in the formation of an engineering crack, and that the effective stress and strain in this finite volume controls the crack formation. This concept leads to the derivation of a new fatigue notch factor, and using this in a Neuber type analysis enables crack formation lives to be predicted.

KEYWORDS

Fatigue crack formation; fatigue notch factor; reversed plastic zone size; strain distribution; strain range; low cycle fatigue; critical volume; Neuber analysis; fatigue life.

INTRODUCTION

In many situations, in an effort to minimise catastrophic failure of components, detailed criteria for establishing safe lives have been developed. For example, in aero-engine turbine and compressor discs, the safe life concept is based on limiting the number of cycles to the first crack which can be reliably detected, modified to allow for statistical considerations and factored to incorporate a safety margin (Evans, 1979). Consequently, if this philosophy is accepted, it is necessary to understand the significance of influencing parameters, and attempt to establish methods for predicting the life to first crack.

CRACK FORMATION LIFE

As a matter of practical convenience, it is desirable to define an engineering crack as one which can be detected using low power magnification (Duggan and Sabin, 1977, Duggan, Lowcock and Staples, 1979). For a surface crack this will typically be about 0.1 mm deep and 0.5 mm long. Once a crack of this size has been produced, it will usually be possible to apply fracture mechanics, although notch plastic strain or microstructure may dominate the situation.

The most common method of predicting crack formation life is that of relating the