CRACK GROWTH IN THICK WALLED TUBES OF LOW DENSITY POLYETHYLENE

Dr. A. S. Boyce, Mono Pumps (U.K.) Ltd., London, England
and
Mr. B. S. Owen, Department of Mechanical Engineering,
University of Newcastle-Upon-Tyne, England.

ABSTRACT

Experimental studies are described of the growth of an external circumferential crack in a thick walled cylinder of low density polyethylene loaded by internal pressure. The results are interpreted in the light of similar investigations for a constant load three point bend specimen of the same material using linear elastic stress intensification factors calculated by finite element methods.

A linear relationship is found to exist between the applied stress intensity factor K_o and the time to failure t_f when plotted on a double logarithmic scale for values of $t_f \leqslant 400$ minutes. For greater failure times the relationship is also linear but with a different slope. These results are compared with those for the three point bend specimens in which the Log (K_o) and Log (t_f) relationship is linear and of constant slope over the whole time range investigated. The values of K_o for a given time to failure are however considerably higher for the three point bend specimens. Better correspondence is obtained between the results for the two types of specimens if the dilatation at the crack tip is plotted against t_f. Crack initiation in the three-point bend specimen is also discussed.

KEYWORDS

Crack, tubes, Polyethylene, three point bend, stress-intensity factor, creep function.

INTRODUCTION

The paper examines the applicability of stress intensity factors calculated for a linear-elastic material to crack growth in low density polyethylene. Interest in the problem arose initially due to the failure of thick walled coupling members for polyethylene piping carrying gas under internal pressure.

Experimental investigations performed on fracture in viscoelastic polymers have used thin sheets of material generally with centre or edge-notching and subject to a remotely applied in plane uniform constant stress. In such tests the degree of distortion is far in excess of that in a thick-walled tube specimen. For this reason, constant load plane-strain three-point bend specimens of thickness equal to the wall thickness of the tube specimen were investigated before proceeding with investigation of the more complex thick tube crack growth problem. Crack propagation through the tube is inevitably non axially symmetric and the calculation of true stress intensity factors is difficult if not impossible.

THEORY

By postulating for a viscoelastic sheet a model of instantaneous stress distribution in the vicinity of an advancing crack and equating the rate at which surface energy is consumed to the rate at which tractions normal to the crack path do work, and making the assumption that the stress and displacement distribution is given by the linearly viscoelastic solution, Knauss (1970) obtained the equation:

$$2\Gamma / \pi \sigma_0^2 \quad a(t) \quad = \quad D_{crp} \; (\alpha / \dot{a})$$

where σ_0 is the uniaxial uniform stress applied to the sheet, Γ is the energy to create one unit of new surface, a (t) is the crack length at time t, and α is a small length and D_{crp} is the creep compliance of the material. If

$$\sigma_{go} = \quad (2\Gamma E_0 / \pi a_0)^{\frac{1}{2}}$$

is the Griffith instability stress based on the short time Modulus E_o and a_o is crack length a (t) at time t = o, then:

$$(\sigma_{go} / \sigma_o)^2 \quad = \quad (a(t)/a_o)\Psi(\alpha / \dot{a})$$

where $\Psi = (D_{crp}/D_o)$ and $D_o = D_{crp}$ (t) at time t = o and $D_o = 1/E_o$

$$\text{or} \qquad (K_{go}/K_o)^2 = \Psi(\alpha/\dot{a}) \qquad (1)$$

$$\text{where} \qquad K_{go} = \sigma_{go} \, (\pi a_o)^{\frac{1}{2}}$$

$$K_o = \sigma_o \, (\pi a_o)^{\frac{1}{2}}$$

At the initiation of crack growth $t = t_1$ and $a(t) = a_o$ where t_1 is the elapse of time after application of constant stress σ_o.

For a linear viscoelastic material, the analysis of Williams (1965) using different shaped cavities suggests that in determining critical instability stresses for all types of discontinuity, Young's Modulus E in the Griffith's formula should be replaced by $(2 D_{crp}(t) - D_o)$.

With this approach

$$(\sigma_{go} / \sigma_o)^2 \quad = \quad (2D_{crp}(t_1) - D_o)/D_o$$

$$(K_{go}/K_o)^2 \quad = \quad 2\Psi(t_1) - 1 \qquad (2)$$

equations (1) and (2) provide a means for relating calculated stress intensity factors to the time to initiation of crack growth, t_1.

CRACK GROWTH IN THREE-POINT BEND SPECIMENS

The three-point bend single edge-notch specimen used, Fig. 1, is featured in British Standard DD3: 1971 which utilises the specimen to evaluate plane strain fracture toughness (K_{Ic}) of metallic materials. The standard incorporates the results of a boundary collocation procedure for evaluation of a linear elastic fracture mechanics (L.E.F.M.) plane strain stress-intensity factor, K_I, for the specimen under a constant applied load giving:

$$K_I = (3P_B \; L \; BW) \, . \, (a)^{\frac{1}{2}} \, . \, (1.93 - 3.07\,(a/w) + 14.53\,(a/w)^2 - 25.11\,(a/w)^3 + 25.8\,(a/w)^4) \qquad (3)$$

The specimens were subjected to a constant load as in Fig. 1. until failure occurred.

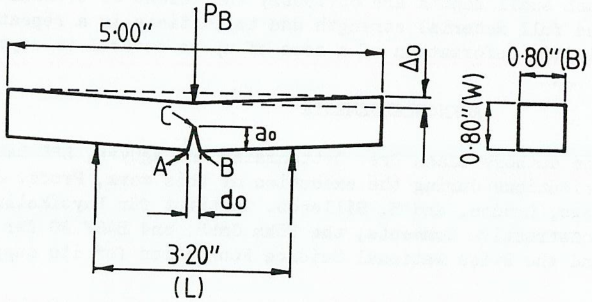

Fig 1. Three point bend specimen

Experimental Procedure

The specimens were cut longitudinally from the outer regions of a four inch diameter bar of a WJG 11 low-density polyethylene of a metal flow index 2 and density 0.921 grm/cc., and milled to size in order to match as far as possible the material properties in the regions where crack growth would occur in both the three point bend and thick tube experiments. Notch depths 0.20 inches and 0.40 inches were used for the notch-bend and thick-walled tube specimens so that low values of load on the three-point bend specimens would produce crack growth to failure within a reasonable time-span. Before notching, the specimens were allowed to normalise for two months at a constant temperature of 20°c. Initial notching procedure for the two types of specimen was the same.

Pre-cracking procedure followed that of Marshall (1970) and consisted of loading each specimen up to a critical load value and holding for ten seconds. The load was then removed and the specimen allowed to recover for ten minutes before the crack growth tests were commenced. For the three-point bend specimens critical load values of 99.2 lbf and 198.2 lbf were applied to the 0.40 inches and 0.20 inches initial notch depth specimens respectively and pre-cracks ranging from 0.02 to 0.04 inches resulted.

The specimens were loaded in an Instron tensile testing machine, displacement of the crosshead being monitored at regular intervals of time. Approximate measurements of opening displacement AB (Fig. 1) of the sides of the propagating crack were taken at various time intervals. The measurements were made using a scale and a magnifying glass to within ± 0.25 mm. A similar procedure was employed to record crack length values, the latter being taken as the mean of the sum of lengths AC and BC (Fig. 1).

Upon application of the predetermined load, the specimen instantaneously took up the form illustrated in Fig, 1 with the crack length still at the original value, a_o, and an initial deflection Δo, of the specimen having taken place. Under continuous application of load an approximately linear relationship was found to exist between the deflection of the specimen, as monitored by the crosshead displacement output data, and the crack opening displacement at the mouth of the crack. For smaller applied load values, a noticeable time lapse occurred before initiation of crack growth, the time lapse, t_1, being dependent upon the applied load. After initiation of crack growth, crack propagation occurred for these load values at an even or slowly increasing rate for a major part of the life of the specimen. At higher applied load values, rapid rates of growth were seen to occur immediately or soon after the application of the required test load.

The lengths of time from application of load to initiation of crack growth and the time to total specimen failure for each specimen investigated were recorded.

Graphical plots of crack length against time at 20°C are shown for the 0.20 in. notch specimen in Fig. 2. Similar curves were obtained for the 0.40 inch notch specimens.

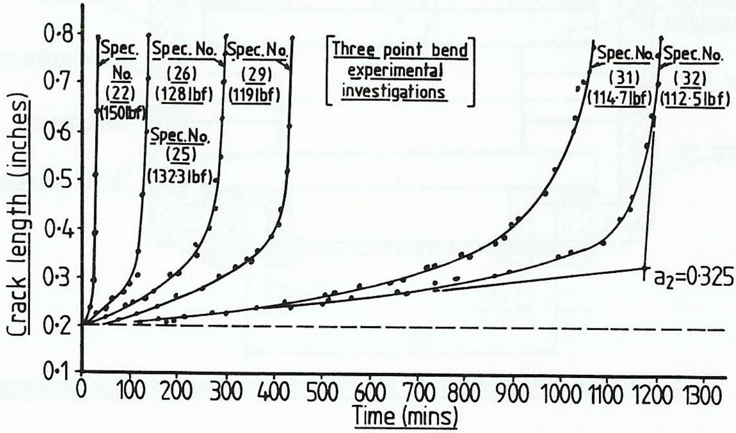

Fig.2. Graphical plot of crack length against time 0·2 inch initial crack depth specimens

Crack Growth Initiation

For a given initial crack length the values of crosshead displacement and crack opening displacement at initiation of crack growth Δ_1 and d_1, were reasonably constant for the range of loads applied, initiation of crack growth occurring for both notch depth and all loads at an approximately constant value of d_1/a_o. Equations (1) & (2) link for a linearly viscoelastic material the time to crack growth initiation, t_1, to the instantaneous stress-intensity factor, K_o, through the creep function $\Psi(t)$ for the material. Using the crosshead displacement, Δ as a measure of strain in the specimen at any instant of time, the creep function at the time of initiation of crack growth was taken as

$$\Psi(t_1) \quad = \quad D_{crp}\,(t_1)/D_{crp}\,(0) \quad = \quad \Delta_1\,/\,\Delta_0$$

where Δ_o is the instantaneous (elastic)displacement of the three-point bend specimen upon application of load. Values of K_o were calculated using equation (3).

In order to establish a value for K, the stress-intensity factor present for Griffith-type instantaneous unstable growth upon load application, it was observed that the most rapid failure experienced in the range of three-point bend

experiments covered occurred for specimen 1, total specimen failure being achieved in 75 seconds. The applied load value used on this specimen was the highest value at which the predetermined applied load was attained and held constant for a period of time before failure. Higher pre-required loads were not reached before total specimen failure occurred. Accordingly, the value of K_o evaluated for specimen 1 was taken as that causing instantaneous rapid failure by fracture and assigned to K_{go}. Double logarithmic plots of $\Psi(t_1)$ against $(K_{go}/K_o)$are shown in Fig. 4 along with the theoretical curves of equations (1) and (2). Fig. 4 shows that equation 2 appears to be a reasonably good approximation to experimental values for medium rates of specimen deformation, the deviation at higher and low values of t_1 being due to the difficulty in these cases of determining the exact moment of crack growth initiation.

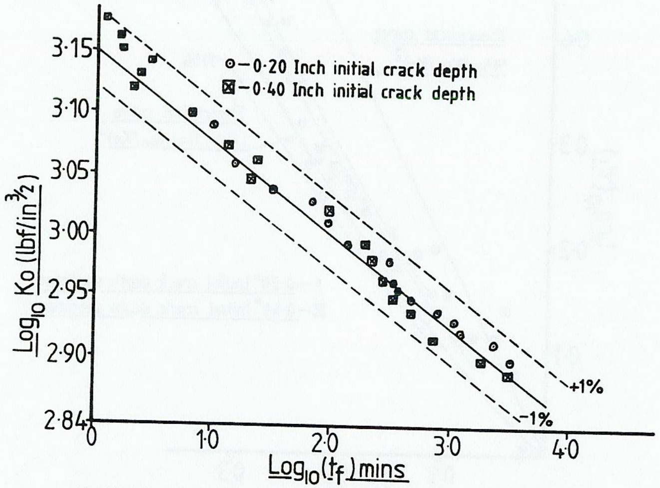

Fig 3 Graphical plot of $\log_{10}$Ko against $\log_{10}(t_f)$ for three point bend specimen

Specimen Failure

Examination of Fig. 2 and a similar curve for the 0.4 in notch reveals that a relatively sharp change in the slope of the curves occurs just prior to specimen failure, after a period of slowly increasing rate of crack growth. Since the initial and latter portions of the curves were essentially linear, these portions of the curves were extrapolated as is shown in the example of Fig. 2 to intersect at C. The value a_2 of the crack length at C was then read off and values of stress-intensity factors, K_2, calculated using equation (3). This method was repeated for all specimens giving a mean value for K_2 for 1390 lbf/in$^{3/2}$ with a scatter of ± 10 percent. This mean value of K_2 was only slightly less than the K value, 1485 lbf/in$^{3/2}$, found in the experimental investigations to result in an instantaneous rapid crack growth rate of the order obtained in the latter stages of the specimen data curves illustrated in Fig. 2.

To establish a link between the time to specimen failure and the initial specimen geometry and applied load, values of applied stress-intensity factor, K_o, were plotted against time to failure on a double logarithmic scale for each specimen tested (Fig. 3). A linear relationship was found to exist.

The near constant values of (d_1/a_o) suggest that the criteria for initiation of crack propagation is crack tip strain. Following initiation, crack growth continues at a rate depending upon the applied stress-intensity factor, K_o, the time to failure being linked to K_o by a double logarithmic relationship. Good agreement between the K2 values obtained suggests that a critical value of stress-intensity factor is necessary to produce unstable crack growth in three-point bend specimens of low-density polyethylene.

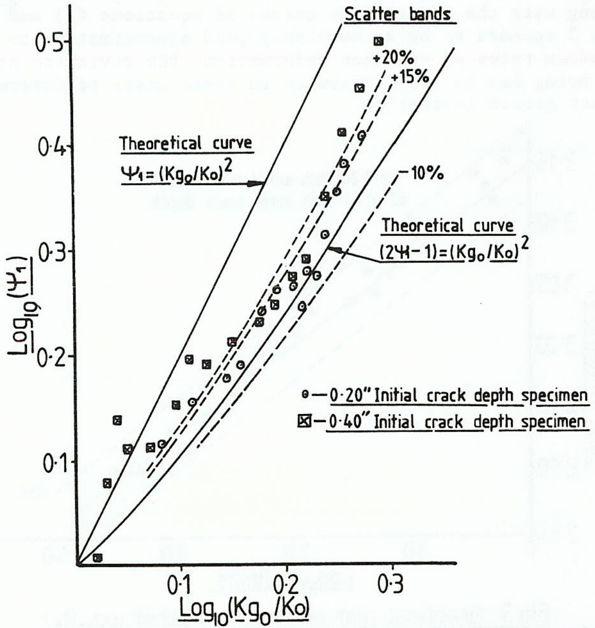

Fig.4. Crack growth initiation in three-point bend specimens
Graphical plot of $\log_{10}(\Psi_1)$ against $Log_{10}(Kg_o/Ko)$

GROWTH OF AN EXTERNAL CIRCUMFERENTIAL CRACK IN A THICK-WALLED TUBE LOADED BY INTERNAL PRESSURE

The nine inches long tube specimens were prepared from a four inch diameter rod of a WJG 11 low density polyethylene machined slowly out to a bore of 2.4 inches to avoid excessive heat dissipation affecting material properties. The tube was notched to an initial notch depth of 0.150 inches or 0.350 inches uniformly produced around the whole external circumference and at a distance of 4.50 inches from the end of the tube specimen. After completion of the initial full circumference notch a "starter" notch of a further 0.050 inches depth was produced over an arc of 120 degress of the external diameter. Each specimen was stored at a controlled temperature of 20 degrees centigrade for two months before being used in a pressure test. The arrangement for applying internal pressure loading is shown in Fig. 5, axial load being transmitted through the thrust rods. Attempts to monitor the propagation of the crack through the wall of the tube using the reading of three strain gauges mounted on the inside of the tube adjacent to the crack were unsuccessful. The crack opening displacement was measured using the Welwyn Electric 101 series clip gauge specified in BS DD3: 1971 "Method for Plane Strain Fracture Toughness Evaluation." The gauge was mounted on two knife edges as shown in Fig 5 and was connected to a Bruel and Kjoer type 1516 unit and pen-recorder. Pre-cracking the tube specimens was carried out in a similar fashion to the three-point bend specimens. The requisite internal pressure being held constant for a period of ten seconds after which the specimens were allowed to recover for ten minutes before the crack growth tests were commenced. Pre-cracks in the range 0.002 to 0.005 inches were achieved.

A total of nineteen tests were performed at $20^{\circ}C$ at different pressures and crack depths, sixteen being carried out on pre-cracked specimens of 0.20 and 0.40 inches initial notch depths and three with initial notch depths of 0.35, 0.30 and 0.225 inches.

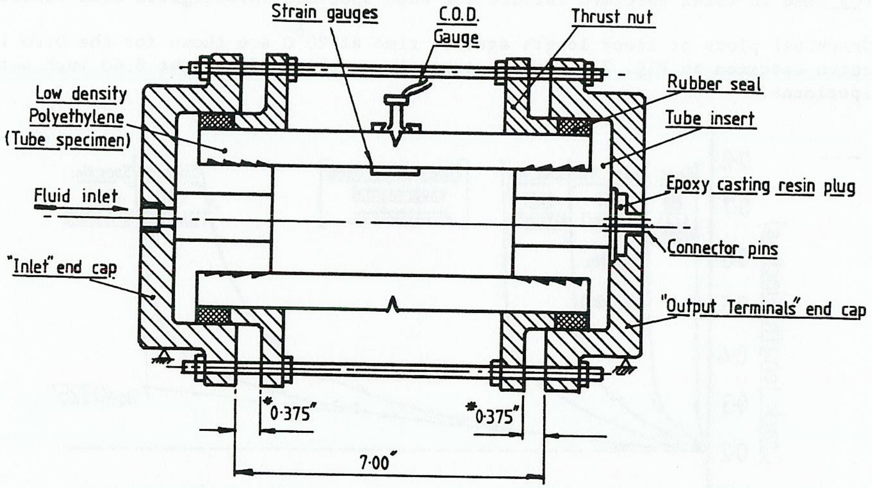

Fig 5. General arrangement of assembled polyethylene tube specimen

Analysis of Results

Figure 6 illustrates for a range of values of K_o the variation of crack opening displacement with time. The values of K_o were determined using an axi-symmetric finite element program by the third node method of Kobayashi (1969) and the extrapolation method of Anderson (1971). For a given crack depth the values given by the two methods differed by less than 3% and in all cases the mean of the two values was used. As in the case of the three-point bend specimen, the rate of deformation in the region of the crack, as indicated by the crack opening displacement, varies considerably for a relatively small difference in loading. The variation time to failure t_f with the stress intensity factor K_o is shown in Fig. 7. With the double logarithmic scale a linear relationship results with a distinct change of slope around $t_f = 400$ mins.

Also shown on the same diagram are the results for the three point bend tests which give values of K_o for the same time to failure t_f some fifteen to twenty times those for the tube tests. Since the value of K_o does not appear to provide a basis for comparison of the two sets of results, the volumetric dilatation C_{Do} in the crack tip region was evaluated and plotted on Fig. 7. Despite good agreement of the results within each specimen type (less than ± 10% scatter) a factor of approximately 3 to 1 results at short failure times between the three-point bend and thick tube C_{Do} values.

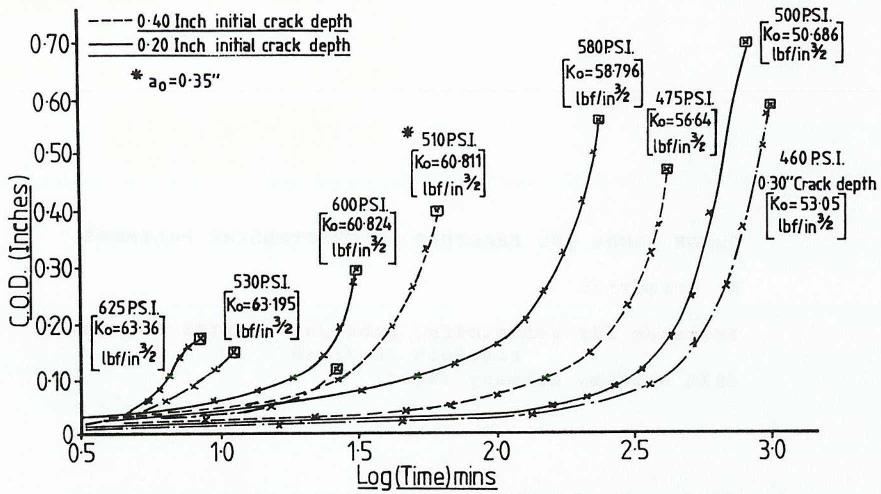

Fig.6 Graphical plot of C.O.D. against $\log_{10}$ (Time) for decreasing Ko values 0·20 inch and 0·40 inch initial crack depth specimens output data.

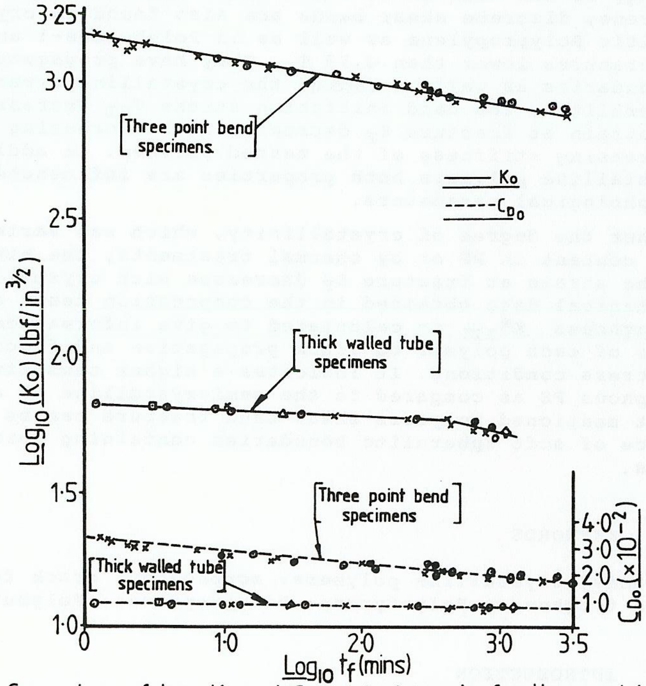

Fig.7 Comparison of $\log_{10}$ Ko and C_{Do} against $\log_{10} t_f$ for three-point bend and thick walled tube specimens

DISCUSSION

The value of the stress intensity factor fails to relate the times for the achievement of critical conditions for the two types of specimen tested. The concept of a critical volumetric dilatation in the crack tip region provides a better correlation. The higher values of dilatation at failure for the three point bend specimen might be ascribed to the triaxial stress system in the region of the crack tip as compared with the tube specimen in which the radial stress in the crack tip region is compressive. Williams (1965) for example has shown that fracture growth in a spherical cavity occurs when the flaw size is twice as large as that required for fracture growth in a cylindrical cavity.

CONCLUSIONS

For the three point bend specimens, crack tip strain is the criterion for crack growth initiation, which occurs after a time lapse given approximately by equation (2). Unstable crack growth occurs at a critical value of the stress intensity factor.

A linear relationship exists between the applied stress-intensity factor K_o and the time to failure t_f, when plotted on a double logarithmic scale.

For the thick walled tube specimens under constant internal pressure, a similar relationship between K_o and t_f is found for $t_f \leqslant 400$ minutes, a change of slope occurring for higher failure times. For corresponding failure times the values of K_o are small compared with the three point bend values. Better agreement is achieved if t_f is plotted against the dilatation in the crack tip region.

ACKNOWLEDGEMENTS

The work described in this paper formed part of a submission by one of the authors (A. S. Boyce) for the degree of Ph.D. at the University of Newcastle-Upon-Tyne. The provision of facilities by the Department of Mechanical Engineering of the University is gratefully acknowledged.

REFERENCES

Anderson, Ruggles and Stibor (1971) "Use of finite element computer programs in fracture mechanics" Int. Jnl. Fr. Mechs, Vol. 7 No. 1.

Knauss W. G. (1970) "Delayed failure - the Griffith problem for linearly viscoelastic materials" Jnl. Fr. Mechs, Vol. 6.

Kobayashi, Iida, Simon and Maiden (1969) "Applications of finite element analysis method in two dimensional problems in fracture mechanics" A.S.M.E. (Pressure vessels and piping division), Winter Meeting, Los Angeles.

Marshall, Culver and Williams, J.G. (1970) Plastics and Polymers, 95.

Williams, M.L. (1965) "Initiation and growth of viscoelastic fracture" Intl. Jnl. Fr. Mechs., Vol 1.