CONSTITUTIVE RELATIONS INCLUDING DUCTILE FRACTURE DAMAGE.
APPLICATION TO CRACKED BODIES.

G. Rousselier

EdF, Dept. Etude des Matériaux, Les Renardières
77250 - Moret-sur-Loing, France

ABSTRACT

Local fracture criteria and node relaxation techniques are extensively used to model ductile fracture at the tip of a crack. This is only an approximation as the influence of damage on the stress-strain relations, via nucleation and growth of voids, implying volumic plastic deformation, is not considered. To account for this, constitutive relations including ductile fracture damage are developed for finite transformation of plastic dilatant materials. These relations are derived from the macroscopic viewpoint, according to an extended normality rule.

In case of isotropic damage, an exponential dependence of ductile fracture damage on stress triaxiality is demonstrated, and the occurence of material instability $\dot{\sigma}\dot{\varepsilon}^p < 0$ is shown on various examples. A finite element analysis of a three-point bend specimen is performed, including some finite deformation effects. Large volumic plastic strains localize in the most severely damaged elements at the crack tip, resulting in stable crack growth without it being necessary to postulate a local fracture criterion nor to release the nodes. The present analysis could be readily extended to angled crack extensions and 3D-problems, without extensive modifications of the existing finite element models.

KEYWORDS

Constitutive relations, damage, ductile fracture, finite transformation, material instability, stable crack growth, stress triaxiality, void growth, volumic plastic deformation.

INTRODUCTION

The modelling of the highly strained zones at the center of a tension specimen or at the tip of a notch or crack requires the development of constitutive relations including ductile fracture damage, i.e. void nucleation, growth and coalescence, specially allowing volumic plastic deformation. The constitutive relations may be constructed from the microscopic level (Gurson, 1977) but the components : void behaviour, void-void interaction, etc..., are badly known, and the transition to the macroscopic level needs drastic simplifying hypotheses. An alternative approach, assumed in the present work, is exclusively macroscopic.

## CONSTITUTIVE RELATIONS

Strain hardening and ductile fracture damage are supposed to be isotropic, characterized by scalar internal variables $\alpha$ and $\beta$ respectively. This is a reasonable assumption under circumstances of interest ; further justifications are given by Rousselier (1979). The dissipated power inequality is :

$$\frac{\sigma_{ij}}{\rho} \cdot \mathcal{D}^p_{ij} + A\,\dot{\alpha} + B\,\dot{\beta} \geqslant 0 \qquad (1)$$

where $\mathcal{D}^p$ is the plastic deformation rate, $A(\alpha)$ and $B(\beta)$ the work conjugates of the internal variables rates. An extended normality rule is assumed, i.e. there exists a convex plastic potential $F(\sigma/\rho, A, B)$ and the rates are oriented in the direction of the external normal to the yield surface :

$$\mathcal{D}^p = \lambda \frac{\partial F}{\partial(\sigma/\rho)} \qquad \dot{\alpha} = \lambda \frac{\partial F}{\partial A} \qquad \dot{\beta} = \lambda \frac{\partial F}{\partial B} \qquad (2)$$

Damage is introduced with a term depending only on the first invariant $\sigma_m = \sigma_{kk}/3$ of the stress tensor, added to the usual von Mises form $F = (J_2)^{1/2} + A$ :

$$F\left(\frac{\sigma}{\rho}, A, B\right) = \left[J_2\left(\frac{\sigma}{\rho}\right)\right]^{1/2} + A + B\,g\left(\frac{\sigma_m}{\rho}\right) \qquad (3)$$

Let $d^p$ and $s$ be the deviatoric parts, $\mathcal{D}^p_m$ and $\sigma_m$ the spherically symmetric parts of $\mathcal{D}^p$ and $\sigma$ respectively. The constitutive relations (2) give :

$$d^p = \lambda \frac{s}{2[J_2(\sigma)]^{1/2}} \qquad \mathcal{D}^p_m = \lambda \frac{B}{3} g'\left(\frac{\sigma_m}{\rho}\right) \qquad (4)$$

$$\dot{\alpha} = \lambda \qquad \dot{\beta} = \lambda\, g\left(\frac{\sigma_m}{\rho}\right) \qquad (5)$$

In ductile fracture the damage parameter $\beta$ is directly related to the change of density of the metal, thus $\beta = \beta(\rho)$ . Expressing $\mathcal{D}^p_m$ and $\dot{\beta} = \beta'(\rho)\,\dot{\rho}$ in the mass conservation law $\dot{\rho} + 3\rho\mathcal{D}^p_m = 0$ , according to (4) and (5), yields $g'/g = -1/\rho B \beta'$ ; the two sides of this equation are functions of distinct variables, therefore they are a constant of dimension $1/\sigma$, say $C/\sigma_0$, where $\sigma_0$ is the yield stress. The integration of the left-hand side gives :

$$g\left(\frac{\sigma_m}{\rho}\right) \equiv D \exp\left(\frac{C\sigma_m}{\rho\sigma_0}\right) \qquad (6)$$

where D is the constant of integration. The exponential effect of $\sigma_m$ is demonstrated ; this result is similar to that obtained by Mc Clintock (1968), Rice and Tracey (1969) in a distinct way. Still there is a difference : the density $\rho$ appears in the exponential. In usual plastic theories $\rho$ is supposed to be constant (unity), but this is just untrue in case of ductile fracture damage.

One of the two functions $\beta(\rho)$ or $B(\beta)$, related by $C\rho B\beta' = -\sigma_0$, must be chosen to match theoretical or experimental results. A possible choice is :

$$\beta = \ln\left(1 + \frac{1-\rho}{\rho f_0}\right) \qquad (7)$$

where $f_0$ is the initial void volume fraction. As the metal of the matrix is supposed to be incompressible, this choice gives (Rousselier, 1979), with $C = 3/2$ and $D = 0.49$ :

$$\frac{\dot{R}}{R} = 0.283\, \mathcal{D}^p_{eq} \exp\left(\frac{3\sigma_m}{2\rho\sigma_0}\right) \qquad (8)$$



which is the void growth rate obtained by Rice and Tracey (1969). Equation (7) is assumed hereunder.

Consider the homogeneous transformation of an element of material submitted to a triaxial stress tensor : $\sigma_{22} = \sigma_{33} = k\,\sigma_{11}$ , k constant < 1 . Some resulting stress-strain and density curves are given in Fig. 1. After a certain amount of plastic deformation, depending on stress triaxiality, material instability $\dot{\sigma}\dot{\varepsilon}^p < 0$ takes place. The damage-related softening overcomes the strain-hardening of the matrix material. The deformation then goes on under decreasing stress till complete decohesion of the element of material ( $\sigma = \rho = 0$ ) . This is not the case with the infinitesimal strain approximation ( $\rho = 1$ ) .

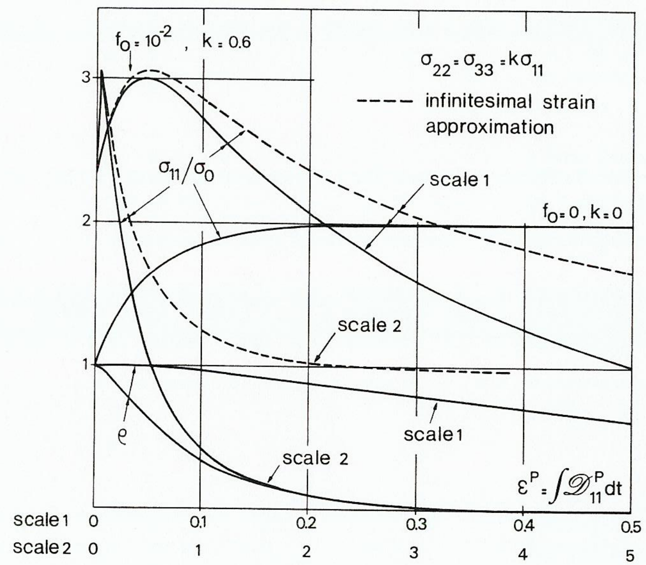


Fig 1 : Stress-strain and density curves with ductile fracture damage.

The effects of various stress triaxialities and various initial void volume fractions $f_0$ are emphasized in Fig. 2, where the plastic strain at instability is plotted versus $\sigma_m/\sigma_{eq} \equiv (1+2k)/3(1-k)$ (that has to be distinguished from $\sigma_m/\sigma_0$ ; $\sigma_{eq} = [3J_2(\sigma)]^{1/2}$ ).

## FINITE ELEMENT ANALYSIS OF A CRACKED SPECIMEN

The analysis of ductile fracture and stable crack growth in a three-point bend specimen (W = 25 mm, a/W = 0.5) has been performed with a 2D infinitesimal strain finite element model (constant plane strain triangular elements). Some finite transformation effects are included : namely, $\sigma/\rho$ is substituted for $\sigma$ into the infinitesimal strain constitutive relations ; $\rho$ is given by equation (7). The strain-hardening curve is that of Fig. 1, with $\sigma_0 = 500$ MPa, and the elastic constants are E = 200 GPa, $\nu = 0.3$.

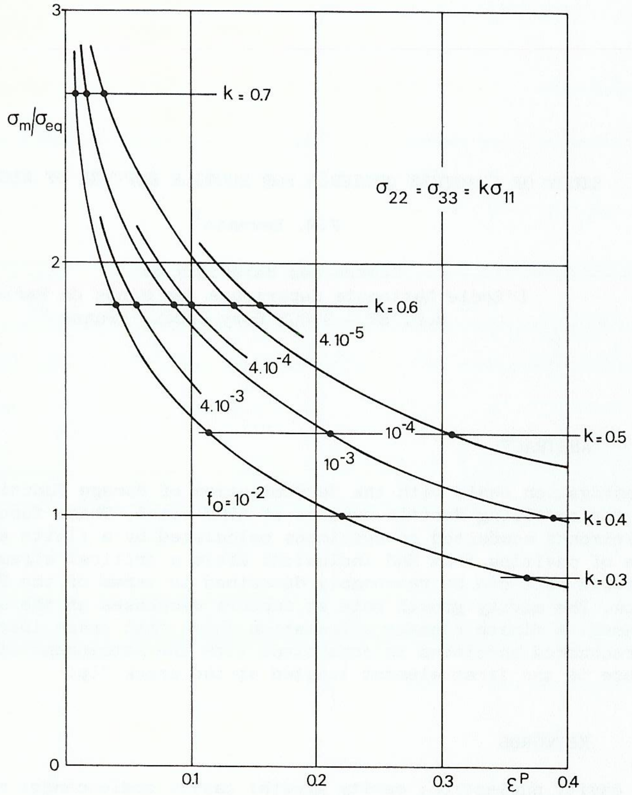


Fig 2 : Plastic strain at material instability versus $\sigma_m/\sigma_{eq}$

The deformation of the crack tip zone is shown in Fig. 3. In the two most deformed elements the damage increases rapidly. At some point, corresponding with the maximum of the local stress-strain curves, as in Fig. 1, the deformation of these elements increases abruptly, and the stresses decrease according to the curves I and II of Fig. 4. This quasi-rupture of the two elements may be identified with the coalescence stage of the ductile frature process. The node $\omega$ , Fig. 3, is no longer bounded to node C (initial crack tip) so stable crack growth occurs naturally, by localization of deformation, resulting from the constitutive relations only, without it being necessary to define a critical state nor to release the nodes. A further stage of stable crack growth is shown in Fig. 3 (four nodes, d = 0.748 mm). Lastly, the effects of various initial void volume fractions $f_0$ on the load-displacement curves is shown in Fig. 4.



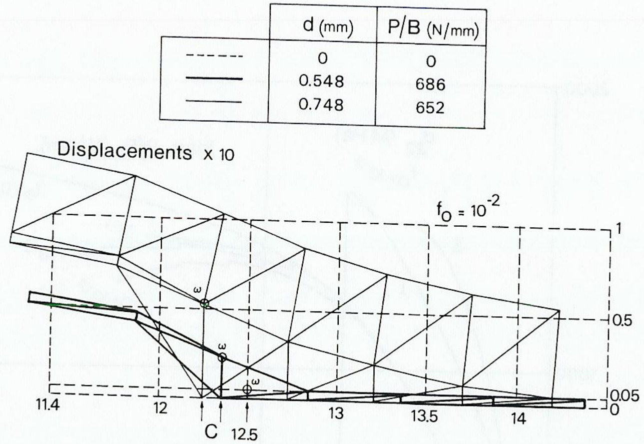


Fig 3 : Localization of deformation in the crack tip elements, by ductile fracture damage, resulting in stable crack growth.

CONCLUSIONS

The constitutive relations including ductile fracture damage and some finite transformation effects provide an attractive new model for the treatment of ductile fracture and stable crack growth problems. It is in better agreement with the physical behaviour of the metal, simpler, and probably cheaper than usual node relaxation models. A simple adaptation of the subroutines giving the plastic strains in the existing finite element models would allow the treatment of angled crack extensions and 3D problems.

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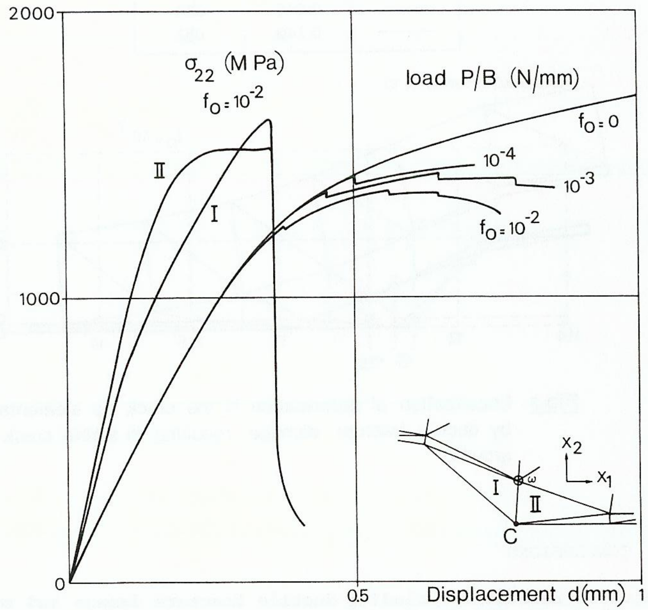


Fig 4: Load-displacement curves of a three-point bend specimen, with ductile fracture damage. Stress curves in two crack tip elements showing the local instability. (Crack tip geometry different from that of load curves)