AN INCREMENTAL CRACK GROWTH MODEL FOR HIGH TEMPERATURE RUPTURE IN METALS

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ABSTRACT

A model is developed which predicts that the creep crack growth rate is dominated by the parameter C*, which is the time dependent equivalent of the J integral. Since creep damage is associated with grain boundaries, it is assumed that the crack advances as a series of discrete steps. Experimental evidence exists to support this view. Approximate calculations are made of the transient time to reach the stress state described by C* and comparison made between this time and the time for the crack to advance a discrete step. Conditions are determined when the transient time is considerably smaller than the crack advance time.

KEYWORDS

Crack growth, creep rupture

INTRODUCTION

Rupture in metals operating at high temperatures can occur either by bulk rupture, in which diffusive hole growth leads to the formation of continuous damage, or by the propagation of an established crack. Once the crack starts to propagate, a steady state tends to set in in which the crack grows at a constant rate at some fixed level of the load. It is this phenomenon that is the subject of the paper. Experiments on creep crack growth have been carried out by a number of investigators [Ellison and Harper (1978)]. Apparently, the creep crack growth rate is often found to be dependent on the C* parameter, and also it has been observed by Pilkington (1979) that the crack advances in discrete steps.

Hui and Riedel (1979) have obtained asymptotic results for steady quasi-static crack growth in an elastic-power law creeping material with the remote stress levels controlled by K, the elastic stress intensity factor. The leading singular term in the expression for stress is proportional to $r^{-1/(n-1)}$ if $n>3$ where r is the distance from the current crack tip and n is the creep index as in

$$\dot{\varepsilon}^{c}_{ij} = \frac{3}{2}\,\dot{\varepsilon}_o(\bar{\sigma}/\sigma_o)^{n-1}\;\sigma'_{ij}/\sigma_o \tag{1}$$

where $\dot{\underset{\sim}{\varepsilon}}^c$ is the creep strain, $\underset{\sim}{\sigma}'$ is the stress deviator, $\bar{\sigma} = (3/2\;\sigma'_{ij}\;\sigma'_{ij})^{\frac{1}{2}}$ and σ_o and $\dot{\varepsilon}_o$ are constitutive parameters. The amplitude of the singularity is determined completely by the crack growth rate, $\dot{a}$, and the material properties. There is no free parameter that has to be adjusted to match far field conditions, so the near tip stress and strain field depend on the remote loads only by those properties of the material set the crack growth rate, i.e. through the fracture criterion. Furthermore, the singularity is stronger than that which would eventually develop around a stationary crack in the elastic-power law creeping material, the well known HRR singularity in which stress is proportional to $r^{-1/(n+1)}$ [Hutchinson (1968), Rice and Rosengren (1968)]. Due to the rapid fall off in stress, it seems likely that the Hui-Riedel singularity dominates over a very small near tip region and that, as a consequence, other less singular or nonsingular terms in the expression for stress could play an important role in creep crack growth. The Hui and Riedel result is obviously important, but it does not give a very clear indication of the load parameters that are likely to control creep crack growth rate. In addition, the assumptions on which the analysis is based are also most accurate when crack growth occurs by rapid repetition of very small steps. This will seem to the observer of the macroscopic to be most like the continuous crack growth assumed by Hui and Riedel. If crack growth occurs by large steps with relatively long periods of time intervening the Hui-Riedel analysis will not be accurate, since steady state stress and strain fields might eventually develop around the stationary cracks. If the time between steps of crack advance are sufficiently long, most of the damage around the stationary crack tip will occur in the stationary crack steady state tip field. This means that the damage increase rate and consequently the fracture criterion will depend on C*, the parameter that characterizes the steady state fields around the stationary crack. The parameter C*, introduced by Landes and Begley (1976), is simply the J integral of Rice (1968) phrased in terms of strain rate and stress rather than strain and stress. The model of incremental crack growth leading to C* controlled growth will only be relevant if the steady state field sets in around the stationary crack well before the next step of crack advance. An approximate model is used in the next section in an attempt to determine the magnitude of the transient time relative to the time elapsed between increments of growth.

INCREMENTAL CRACK GROWTH MODEL

The model developed in this section is for crack propagation in plane strain in a metal at a temperature sufficiently high to cause creep. The structure is considered to be loaded relatively quickly at operating temperature and the load maintained at a fixed level for a long time. While the load is being raised to operating levels, the strain rates are considered to be sufficiently high so that creep strains can be neglected and elastic-plastic response dominates the structural behavior. The operating load is such that only small scale yielding occurs at the tip of the crack in the structure. Creep strains accumulate and the strain rate in the material tends towards a steady state dominated by creep and determined by the creep index. It is assumed that no crack propagation occurs until well after the steady state sets in. Analyses for this situation have been provided by Riedel and Rice (1979), Riedel (1979), and Leckie and McMeeking (1980). Riedel and Rice consider the case of elastic response during loading followed by elastic-power law creep deformation. The steady state straining involves purely creep strain rate with near tip stresses proportional to $r^{-1/(1+n)}$ as in the HRR singularity. The amplitude of the singularity is proportional to C*. Riedel (1979) modified the analysis of Riedel and Rice to include effects of primary and tertiary creep over and above the power law secondary creep. Leckie and McMeeking (1980), however, considered elastic-perfectly plastic response during loading followed by elastic-perfectly plastic-power law creep response. The strain rate was considered to be the sum of elastic, plastic, and creep terms. The elasticity was linear and isotropic and the von Mises yield criterion was used for the nonhardening plastic flow. The creep rate was of the power law form as given by (1). With this constitutive law the steady state leads to the presence of a small zone at the crack

tip in which the stresses are at yield and in which both creep and plastic straining occur. The creep and plastic strain rates, both being parallel to the stress deviator, are indistinguishable. This plastic zone is contained within a region where only creep straining occurs. This situation is analogous to small scale yielding at cracks in the time independent plasticity case and so, following similar reasoning in that case in which the plastic zone is considered to be a crack tip boundary layer, the stresses in the purely creep zone some distance from the crack tip are assumed to be proportional to $r^{-1/(1+n)}$. Leckie and McMeeking determined the size of the plastic zone in an approximate way by finding the locus of points at which the stress of the HRR singular field is at yield. They found the maximum radius of the plastic zone to be about $0.2(\sigma_o/\sigma_y)^{n+1}(C^*/\sigma_o\dot{\varepsilon}_o)$ where σ_y is the time independent uniaxial yield stress. The time t_T that elapses between the application of the load and the setting in of steady state was also estimated to be $(\sigma_y/\dot{\varepsilon}_o E)(\sigma_o/\sigma_y)^n$ where E is Young's modulus. As discussed by Leckie and McMeeking (1980), this is likely to be a very small fraction of the life of a component.

Consider now initiation and continuation of rupture at the tip of the crack. Attention will be restricted to cases where the growth initiates well after the steady flow sets in. Since crack growth occurs by the diffusion aided growth of holes on grain boundaries, it is assumed that propagation occurs by a finite increment whose length is dependent on the material and possibly on temperature and C* level. It is likely that the length of the growth step will be a grain diameter or so, since the phenomenon of creep rupture is associated with grain boundaries. The holes grow continually but their presence is unlikely to have much far reaching effect on the deformation of the component until shortly before the coalescence of the holes with the crack. This coalescence probably occurs relatively quickly, so the effect will be more like an instantaneous propagation of the crack than continuous growth. Thus the growth is accommodated immediately by predominantly time-independent elastic-plastic deformation and then creeping will commence around the new stationary crack tip. The flow in the structure will tend towards the steady state field, associated now with the longer crack, and the stresses somewhat remote from the tip will be proportional to $r^{-1/(1+n)}$. If this steady state arises well before the next increment in crack growth then the rate of propagation of the crack will depend on C* since, as Leckie and McMeeking (1980) have shown, the crack tip opening rate during steady state will be proportional to C^*/σ_y, so the damage rate will increase monotonically with C*. If, however, the next step of propagation occurs before or soon after steady state sets in, the rate of growth may or may not depend on C* and a detailed analysis of the transient fields would be required to determine what parameters are likely to control the crack growth rate. If the time between the crack growth increments is short, then the process will seem most like continuous growth and the analysis of Hui and Riedel (1979) will be the relevant method.

Since the relevance of the incremental crack growth model depends on whether the steady state strain rate fields set in quickly around the new crack tip, it is important to estimate the time elapsed during transient straining after each step of crack growth. This could be done by using the elastic-creep models of Riedel and Rice (1979) or Riedel (1979). However, the model of Leckie and McMeeking (1980) will be used which should give a similar result. Thus the response to the increment of crack growth is entirely elastic-plastic with the plastic strain approximated by that arising from a nonhardening law with a von Mises yield criterion. The crack first grows through the plastic zone in which the stress released by the separation of the crack will vary from $3\sigma_y$ at the original crack tip down to about σ_y at the plastic zone boundary. If the crack grows to this point and further, the stress released is assumed, according to the model of Leckie and McMeeking to be given by

$$\sigma_{yy} = \sigma_o(C^*/r_o\,\sigma_o\,\dot{\varepsilon}_o)^{1/(1+n)}\,g_{yy}$$

where r_o is the distance from the prior crack tip and g_{yy} is a parameter of order unity that can be evaluated from the HRR singularity. Since the model will be most accurate for intermittent steps of large amounts of growth, it will be assumed that the crack grows through the plastic zone and well beyond. The component is assumed to be loaded at levels that cause small scale plastic zones, so the stresses around the new crack tip can be estimated by first calculating the stress intensity factor K_G associated with the tractions wiped out by the crack growth and adding the associated stresses to the existing stresses. Of course, this will raise the stresses in some material above yield, and this material is considered to form the new plastic zone. The actual evaluation of K_G can be omitted, since it is found not to play a role.

The step of crack growth has been assumed to exceed the size of the initial plastic zone, and it is also helpful to assume that the active plastic zone initially created at the new crack tip is much smaller than the step of crack growth. This allows the existing stress around the plastic zone of the new tip to be approximated as a homogeneous stress with a magnitude given by the stresses at the position of the new crack tip but prior to growth. With the additional stresses due to growth, the resulting stress outside the active plastic zone is

$$\sigma_{yy} = \left(\frac{C^*}{\Delta a\ \sigma_o\ \dot{\varepsilon}_o}\right)^{1/(n+1)} \sigma_o\ g_{yy} + \frac{K_G}{\sqrt{2\pi r}} h_{yy}(\theta) \tag{2}$$

where r, θ are polar coordinates measured from the new crack tip and $h_{yy}(\theta)$ of order unity, is the angular function for the mode I elastic crack tip stresses. The size r_p of the new plastic zone can be estimated by setting $\sigma_{yy} = \sigma_y$ and dropping $h_{yy}(\theta)$ so that

$$\frac{1}{\sqrt{r_p}} = \frac{\sqrt{2\pi}\ \sigma_y}{K_G}\left[1 - \left(\frac{C^*}{\Delta a\ \sigma_o\ \dot{\varepsilon}_o}\right)^{1/(n+1)} \frac{\sigma_o}{\sigma_y}\right] \tag{3}$$

The initial rate of change of stress can be estimated, following Leckie and McMeeking (1980), as $\dot{\sigma} = -E\ \dot{\varepsilon}_o(\sigma_y/\sigma_o)^n$ where an angular function presumed to be of order unity has been ignored. As in Leckie and McMeeking the initial rate of change of the plastic zone is estimated as

$$\dot{r}_p = -\left(\frac{\dot{\sigma}}{d\sigma/dr}\right)_{r=r_p}$$

which in this case leads to

$$\dot{r}_p = -2\sqrt{2\pi}\ E\ \dot{\varepsilon}_o(\sigma_y/\sigma_o)^n\ r_p^{3/2}/K_G \tag{4}$$

and, using (3),

$$\dot{r}_p/r_p = -\frac{2\ \dot{\varepsilon}_o(\sigma_y/\sigma_o)^n\ (E/\sigma_y)}{1 - \frac{\sigma_o}{\sigma_y}\left(\frac{C^*}{\Delta a\ \dot{\varepsilon}_o\ \sigma_o}\right)^{1/(n+1)}} \tag{5}$$

The denominator cannot be zero since that would be the case of Δa being equal to the initial plastic zone size. The ratio $\dot{r}_p/r_p$ is taken as an estimate of the inverse of the time for the transient to die away, since during steady state straining a much smaller plastic zone exists. The transient time is normalized by the time between steps of growth t_G which is expressed as $\Delta a/\dot{a}$ where $\dot{a}$ is the macroscopic crack growth rate. Thus

$$\frac{t_T}{t_G} = \frac{[1 - (C^*/\Delta a\ \dot{\varepsilon}_o\ \sigma_o)^{1/(n+1)}\ (\sigma_o/\sigma_y)]\ \dot{a}}{2\dot{\varepsilon}_o(E/\sigma_y)(\sigma_y/\sigma_o)^n\ \Delta a} \tag{6}$$

Note that if $\Delta a \to \infty$, the result for t_T is as in Leckie and McMeeking (1980) for the transient time after initial loading of a stationary crack. Since a large growth step would take the tip into material stressed at a relatively low level, the analogy to the loading of a crack in unstressed material seems appropriate.

The magnitude of t_T/t_G will determine whether C* is unambiguously the parameter determining the rate of growth, since a small value of the ratio will validate the intermittent growth model developed here. Consider the plane strain center cracked panel in tension with crack length to width ratio 0.5. In this case in steady state $C^*/\sigma_o\ \dot{\varepsilon}_o = .833a(\sqrt{3}\ \sigma/\sigma_o)^{n+1}$ where a is the crack length and σ is the applied stress so

$$\frac{t_T}{t_G} = \frac{[1 - \sqrt{3}\left(\frac{.833a}{\Delta a}\right)^{1/n+1} (\sigma/\sigma_y)]\ \dot{a}}{2\dot{\varepsilon}_o(E/\sigma_y)(\sigma_y/\sigma_o)^n\ \Delta a} \tag{7}$$

Choosing fairly representative values of $E/\sigma_y = 10^3$, n = 5 and $\sigma/\sigma_y = .05$ and making the arbitrary choice $\sigma_o/\sigma_y = 1$ so that the value of $\dot{\varepsilon}_o$ is left to characterize the creep law or (7) becomes

$$\frac{t_T}{t_G} = \frac{[1 - .084(a/\Delta a)^{1/6}]\ \dot{a}}{2000\dot{\varepsilon}_o\ \Delta a} = \alpha\left(\frac{a}{\Delta a}\right) \frac{\dot{a}}{\dot{\varepsilon}_o\ \Delta a}$$

Since the factor in brackets in the numerator must be much less than unity to permit the approximations used implies that $\Delta a/a >> 3.5 \times 10^{-7}$. The function $\alpha(a/\Delta a)$ is tabulated for some representative values.

a/Δa	α
10^6	$.1 \times 10^{-3}$
10^5	$.2 \times 10^{-3}$
10^4	$.3 \times 10^{-3}$
10^3	$.4 \times 10^{-3}$
10^2	$.4 \times 10^{-3}$
10	$.4 \times 10^{-3}$

If $\dot{a}/\dot{\varepsilon}_o\ \Delta a$ is less than 10, then t_T/t_G is less than 1%. If $\dot{a}/\dot{\varepsilon}_o\ \Delta a$ is less than unity, then t_T/t_G will be so small that even if it has been grossly underestimated most of the time will be spent in steady state creeping around the stationary crack tips. Thus C* will determine the crack growth rate. In a case where $\dot{\varepsilon}_o = 10^{-4}$ per hour and $\dot{a} = 10^{-4}$mm/hr, then C* dominated growth would definitely arise for step sizes greater than 1mm and possibly for those greater than .1mm.

DISCUSSION

From the foregoing results it seems possible in some cases that transient fields around temporarily stationary crack tips die away sufficiently quickly for the steady state straining around the stationary crack to play a dominant role in the creation of damage of the crack tip. This would explain why some creep crack growth rates correlate with C*. Of course, much more accurate analysis is required to clarify this. The deficiencies in the model just discussed pertain not only to those that might be reduced by using, say, finite element analysis on the problem as formulated, but also to the constitutive laws on which the calculations were based. The phenomenon of plastic flow at high temperatures is not as straight forward as the simple flow law adopted, especially when reversals of strain rate near the growing crack tip are considered. However, basic features of the behavior are present in the elastic-perfectly plastic power-creep laws used.

It is possible that creep crack growth in a given material might occur at some temperatures or loads with $t_T/t_G << 1$ and in other circumstances with t_T/t_G not much smaller than unity. Since the different circumstances might pertain to operation and laboratory conditions, it is important to determine the operating circumstances and duplicate these in the laboratory. Furthermore, the experiment should not induce failure by the spread of diffuse rather than crack tip damage where the operating conditions cause rupture by crack growth.

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