TWO STRESS INTENSITY FACTOR CALCULATION METHODS AND SOLUTIONS FOR VARIOUS THREE-DIMENSIONAL CRACK PROBLEMS

PALUSAMY, S.S. *, HELIOT, J. **

* WESTINGHOUSE ELECTRIC CORPORATION, P.O. BOX 355, PITTSBURGH, PENNSYLVANIA 15236, U.S.A.

** CREUSOT-LOIRE, 15 PASQUIER, 75383 PARIS, CEDEX 08 FRANCE.

ABSTRACT

The paper reports on two stress intensity factor calculation methods for three dimensional crack configurations. They have been used in the analysis of the same problems : surface cracks in plates and cylinders, corner crack in a nozzle. The results are compared.

KEYWORDS

Stress intensity factor, calculation method, finite element, boundary integral equation, crack, cylinder, plate, nozzle.

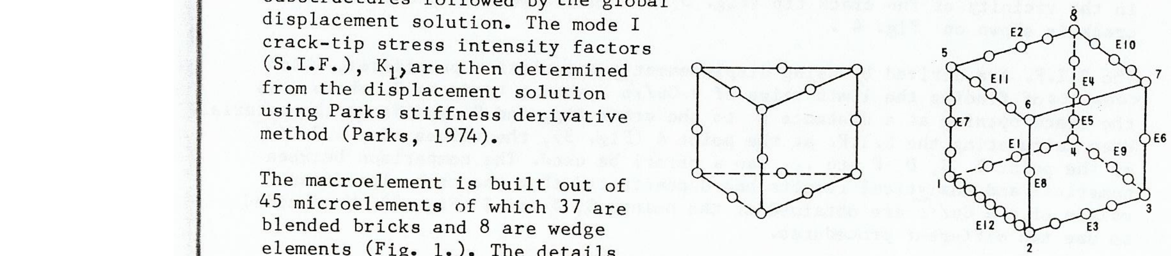
INTRODUCTION

The paper reports on linear elastic fracture mechanics analysis. The work was performed by two companies mainly in a joint R. and D. program. They have used two independent methods. Many crack problems were solved twice. The methods are presented with some details. A lot of previously published results are analysed and are compared.

FIRST METHOD

The first method is based upon three-dimensional finite element (F.E.) calculation and the macroelement technique developed by (Hall, 1979). It consists of dividing the flawed three-dimensional structure into two or more substructures and modeling the region containing the flaw by one or more macroelement substructures. The solution process begins by obtaining a condensed stiffness matrix for each of the substructures followed by the global displacement solution. The mode I crack-tip stress intensity factors (S.I.F.), K_I, are then determined from the displacement solution using Parks stiffness derivative method (Parks, 1974).

The macroelement is built out of 45 microelements of which 37 are blended bricks and 8 are wedge elements (Fig. 1.). The details of these microelements are contained in (Hall, 1979) and it



(a) 45 d.o.f. WEDGE (b) VARIABLE d.o.f. BLENDED BRICK

Fig. 1

suffices to know that the wedges have 45 degress of freedom(DOF) and the D.O.F. of blended bricks can be varied subject to minor restrictions specified in(Hall, 79).

The undeformed macroelement shown in Fig. 2 contains a built-in quarter-elliptical crack. The region surrounding the crack tip is modeled by a channel of 28 blended bricks enabling the analyst to vary the density of nodes to achieve a desired combination of accuracy and cost. The total D.O.F. corresponding to the choice of minimum density is 1656.

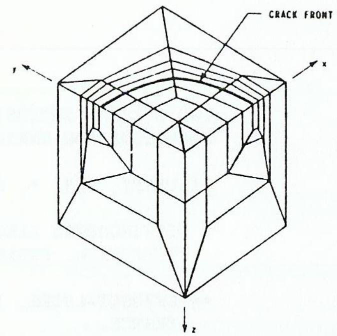


Fig. 2

The principal characteristics of the macroelement are the following : it is compatible with the 20-nodes isoparametric element. It has the option to vary crack tip region nodal density. It is parametrically defined so as to allow curved faces. It permits a wide variety of crack surface loadings (any combination of a bivariable cubic polynomial) and it significantly reduces the man-time needed to formulate the finite element model.

The first verification of this method was made with reference to a plate containing a semielliptical surface flaw of aspect ratio 5, fractional through-wall depth 0.6 and subjected to remote uniform tension loading. It has been shown that the K values obtained by macroelement technique agreed with those of (Smith, 1973) and of (Raju, 1979) within 8 and 3 percent respectively.

SECOND METHOD

The second method was set up and assessed by solving some problems, the solution of which is known in a closed form. The results were compared to the exact solution. A few procedures were tested and a simple, accurate and reliable procedure was chosen ; the accuracy of the results has been checked.

This method consists of an elastic analysis of the cracked body by using the boundary integral equation method and the the "Equations Integrales Tridimensionelles E.I.T.D." program (Lachat, 1975) ; which was written by Centre Technique des Industries Mecaniques, C.E.T.I.M. (France). The outer surface of the body is meshed with 6 and 8 nodes isoparametric elements and quarter points elements in the vicinity of the crack tip (Fig. 3). A typical mesh in the plane of the crack is shown on Fig. 4 .

The S.I.F. are derived by using displacement extrapolation procedures, which consists of finding the limit value of $y = Cu/\sqrt{\rho}$,when ρ tends to 0, where u is the crack opening at a distance ρ to the crack tip, and C depends on the material. When calculating the S.I.F. at the point A (Fig. 3), the values of $y = Cu/\sqrt{\rho}$ at the point B, C, D, F etc ... can a priori be used. The comparison between numerical and analytical results has demonstrated that the most significant values of $y = Cu/\sqrt{\rho}$ are obtained at the points C, D and E (Fig. 3). We decided to use two different procedures.

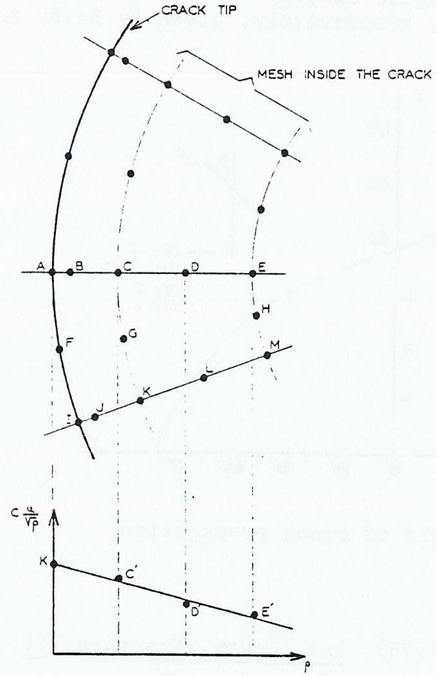


Fig. 3

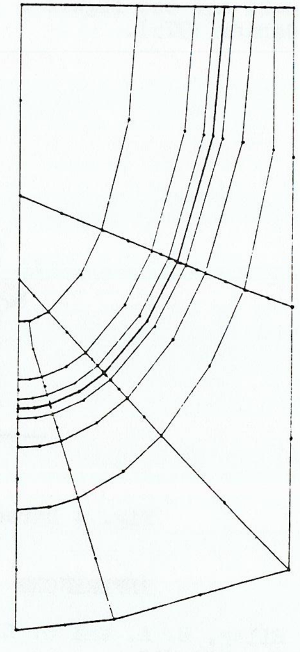


Fig. 4

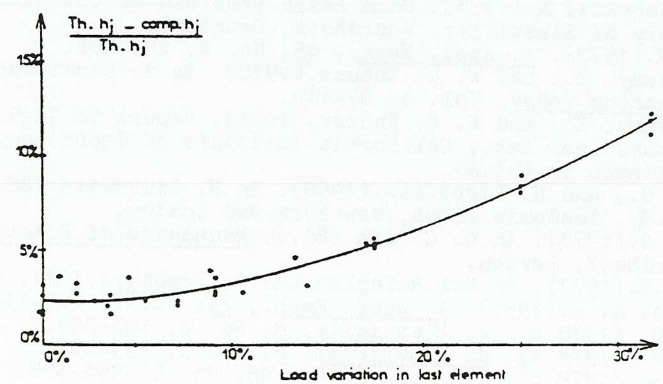


Fig. 5

The first one consists of considering the straight line defined by the three points C', D', E' , and by a least square fitting ; the second one considers the parabola $y = d\rho^2 + e\rho + f$ defined by the three points. The intersection of the straight line or of the parabola with the y axis gives the K value. To calculate the K value at the point F (Fig. 3) the first procedure can be used by considering the straight line corresponding to the points G and H. The two procedures give results whose accuracy is usually similar. The main interest of using the two procedures is to compare the results and to verify that the difference is less than 1 or 2 %.

The procedures were tested by calculating K values along elliptical cracks in infinite solids aspect ratios 1, 1/3, 1/10 . The cracks were subjected to pressures $\sigma = x^i \; y^j$). The comparison, between the calculated value of K denominated K comp. and the theoretical value K_t showed that ($K_t - K_{comp})/K_t$ at the point A (Fig. 3) depends mostly on ($\sigma_A - \sigma_C$)/ σ_A as shown on the fig. 5 : σ_A and σ_C are the values of the pressure applied on the crack at the point A and C respectively (Fig. 3).

The method and procedure were validated by calculating the S.I.F. along a penny shaped crack in an infinite solid subjected to stress x°, $|x|$, x^2 , $|x^3|$, x^4 . These calculations (Héliot, 1979 b) evidenced what mesh refinement was necessary. Similar calculations were performed on embedded elliptical cracks (Héliot, 1980).

The B.I.E. method and the above procedures can be used for any cracked body, it is simple, accurate and not very costly (100 to 200 man hours and 100 to 800 seconds CPU of CDC 7600 computer) Alternative procedures have been used with B.I.E. method ; the stress extrapolation and the energy release rate procedure ; the first one is not accurate, the second is more accurate than the displacement extrapolation procedure but its use is more difficult and more costly.

S.I.F. SOLUTIONS FOR SURFACE - CRACKED PLATE ; BATELLE BENCHMARK PROBLEM N°1

S.I.F. solutions were obtained for semi-elliptical cracks in a plate as defined in(Hulbert, 1977). The aspect ratio was a/c = 0.5. The crack depth, a, was 25 and 75 percent of the thickness t. The two methods were used.

The crack was subjected to a pressure : $\sigma = \sigma_0 + \sigma_1 \; y/t$ (1)
where y is through the thickness coordinate with the origin on the side of the plate containing the crack mouth (Fig. 6).

The resulting S.I.F. can be expressed by :

$$K(\phi) = \sqrt{\frac{\pi a}{Q}} \left[\cos^2\phi + \frac{a^2}{c^2}\sin^2\phi\right] \left[\sigma_0 \, h_0(\phi) + \frac{a}{t}\sigma_1 \, h_1(\phi)\right] \quad (2)$$

$Q = \left[E(k)\right]^2$; $k = 1 - (a/c)^2$; E (k) is the complete elliptic integral of the second kind. The value of E (k) is 1.211, 1.114, 1.016 when a/c is 0.5, 1/3 and 0.1 respectively.

$h_o(\phi)$ and $h_1(\phi)$ are the calculated influence functions

The first method was used with a 1656 D.O.F. macroelement. The length and width of the plate was 1.524 and 0.762 respectively. The thickness of the plate was 0.1524 and 0.0508. The second method used similar dimensions but one more case a/t = 0.50 was solved. The results (Mc Gowan, 1977; Héliot, 1979 a) are compared on the Fig. 6 and 7. The difference between the corresponding curves is less than 8%.

A recent report issued by the Society for Experimental Stress Analysis (Mc Gowan, 1980) compared these solutions to others and concluded that they were valid.

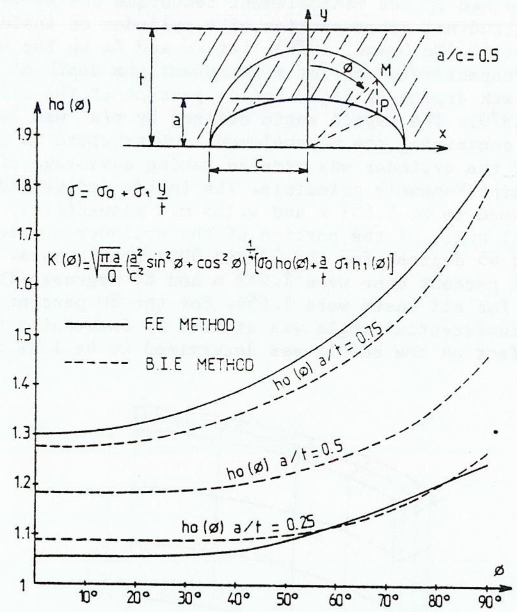


Fig. 6

Semi elliptical surface crack in a flat plate

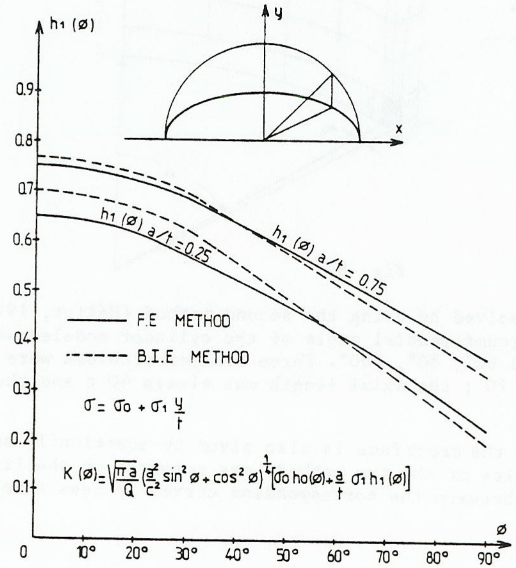


Fig. 7

Semi elliptical surface crack in a flat plate

S.I.F. SOLUTIONS FOR SEMI-ELLIPTICAL SURFACE CRACK IN A CYLINDER

S.I.F. solutions were obtained by the macroelement technique for semi-elliptical surface flaws in the longitudinal cross-section of a cylinder of inside radius R_i to thickness t ratio of 10 (Mc Gowan, 1979). Let 2c and 2a be the major and minor axes of the flaw, respectively and let a represent the depth of the semi-elliptical flaw. Three crack depths of 25,50 and 80 percent of the cylinder wall were studied (Mc Gowan, 1979). The aspect ratio defined by c/a was 3.
The finite element model containing the macroelement substructure is shown in Fig. 8. Only a portion of the cylinder was modeled taking advantage of the symmetry condition and Saint-Venant's principle. The inside radius and thickness of the cylinder were assumed to be 1.651 m and 0.165 m., respectively. The axial length and circumferential angle of the portion of the cylinder modeled were, respectively, 1.753 m and 45 degress for the 25 and 50 percent cases. The respective values for the 80 percent case were 2.438 m and 60 degrees. The macroelement degrees of freedom for all cases were 1.656. For the 80 percent case, the influence of the circumferential angle was studied by increasing the angle to 90 degrees and the effect on the result was determined to be less than 2 percent.

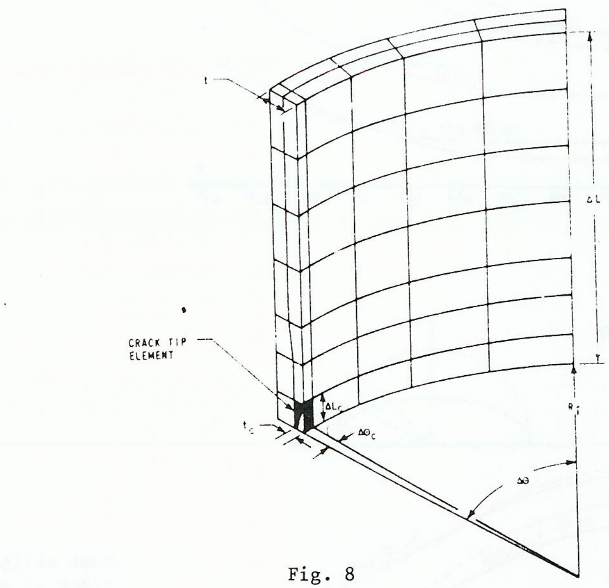


Fig. 8

The same problems were solved by using the second method (Héliot, 1979 b). But the axial length and circumferential angle of the cylinder modeled were respectively 8 t, 12 t, 16 t and 60°, 80°, 180°. Three similar problems were also solved with aspect ratio c/a = 10 ; the axial length was always 40 t and the circumferential angle 180°.

The pressure applied on the crack face is also given by equation(1) and the S.I.F. by (2). The results of the two methods are compared on the figures 9,10 and 11. The difference between the corresponding curves is less than 9 %.

SEMI ELLIPTICAL SURFACE CRACK IN A CYLINDER

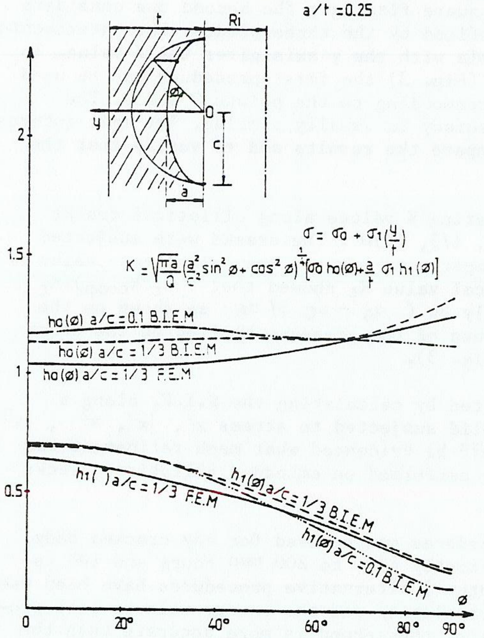


Fig. 9

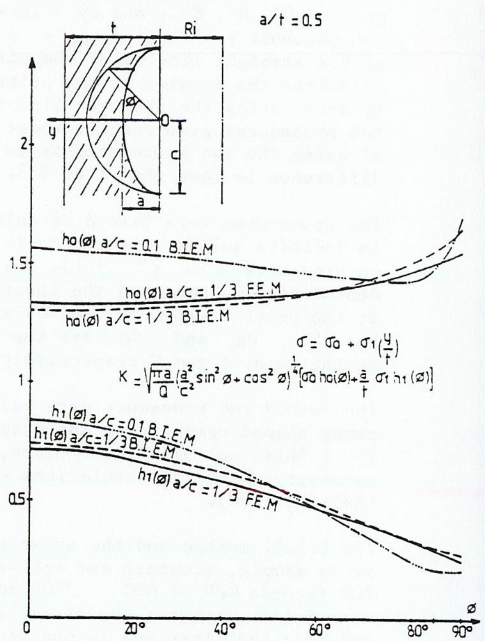


Fig. 10

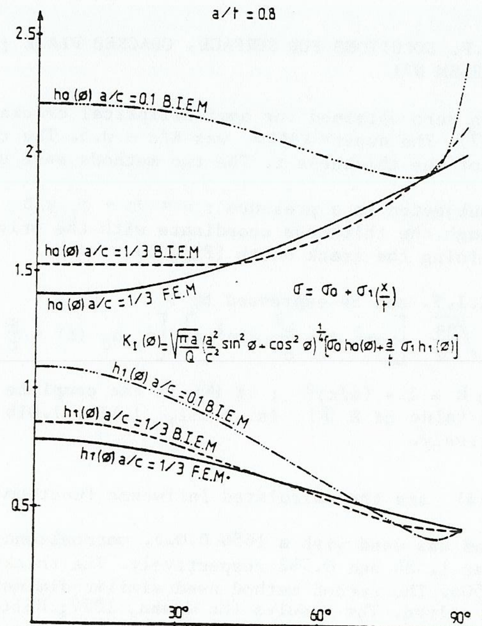


Fig. 11

S.I.F. SOLUTIONS FOR A CORNER CRACKED HOLE IN A PLATE, BENCHMARK PROBLEM N°2.

The macroelement technique was applied to one of the corner cracked hole Benchmark problems (Palusamy, 1979). The crack was assumed to be quarter-circular of radius a in the corner of a hole of radius R in a plate of height 2H width 2W and thickness t. The dimensional values a,R,H,W and t were chosen to be 12.5 mm. 25 mm., 315 mm., 187,5 mm. and 25 mm., respectively. The value of a/R as well as a/t for this geometry is 0.5.

Both remote and crack surface loadings were considered. The remote loading consisted of a uniform tension, σ_0 , at the edge of the plate in a direction normal to the plane of the crack. The crack surface loading was represented by

$$\sigma = \sigma_{00} + \sigma_{10}\ x/t \qquad (3)$$

Where σ_{00} and σ_{10} are arbitrary stresses and x and y are coordinates rotated through 45 degree with respect to the plate surface. See Fig. 12.

The S.I.F. for the remote tension loading is given by

$$K = \frac{2}{\pi}\ \sigma_0\ \sqrt{\pi a}\ h_0\ (\theta) \qquad (4)$$

And the S.I.F. for the crack surface loading is given by

$$K = \frac{2}{\pi}\ \left[\sigma_{00}\ h_{00}\ (\theta) + \frac{a}{t}\ \sigma_{10}\ h_{10}\ (\theta)\right] \qquad (5)$$

The same calculation was performed also by using the B.I.E. method. The results are compared on the Fig. 13. The difference between the corresponding curves is less than 6 percent except at the point θ = 90° where it is 12 percent.

The work of the authors is presently continued by solving problems of corner cracks in nozzles.

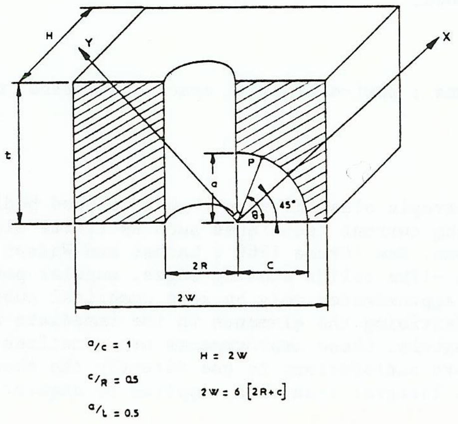


Fig. 12

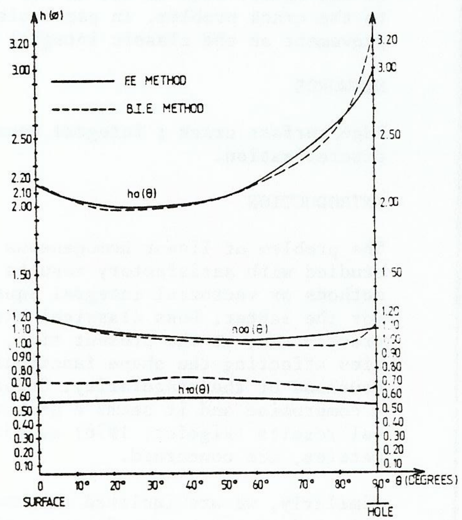


Fig. 13
Corner cracked hole in a plate

CONCLUSION

Many 3-D L.E.F.M. analysis results were presented and most of them were obtained by using two independent methods. The agreement between the results is good, they can be considered as reliable, and they are now used in engineering applications.

ACKNOWLEDGEMENTS

These results were obtained under cooperative research programs between Westinghouse, Framatome, Commissariat à l'Energie Atomique (CEA) and Electricité de France (EDF), except for the a/c = 0.1 semi-elliptical cracks, which were calculated under a cooperative program between CEA/DSN and Framatome.

The author of the macroelement technique acknowledges the contribution made by Dr. M. Raymund and Mr. J.F. Petsche in the computation of numerical results ; the author of the B.I.E. work acknowledges the contribution made by Mr. Flamand and Mr. Large.

REFERENCES

Hall, C.A., Raymund, M. and Palusamy S. (1979) Int. J. of Fracture, 15,1979, pp. 231-245.

Héliot, J., Labbens, R.C. Pellissier-Tanon, A. (1979 a) Int. J. of Fracture, 15, (1979), pp. 197-202.

Héliot, J., Labbens, R.C.,Pellissier-Tanon A. (1979 b) ASTM STP 677 pp. 341-364.

Héliot, J., Labbens, R., Pellissier-Tanon, A. (1980) A.S.M.E. Pressure Vessel and Piping Conference, San Francisco, Aug 12-15,1980.

Hulbert, L.E. (1977). Int J. of Fracture, 13 pp.87-91-1977.

Lachat, J.C. and Watson, J.O. (1975). In Symposium on Boundary Integral Equation Method, A.S.M.E. (1975) pp. 85-100.

Mc. Gowan J. J. and Raymund M. (1978), WCAP-9318 Westinghouse Electric Corporation, Pittsburgh Pa. April 1978.

Mc. Gowan J. J. and Raymund M. (1979). ASTM STP 677., pp. 365-380.

Mc. Gowan J. J. (1980). A critical evaluation of numerical solutions to the "Benckmark" surface flaw problem. Editor SESA, PO. Box 277 Saugatuck Station Westport, Connecticut 06880.

Memoires Techniques du Cetim n° 25, CETIM, 52 avenue Felix Louat, 60304 SENLIS (FRANCE).

Palusamy S.S. and Raymund. M. (1979). XIII th National Symposium on Fracture Mechanics, ASTM, Philadelphia June 1979.

Parks, D.M. (1974). Int. J. of Fracture, 10 pp. 487-502. (1974).

Raju I.S. and Newman J.C. (1979). Journal of Engineering Fracture Mechanics, V.4, 1979, pp. 817-829.

Smith, F.W. and Sorensen, D.R. (1976). Int J. of Fracture, V12,1979, pp. 47-57 Also see : Colorado State Univ. Tech. report 4, NASA grant NGL-06-002-063, 1973.