THE RELATIONSHIP OF COMPLIANCE CHANGES DURING FATIGUE LOADING TO THE FRACTURE OF COMPOSITE MATERIALS

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ABSTRACT

The present work is based on measurements of the change in engineering stiffness values induced by the development of damage in composite laminates during quasi-static or cyclic (fatigue) loading. It is found that these changes are related to the individual details of the damage events in the laminates, as well as to the residual strength and life of the laminates. It appears that stiffness changes can also be used to relate composite material behavior under cyclic loading to its behavior under quasi-static loading. Results are reported for both notched and unnotched laminates.

KEYWORDS

Stiffness; composite materials; fracture; compliance; damage.

THE RELATIONSHIP OF COMPLIANCE CHANGES TO FRACTURE

It has been known for some time that the stiffness of composite materials changes under both quasi-static and (especially) cyclic loading (Henneke, 1979; O'Brien, 1977; O'Brien, 1978; Stinchcomb, 1978; Stinchcomb, 1979). This stiffness change is important in a direct way since it affects deflections and vibration characteristics, and may cause an engineering component to "fail" in the sense that dimensional changes or dynamic response may fail to meet design requirements after some period of loading. In the last few years it has also been suggested that stiffness changes can be an excellent direct quantitative indicator of the internal integrity of composite laminates. It has been found that changes in stiffness - or compliance, its inverse - can be used to detect damage development, to discriminate types of damage, and to relate the damage detected to residual response of the laminates (Henneke, 1979; O'Brien, 1977; O'Brien, 1978; Stinchcomb, 1979). In the present paper we will present the part of that experience which relates to fracture. We will begin with a discussion of the relationship of compliance changes to fracture and then discuss specific data for uniform stress (unnotched specimens) and non-uniform stress (notched specimens) fields.

Laminate fracture is generally preceded by several types of damage in laminates which have some combination of "on-axis" and "off-axis" plies, the "axis" referring to a principle loading direction. Depending upon the ply orientations and the stacking sequence, as well as the loading, this damage can be typified by matrix cracking, delamination and debonding, and fiber fracture for the high-modulus fiberous composites of interest here. The laminate fails when the plies with the lowest angle to the load axis are no longer able to carry the applied load, a situation we will call last ply failure. It is obvious that fiber fractures will lower the ply strengths in a direct and important way. However, it is also true that relatively few fibers fracture in the lowest angle plies until last ply fracture is incipient (O'Brien, 1977; Whitcomb, 1979). For the most part, compliance changes prior to fracture are caused by the development of matrix damage. The quantitative relationship of compliance changes to last ply failure comes about through the mechanics of the stress redistribution that occurs in the laminates due to these damage events.

Let us consider the damage events as they would occur during a quasi-static loading or a cyclic fatigue test: The first damage events to occur will be matrix cracks in the plies with large off-axis orientations in tensile loading (or smallest off-axis orientations under compression). These matrix cracks form, generally, in planes which are frequently parallel to the fibers in a given ply and perpendicular to the major loading direction. During the course of earlier work we have discovered a particular aspect of this transverse crack formation which is very helpful to our effort to relate it to stiffness change and to the stress redistribution which affects strength. We have discovered that transverse cracks form in a very specific way resulting in a crack pattern that is stable in the sense that crack formation virtually stops once that pattern is formed. We have come to call that pattern a "characteristic damage state" or CDS for short. The CDS is a laminate property in that it does not depend on the load history, residual stress, moisture related stress, or other extensive variables; it is entirely controlled by the properties of the individual plies of a laminate and by the stacking sequence of those plies. The CDS consists of regularly spaced cracks in each of the off-axis plies. The spacing of the cracks is determined by the rate at which stress is transferred into a cracked ply as a function of distance from the crack, as explained in the references by Masters (1978), Reifsnider (1977) and Stinchcomb (1979).

An example of such a CDS for a $[0,90,\pm45]_s$ laminate constructed from AS 3501 graphite epoxy is shown in Fig. 1 along with a pattern predicted by an analysis developed by one of the authors, also discussed in the references (Reifsnider (1977) and Stinchcomb (1979)). The pattern in Fig. 1 (a) was traced from a replica of the edge pattern of cracks observed in a specimen after cyclic loading for one million cycles at a maximum stress level of about two thirds of the ultimate strength (R=0.1). The pattern shown in Fig. 1 (b) was predicted from a simple analysis which has been used to successfully predict the CDS for more than a dozen other cases (Stinchcomb, 1979). The value of such a state in the present context is that it provides a well-set mechanics analysis in the same way that the single crack acts as a generic damage state for homogeneous materials. The regions in the neighborhood of the transverse cracks in the CDS support less load, thereby reducing the contribution of the cracked ply to the laminate stiffness. The reduction is proportional to the original stiffness of the cracked ply the number of cracks per inch that form in that ply, and the number of those plies compared to the number of other plies in the laminate. Hence, the stiffness change is a laminate property which depends on the material and orientation in each ply, the number of plies of each type and the stacking sequence of the plies - exactly those factors which determine the CDS. For example, consider a $[0,90_3]_s$ glass-epoxy laminate loaded quasi-statically. The total change in stiffness due to the cracking of the 90 deg plies during the formation of the stable CDS pattern is about 34 percent. Using simple laminate theory and reducing the stiffness of the cracked ply, a value of about 30 percent is calculated.

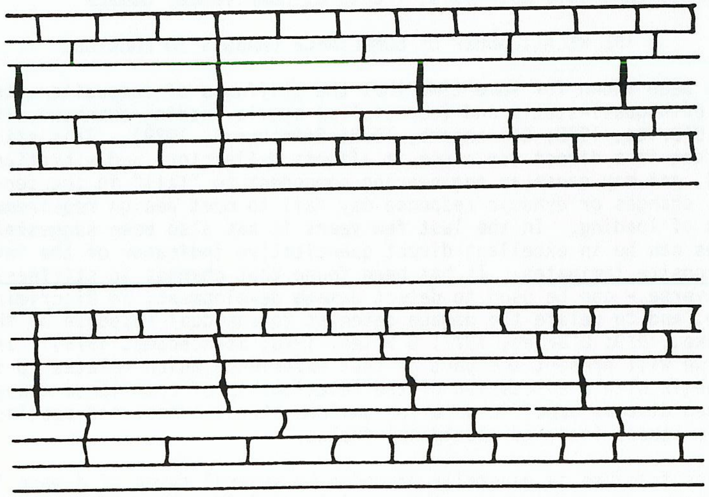

Fig. 1. Predicted (a) and observed (b) characteristic crack patterns in a $[0,90,\pm45]_s$ graphite epoxy laminate.

The next step is to establish the relationship of this stiffness change to fracture. As mentioned above, that relationship is provided by the mechanics of internal stress redistribution. At the global level, when one laminate ply develops matrix cracks (or other types of damage) the load that was being carried by that ply is transferred to an unbroken ply. Table 1 illustrates that process for a $[0,90,\pm45]_s$ graphite epoxy laminate. As transverse cracking occurs in the 90 deg plies, the stiffness of those plies in directions perpendicular to the cracks ($\bar{Q}_{11}$ and $\bar{Q}_{12}$) are reduced. If they are reduced to zero, the longitudinal stiffness of the laminate changes by 3.7 percent as shown in the second entry of the Table. The axial stress in the 0 deg plies increases by about 3.1 percent because of the load dropped by the broken plies, while the transverse stress in that ply drops to nearly zero. (The calculation was made for an applied stress of 6.89 mPa.) The important concept here is that if the precise nature of damage events is known and understood, then the mechanics of those events will relate the measurable stiffness change to the internal stress redistribution which controls strength. (This principle is also a basic premise of fracture mechanics for single flaws.)

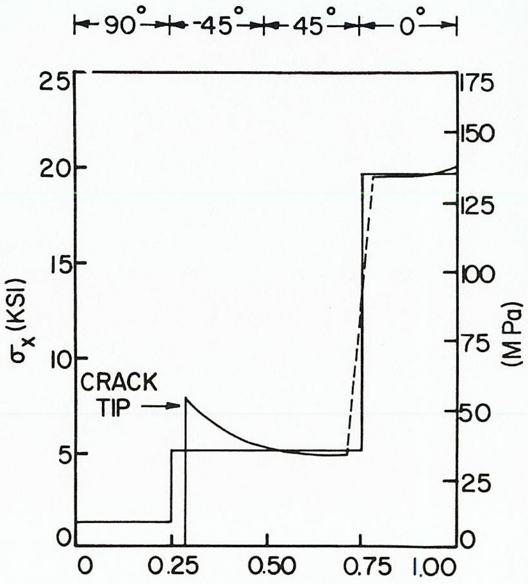

Fig. 2. Axial normal stress distribution through the thickness of a $[0,\pm45,90]_s$ laminate with a crack in the 90 deg plies.

The stress redistribution which relates stiffness change to fracture may also occur on a local rather than global scale. Figure 2 shows a three dimensional finite difference solution of the axial stress fields in the vicinity of a crack through the thickness of all off-axis plies in a $[0,\pm45,90]_s$ specimen interior. The stress in the 0 deg ply is elevated above the global value by the off-axis ply crack. Figure 3 shows another stress component for that situation, an interlaminar shear. In general, when matrix cracks develop, interlaminar shear and normal stresses also develop in the neighborhood of the matrix crack interior boundaries. These stresses contribute to ply delamination and to the coupling together of transverse cracks by limited delamination. Crack coupling is shown in Fig. 4 for the $[0,\pm45,90]_s$ laminate discussed earlier, just prior to failure.

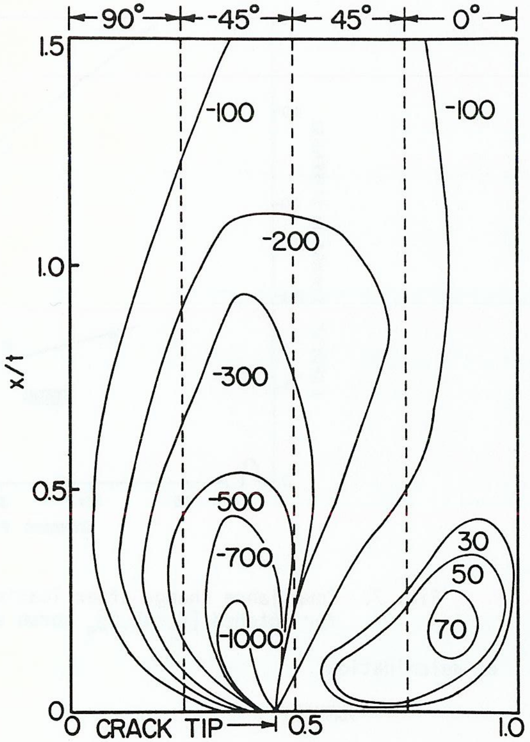

Fig. 3. Interlaminar shear stress distribution for the laminate shown in Fig. 2.

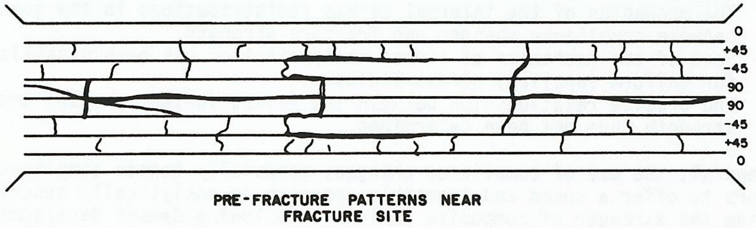

Fig. 4. An example of crack coupling in a laminate just prior to fracture initiation.

The scenerio is similar for other damage mechanisms. If the 90 deg plies delaminate from the 0 and ±45 deg plies in a $[0,\pm45,90]_s$ laminate, the stress redistribution causes a change in stiffness and ply stresses as shown in the third data set in Table 1. The stiffness of the "laminate" (which now consists of two 90 deg plies acting together and the [0,±45] parts of the laminate acting as a unit, both pulled to a common strain) is reduced to 50.78 GPa, a 10 percent reduction. (Other stiffnesses are not defined.) The stress components in the other plies change according to the relaxation of the lateral constraint formerly imposed by the 90 deg plies. Most importantly, the axial stress in the 0 deg plies

increases by about 9.3 percent, an increase which will affect the residual strength of the laminate. If such behavior is local, these changes would also be local, but the consequences would be just as great (or greater) if the local stress gradients are significantly detrimental.

Having established that the changes in compliance during damage development are related to fracture through the stress redistributions that accompany specific damage mechanisms and induce stress increases in the lowest angle ply or plies, we now examine some data for uniform and nonuniform stress situations corresponding to unnotched and notched specimens, respectively.

COMPLIANCE CHANGES AND FRACTURE UNDER UNIFORM STRESSES

Uniform tensile stresses produce a regular array of cracks called a characteristic damage state, as described earlier. Figure 5 shows the relationship of stiffness

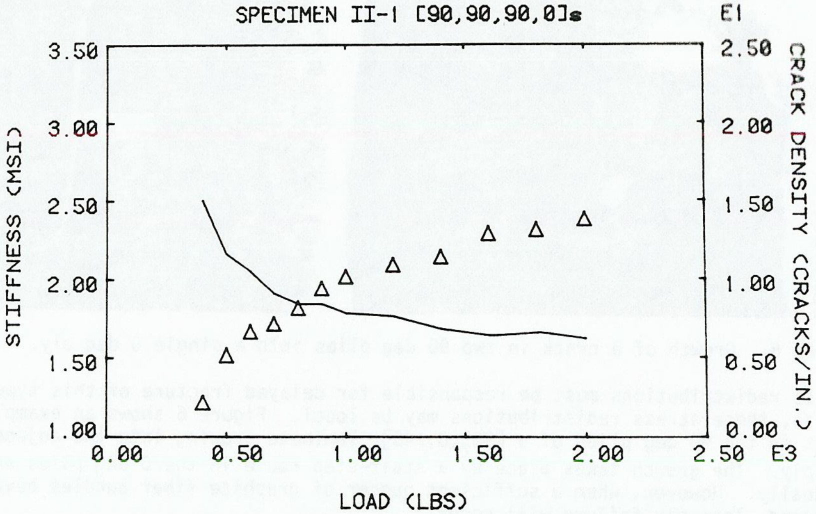

Fig. 5. Comparison of the number of transverse matrix cracks and the stiffness change as a function of loading.

change to the formation of such cracks for a $[90_3,0]_s$ glass epoxy laminate. The correlation between the number of cracks that have formed and the change in stiffness is very close. This type of damage which develops throughout a specimen in uniform stress fields is most easily detected and quantitatively characterized by global stiffness change measurements. The relationship of this type of crack formation to final strength is complex, although the principle of stress redistribution is thought to be controlling. It can be established that damage in off-axis plies does have an influence on fracture of the 0 deg plies from two simple observations. First, in the 0 deg direction creep strain during constant load tests is essentially zero. Second, in laminates which have off-axis plies, delayed fracture is commonly observed, i.e., if a load which is close to the ultimate quasi-static loading strength is maintained on the laminate, it will break after some period of time (Henneke, 1979). It is known that cracks form at such constant loads and that cracks in different off-axis plies do couple together in such a situation in some cases (Henneke, 1979). Since there is no evidence that the 0 deg plies fail alone under constant stress, evidently the cracking of the off-axis plies and the attendent

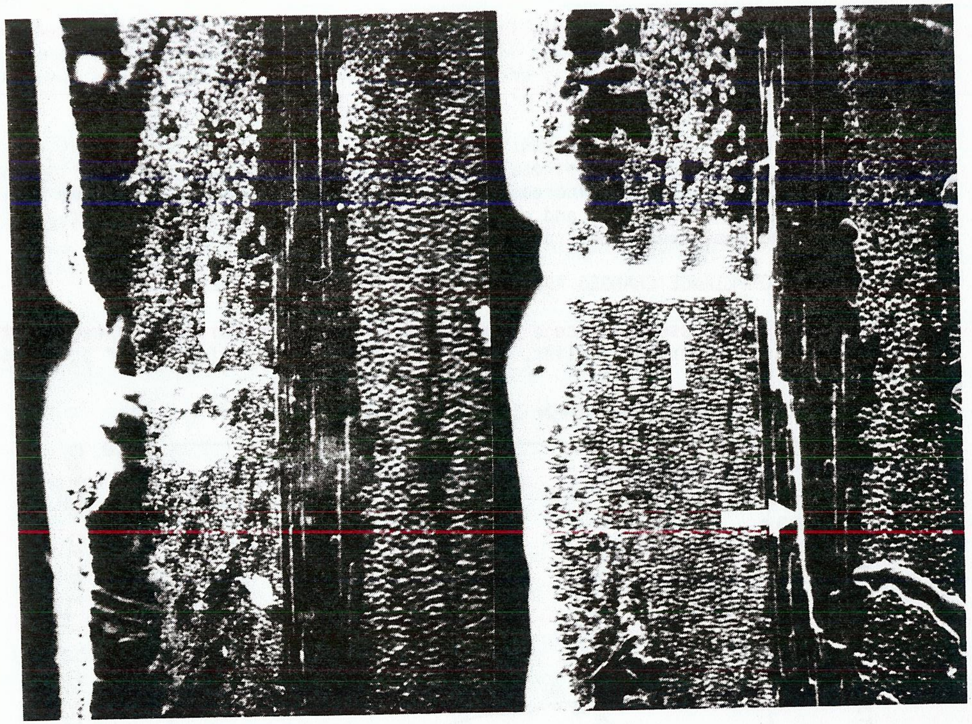

Fig. 6. Growth of a crack in two 90 deg plies into a single 0 deg ply.

stress redistributions must be responsible for delayed fracture of this type. Of course, these stress redistributions may be local. Figure 6 shows an example of a crack in the 90 deg plies of a $[90_2,0,\pm45]_s$ laminate growing into the adjacent 0 deg ply. The growth takes place by a stair-step route in the 0 deg plies and occurs gradually. However, when a sufficient number of graphite fiber bundles have been ruptured, last ply failure will occur.

COMPLIANCE CHANGES AND FRACTURE UNDER NONUNIFORM STRESSES

For notched specimens compliance changes are generally large in the region of the notch and such changes are quite sensitive to damage development. Figure 7 shows the relationship of such changes to final strength for boron epoxy and boron aluminum specimens. The compliance was measured with an extensometer which was placed across the center hole in the specimens during cyclic loading with an amplitude which was about two thirds of the ultimate strength in tension (R=0.1). Stiffness changes over a 25 mm gage length which included the 6.25 mm diameter hole were commonly as high as 60 percent. It should be noted that residual strength for notched composites is not a monotonic function of load or cycles of load in many cases. Damage in the neighborhood of a notch frequently has the effect of relaxing the geometric constraint induced by the notch causing the residual strength to increase to essentially the ligament or net section strength. Further damage will reduce the residual strength. Stiffness change is monotonic during that process making it a convenient monitoring scheme for notched damage development even when the strength is increasing. If damage in the neighborhood of the notch includes delamination, the transverse Poissons' strain will change also. Changes in Poissons' ratio are excellent indicators of delamination.

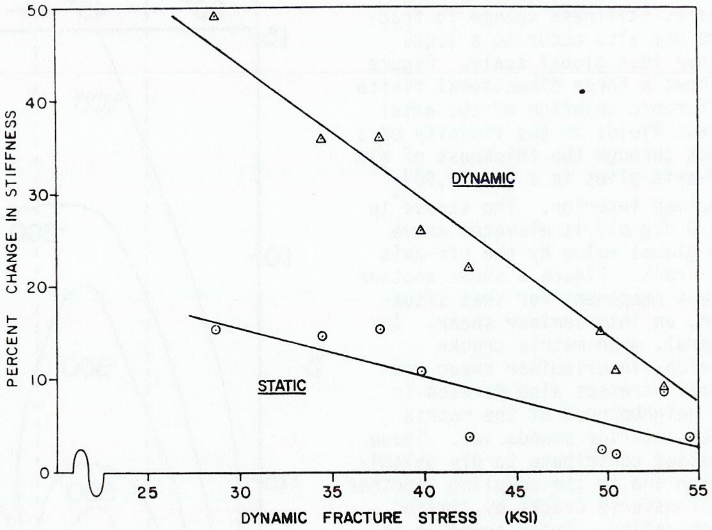

Fig. 7. Compliance change after loading as a function of fracture strength for notched $[0,\pm45,0]_s$ boron epoxy laminates.

SUMMARY

It has been shown that compliance changes in high modulus composite laminates are directly and quantitatively related to the fracture of those laminates. Specific points of general importance include the following observations.

1. Compliance changes are caused by damage events which bring about both global and local redistributions of stress.
2. The redistributions of stress determine the residual strength of the laminate.
3. The mechanics of the internal stress redistributions is the quantitative link between compliance changes and fracture strength.
4. Some of the mechanics of stress redistribution has been established, especially for uniform (applied) stress fields.
5. The precise relationships between the redistributed stresses and final fracture strength have not been determined.

In general, the use of compliance changes, especially tensor compliance changes, appears to offer a sound and tractable approach to analytically describing and predicting the strength of composite laminates following damage development. While this type of philosophy has only just recently been suggested by the authors and is quite incompletely developed at this point, the early indications suggest that more development of such a concept is warranted.

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TABLE 1 Laminate Analysis Results for $[0,90,\pm45]_s$ Laminate*

Laminate Condition	Engineering Moduli					Ply Stresses (MPa)			
	E_x	E_y	G_{xy}	ν_{xy}		-45°	+45°	90°	0°
Initial Properties	55.5 GPa	55.5 GPa	21.3 GPa	0.302 GPa	σ_x	3.9	3.9	1.0	15.7
					σ_y	2.2	2.2	-4.4	3.4
					τ_{xy}	-2.6	2.6	0	0
$\overline{Q}_{11}$ and $\overline{Q}_{12}$ reduced to zero in 90° plies	53.4 GPa	55.8 GPa	21.3 GPa	0.289 GPa	σ_x	4.1	4.1	0	16.3
					σ_y	2.4	2.4	-4.7	0.01
					τ_{xy}	-2.7	2.7	0	0
Delamination of 90 deg plies from rest of laminate	50.78 GPa				σ_x	4.0	4.0	0.93	18.6
					σ_y	0.136	0.136	0	-0.27
					τ_{xy}	-1.82	1.82	0	

*applied load of 6.89 mPa

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