ON THE MAXIMUM-ENERGY-RELEASE-RATE CRITERION FOR FRACTURE UNDER COMBINED LOADS

K.C.Hwang,* D.H.Hua* and S.W.Yu*

*Dept. of Engineering Mechanics, Tsinghua Univ., Peking, China

ABSTRACT

In this paper the asymptotic behavior of a deflected crack problem with vanishing branch crack length is investigated. The complex potential functions are expressed in eigen-function series near the tip of the branch crack. The problem is reduced to an infinite set of algebraic equations for the coefficients of the eigen-function series. The numerical solution of these equations truncated gives the fracture criterion and branch angle for combined loads of mode I and mode II . The numerical results are compared with those of the previous investigators.

KEYWORDS

Fracture; fracture criterion; mixed mode fracture; fracture under combined loads; maximum energy release rate.

INTRODUCTION

The fracture of a straight crack under combined loads was first studied by Erdogan and Sih(1963), who suggested the maximum-stress criterion. Sih(1972) proposed also the strain-energy-density criterion. The approach consistent with Griffith's energy-release-rate concept was first mentioned by Erdogan and Sih(1963), and later investigated by many authors. The first attempt seems to have been given by Hussain,Pu and Underwood (1974). But as pointed out by Palaniswamy and Knauss (1976) and Wu (1978 a,b), their results of limit process as branch crack length tends to zero are incorrect. For this asymptotic case, Bilby and Cardew (1975) have given accurate calculations based on the solution of Khrapkov (1971). Lo (1978) has confirmed their results by another method and stated that they agree with those given in the report of Palaniswamy and Knauss (1974). Bilby and Cardew also noted that values read from Chatterjee (1975) were consistent with their own. Palaniswamy and Knauss (1976) avoided the difficulties arising from the branching points by replacing the crack with a smooth contour, so their results, though acceptable, are approximate. Wang (1977) used the eigen-function expansion (Williams, 1957) for the complex potential $\varphi_1(z)$ at the tip of the branch crack. Since the author retained only one term (n=1), the results obtained are also approximate. Wu (1978 a, 1978 b, 1978 c) solved successfully the elasticity problem of a slender Z-crack by matching technique of singular perturbation. Since there is no explicit expression of the Schwarz-Christoffel transformation for mapping of the z-plane with the Z-shaped crack onto the upper half of the complex ζ -plane, the calculation is rather lengthy. In this paper a simpler L-shaped crack is considered. The Williams' eigen-function expansions of the complex potential functions are used, and it is shown that not only the first term but the whole series make contributions to the energy-release-rate as the branch crack length tends to zero. The problem is reduced to an infinite set of algebraic equations for the coefficients of the eigen-function series. The numerical solution of these equations truncated gives the fracture criterion and branch angle for combined loads of mode I and mode II . The numerical results are compared with those of the previous investigators. The method of this paper has been employed earlier by the authors to the anti-shear crack problem (mode III) and results identical to those of Palaniswamy and Knauss (1976) and Wu (1978 b) are obtained.

FORMULATION OF THE PROBLEM

In Fig. 1 is shown a deflected crack with the main crack length r_1, branch length r_2 and deflection angle γ. The mapping function due to Darwin (1950), which transforms the physical z-plane onto the region $|\zeta| \geq 1$ is

$$Z = x + iy = \omega(\zeta) = A\zeta^{-1}(\zeta-\sigma_1)^{\lambda_1}(\zeta-\sigma_2)^{\lambda_2} \tag{1}$$

z plane $\qquad\qquad$ ζ plane

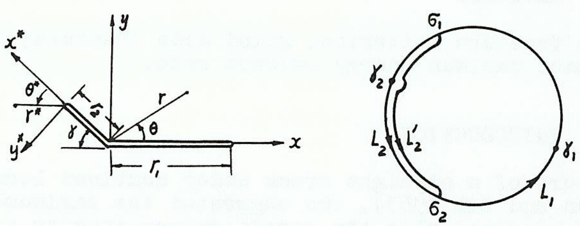

Fig. 1. Deflected crack and its image

Here $\sigma_1 = e^{i\alpha_1}$, $\sigma_2 = e^{i\alpha_2}$ are the images of the deflection points (z=0), and $\lambda_1 = 1-\gamma/\pi$, $\lambda_2 = 1+\gamma/\pi$. From eq.(1) one can obtain

$$\omega'(\zeta) = \frac{d\omega(\zeta)}{d\zeta} = \frac{1}{\zeta g(\zeta)}\omega(\zeta) \tag{2}$$

where, denoting by $\gamma_1 = e^{i\beta_1}$, $\gamma_2 = e^{i\beta_2}$ the images of the tips $z_1 = r_1$, $z_2 = r_2 e^{i(\pi-\gamma)}$ of the main and the branch cracks,

$$g(\zeta) = (\zeta-\sigma_1)(\zeta-\sigma_2)/(\zeta-\gamma_1)(\zeta-\gamma_2) \tag{3}$$

Hussain, Pu and Underwood (1974) gave in Appendix the asymptotic expansions of the various constants in terms of the small parameter $\varepsilon = \alpha_2 - \alpha_1$, as the branch length tends to zero. Denote the complex potential functions (Muskhelishvili 1963) by

$$\begin{aligned} \varphi_1(z) &= \varphi_1[\omega(\zeta)] = \varphi(\zeta) \\ \psi_1(z) &= \psi_1[\omega(\zeta)] = \psi(\zeta) \end{aligned} \tag{4}$$

The traction-free condition on the crack surfaces is

$$\varphi^-(\sigma) + \frac{\omega^-(\sigma)}{\overline{\omega'(\sigma)}}\overline{\varphi'^-(\sigma)} + \overline{\psi^-(\sigma)} = const \tag{5}$$

Here and hereafter we denote by the superscript "-" the boundary value for the region D^- exterior to the unit circle $|\zeta| = 1$. By use of eq. (2), the coeficient $\omega^-(\sigma)/\overline{\omega'(\sigma)}$ in eq.(5) can be evaluated:

$$\omega^-(\sigma)/\overline{\omega'(\sigma)} = -g(\sigma)/\sigma \qquad \sigma \in L_1 \tag{6}$$

$$-e^{-2i\gamma}g(\sigma)/\sigma \qquad \sigma \in L_2 \tag{7}$$

Here L_1 denotes the image of the main crack ($0<\theta<\alpha_1$, $\alpha_2<\theta<2\pi$), while L_2 the image of the branch ($\alpha_1<\theta<\alpha_2$) on the unit circle $L=L_1+L_2$. As $|\zeta|$ tends to infinity, the functions $\varphi(\zeta)$ and $\psi(\zeta)$ have Laurent expansions of the form

$$\varphi(\zeta) = \Gamma A\zeta + \sum_{n=0}^{\infty} A_n\zeta^{-n} \tag{8}$$

$$\psi(\zeta) = \Gamma' A\zeta + \sum_{n=0}^{\infty} B_n\zeta^{-n} \tag{9}$$

where Γ and Γ' can be expressed in terms of constant traction σ_x^∞, σ_y^∞, τ_{xy}^∞ applied at infinity

$$\Gamma = \frac{1}{4}(\sigma_x^\infty + \sigma_y^\infty) \tag{10}$$

$$\Gamma' = -\frac{1}{2}(\sigma_x^\infty - \sigma_y^\infty) + i\tau_{xy}^\infty \tag{11}$$

Multiplying eq. (6) and (7) by $(\sigma-\gamma_1)$ $(\sigma-\gamma_2)$ and summing up their Cauchy integrals over L_1 and L_2, we can obtain the integral equation for $\varphi_1(z)$

$$\begin{aligned} \varphi^*(\zeta) = {} & (\zeta-\gamma_1)(\zeta-\gamma_2)(\Gamma A\zeta + A_0) + A_1(\zeta-\gamma_1-\gamma_2) + A_2 + \\ & + A(\sigma_1\sigma_2\Gamma - \gamma_1\gamma_2\bar{\Gamma}')/\zeta + f(\zeta) \qquad \zeta \in D^- \end{aligned} \tag{12}$$

where

$$\varphi^*(\zeta) = \varphi(\zeta)(\zeta-\gamma_1)(\zeta-\gamma_2) = \varphi_1(z)(\zeta-\gamma_1)(\zeta-\gamma_2) \tag{13}$$

$$f(\zeta) = (1-e^{2i\gamma})(2\pi i)^{-1}\cdot\int_{L_2}\omega(\sigma)(\sigma-\gamma_1)(\sigma-\gamma_2)\overline{\varphi_1'[\omega(\sigma)]}\,d\sigma/(\sigma-\zeta) \tag{14}$$

Now we shall expand eq. (12) into a Power series of $(z-z_2)^{1/2}$ at the tip of the branch crack. From eq.(1) we obtain $z-z_2$ as a power series of $\zeta-\gamma_2$

$$(Z-Z_2)/A = \sum_{n=2}^{\infty}\frac{1}{n!}\omega^{(n)}(\gamma_2)(\zeta-\gamma_2)^n/A \tag{15}$$

where

$$\omega^{(n)}(\gamma_2)/A = \sum_{m+p+q=n}\frac{n!}{m!\,p!\,q!}\left[\frac{d^m}{d\zeta^m}\zeta^{-1}\cdot\frac{d^p}{d\zeta^p}(\zeta-\sigma_1)^{\lambda_1}\frac{d^q}{d\zeta^q}(\zeta-\sigma_2)^{\lambda_2}\right]_{\zeta=\gamma_2} \tag{16}$$

From eq.(15) we can express $\zeta-\gamma_2$ and its powers as power series of $[(z-z_2)/A]^{1/2}$

$$(\zeta-\gamma_2)^j = \sum_{l=j}^{\infty}\tau_{jl}[(Z-Z_2)/A]^{l/2} \tag{17}$$

The calculation of the coefficients τ_{jl} ($j \leq l$), though somewhat lengthy, is simple and direct. Some of their expressions are given here

$$\begin{aligned} &\tau_{11} = \left[\frac{1}{2}\frac{\omega''(\gamma_2)}{A}\right]^{-1/2}, \qquad \tau_{12} = -\frac{1}{6}\frac{\omega'''(\gamma_2)}{\omega''(\gamma_2)}(\tau_{11})^2 . \\ &\tau_{13} = -\frac{1}{2}\frac{(\tau_{12})^2}{\tau_{11}} - \frac{1}{2}\frac{\omega'''(\gamma_2)}{\omega''(\gamma_2)}\tau_{11}\cdot\tau_{12} - \frac{1}{24}\frac{\omega^{(4)}(\gamma_2)}{\omega''(\gamma_2)}(\tau_{11})^3 . \\ &\tau_{jj} = (\tau_{11})^j, \qquad \tau_{j(j+1)} = j(\tau_{11})^{j-1}\tau_{12}, \quad \ldots\ldots \end{aligned} \tag{18}$$

The expansion of the expression (14) for $f(\zeta)$ near $\zeta=\gamma_2$ leads to

$$f(\zeta) = \sum_{n=0}^{\infty}\frac{1}{n!}f^{(n)}(\gamma_2)(\zeta-\gamma_2)^n \tag{19}$$

where

$$f^{(n)}(\gamma_2) = \frac{n!}{2\pi i}(1-e^{2i\gamma})\int_{L_2'}\frac{\omega(\sigma)(\sigma-\gamma_1)}{(\sigma-\gamma_2)^n}\overline{\varphi_1'[\omega(\sigma)]}\,d\sigma \tag{20}$$

and L_2' is a path from σ_1 to σ_2 inside the region D^+ interior to the unit circle $|\zeta| = 1$ (Fig. 1). The function $\varphi_1(z)$ can be expanded at the branch tip (Williams, 1957)

$$\varphi_1(z) = A^{1/2} \sum_{m=0}^{\infty} a_m \left(\frac{z-z_2}{A} \right)^{m/2} \tag{21}$$

Expanding eq. (12) into power series of $(z-z_2)^{1/2}$ with the aid of eq.(17), (19)-(21), and equating the coefficients of like terms, we obtain an infinite set of algebraic equations for a_k (k≥0), which after elimination of a_0 takes the form

$$\begin{aligned} A^{1/2} \sum_{k=1}^{n} a_k \left[(\gamma_2-\gamma_1)\tau_{1(n-k+1)} + \tau_{2(n-k+1)} \right] \\ = \Big[(\gamma_2-\gamma_1)\Gamma A - \frac{A_1}{\gamma_2-\gamma_1} \Big] \tau_{2(n+1)} + \Gamma A \tau_{3(n+1)} + A(\sigma_1\sigma_2\Gamma - \gamma_1\gamma_2\bar{\Gamma}') \Big[\frac{2\gamma_2-\gamma_1}{\gamma_2^3(\gamma_2-\gamma_1)} \tau_{2(n+1)} + \\ + \sum_{j=3}^{n+1} \frac{(-1)^j}{\gamma_2^{j+1}} \tau_{j(n+1)} \Big] + \Big[\frac{1}{2!} f_0''(\gamma_2) - \frac{f_0'(\gamma_2)}{\gamma_2-\gamma_1} \Big] \tau_{2(n+1)} + \sum_{j=3}^{n+1} \frac{1}{j!} f^{(j)}(\gamma_2)\tau_{j(n+1)}, \quad n \geq 1 \end{aligned} \tag{22}$$

Here the summation terms $\sum_{j=3}^{n+1}$ drop out when n=1, and τ_{jl} is defined to be zero when j>l.

ASYMPTOTIC SOLUTION

Since we are aiming in this paper at the branch crack impending problem, the asymptotic behavior with vanishing branch crack length is investigated and all quantities are expanded as asymptotic series of $\varepsilon = \alpha_2 - \alpha_1$. By use of the expressions given in Appendix of Hussain, Pu and Underwood (1974), straight calculation leads to the asymptotic expansions for the coefficients of power series (16) and (17)

$$\begin{aligned} \omega^{(n)}(\gamma_2)/A &\sim \varepsilon^{2-n} \left[\omega_{(0)}^{(n)} + \varepsilon\, \omega_{(1)}^{(n)} + \cdots\cdots \right] \\ \tau_{jl} &\sim \varepsilon^{j-l} \left[\tau_{jl_{(0)}} + \varepsilon\, \tau_{jl(1)} + \cdots\cdots \right] \end{aligned} \tag{23}$$

Only the first term of each asymptotic series will be needed hereafter, and owing to the limited space, only some of them are given below.

$$\omega_{(0)}^{(n)} = - \sum_{p+q=n} \frac{n!}{p!\,q!} J_p H_q \qquad n \geq 0 \tag{24}$$

where

$$J_p = \left[e^{\frac{3}{2}\pi i} \frac{\lambda_1}{2} \right]^{\lambda_1 - p} \prod_{k=0}^{p-1} (\lambda_1 - k), \qquad H_q = \left[e^{\pi i/2} \frac{\lambda_2}{2} \right]^{\lambda_2 - q} \prod_{k=0}^{q-1} (\lambda_2 - k)$$

and the products $\prod_{k=0}^{p-1}$ or $\prod_{k=0}^{q-1}$ are defined eaual to one when p or q vanishes,

$$\begin{aligned} &\tau_{11(0)} = \left(\omega_{(0)}^{(2)}/2 \right)^{-1/2}. \qquad \tau_{12(0)} = -\frac{1}{6} \frac{\omega_{(0)}^{(3)}}{\omega_{(0)}^{(2)}} (\tau_{11(0)})^2. \\ &\tau_{13(0)} = -\frac{1}{2} \frac{(\tau_{12(0)})^2}{\tau_{11(0)}} - \frac{1}{2} \frac{\omega_{(0)}^{(3)}}{\omega_{(0)}^{(2)}} \tau_{11(0)}\tau_{12(0)} - \frac{1}{24} \frac{\omega_{(0)}^{(4)}}{\omega_{(0)}^{(2)}} (\tau_{11(0)})^3. \\ &\tau_{jj(0)} = (\tau_{11(0)})^j. \qquad \tau_{j(j+1)(0)} = j\,(\tau_{11(0)})^{j-1} \tau_{12(0)} \quad \cdots\cdots \end{aligned} \tag{25}$$

As can be seen from eq. (23), the coefficients of power series near branch crack tip $z=z_2$ or $\zeta = \gamma_2$ have different indices in ε, such as 2 -n and j-1 in (23). So the coefficients a_m of Williams' expansion (21) is expected to be asymptotically of the form

$$a_m \sim \varepsilon^{1-m} \left(a_{m(0)} + \varepsilon\, a_{m(1)} + \cdots\cdots \right) \tag{26}$$

In order to obtain the asymptotic espansion of $f^{(n)}(\gamma_2)$ in eq. (20), we map the region D^- exterior to the unit circle in ζ plane onto the lower half of z plane, with the pointe $\zeta = \sigma_1, \gamma_2, \sigma_2$ mapped into $z = -1, -\gamma/\pi, +1$, (Fig. 2). The mapping function will be

$$\frac{z+1}{z-1} \cdot \frac{(-\gamma/\pi) - 1}{(-\gamma/\pi) + 1} = \frac{\zeta - \sigma_1}{\zeta - \sigma_2} \cdot \frac{\gamma_2 - \sigma_2}{\gamma_2 - \sigma_1} \tag{27}$$

By use of the expressions in Appendix of Hussain, Pu and Underwood (1974) and eq. (1), (21), (23), (27) we obtain

$$\frac{1}{n!} f^{(n)}(\gamma_2) \sim \varepsilon^{2-n} A^{1/2} \left(F_{(0)}^{(n)} + \varepsilon F_{(1)}^{(n)} + \cdots\cdots \right) \tag{28}$$

where

$$F_{(0)}^{(n)} = \sum_{m=1}^{\infty} C_{nm}\, \bar{a}_{m(0)} \qquad n \geq 0$$

$$C_{nm} = \frac{m}{\pi} 2^{n-4} e^{\frac{1}{2}(m\gamma + n\pi)i} (1 - e^{-2i\gamma}) \int_{\mathcal{L}_2'} \frac{(1+t)^{\lambda_1}(1-t)^{\lambda_2}}{(t + \frac{\gamma}{\pi})^n} \overline{[\Lambda(\bar{z})]^{\frac{1}{2}(m-2)}}\, dt \qquad n \geq 0,\ m \geq 1 \tag{29}$$

with

$$\Lambda(z) = \frac{1}{4} \lambda_1^{\lambda_1} \lambda_2^{\lambda_2} - \frac{1}{4} (1+z)^{\lambda_1} (1-z)^{\lambda_2} \tag{30}$$

and $\mathcal{L}_2'$ being a path from $z = -1$ to $z = +1$ in $\mathcal{D}^+$, the upper half z -plane, Fig. 2.

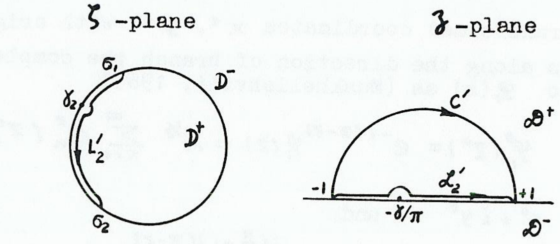

Fig. 2 Mapping of ζ -plane onto z -plane

As $\varepsilon \to 0$, the function $\varphi(\zeta)$ in eq. (8) should approach the well-known result for a line crack without branch, so we have $A_1 \to -A(\Gamma + \bar{\Gamma}')$, $A_n \to 0$ (n ≥2). With all the asymptotic information just obtained we can write down the asymptotic limit (as $\varepsilon \to 0$) of the infinite set of eq. (22)

$$\sum_{k=1}^{n} a_{k(0)} \tau_{1(n+1-k)(0)} = \frac{\mathring{k}}{\sqrt{2}} (\sin\alpha - i\cos\alpha) \tau_{2(n+1)(0)} - \frac{1}{2} \sum_{m=1}^{\infty} \sum_{j=2}^{n+1} C_{jm}\, \bar{a}_{m(0)} \tau_{j(n+1)(0)} \qquad n \geq 1 \tag{31}$$

where $\mathring{k}\sin\alpha$ and $\mathring{k}\cos\alpha$ are the stress intensity factors before the branch crack is propagated

$$\mathring{k}_1 = \mathring{k}\sin\alpha = \sqrt{2A}\, \sigma_y^{\infty}, \qquad \mathring{k}_2 = \mathring{k}\cos\alpha = \sqrt{2A}\, \tau_{xy}^{\infty} \tag{32}$$

The solutions of eq. (31) gives $a_{m(0)}$, m ≥ 1.

ENERGY-RELEASE RATE

As is well known, the stresses at the branch crack before its propagation are

$$\overset{\circ}{\sigma}_\theta = \frac{1}{2\sqrt{2r}}\overset{\circ}{f}_1 \qquad \overset{\circ}{\tau}_{r\theta} = \frac{1}{2\sqrt{2r}}\overset{\circ}{f}_2 \tag{33}$$

where

$$\begin{aligned}\overset{\circ}{f}_1 &= \overset{\circ}{k}_1(1+\cos\gamma)\cos\frac{\gamma}{2} + 3\overset{\circ}{k}_2\sin\gamma\cos\frac{\gamma}{2}\\ \overset{\circ}{f}_2 &= -\overset{\circ}{k}_1\sin\gamma\cos\frac{\gamma}{2} + \overset{\circ}{k}_2(3\cos\gamma-1)\cos\frac{\gamma}{2}\end{aligned} \tag{34}$$

The energy released by the branch propagation will be

$$G(r_2) = \frac{1}{2}\int_0^{r_2}(\overset{\circ}{\sigma}_\theta\,\Delta u^*_\theta + \overset{\circ}{\tau}_{r\theta}\,\Delta u^*_r)\,dr \tag{35}$$

where r* and θ * are the polar coordinates with origin at branch tip (Fig. 1) and

$$\begin{aligned}\Delta u_r &= u^*_r\big|_{\theta^*=-\pi} - u^*_r\big|_{\theta^*=\pi}\\ \Delta u_\theta &= u^*_\theta\big|_{\theta^*=-\pi} - u^*_\theta\big|_{\theta^*=\pi}\end{aligned} \tag{36}$$

For the transformed coordinates x^*, y^* with origin at branch tip and x^*-axis along the direction of branch the complex potential $\varphi^*_1(z^*)$ is related to $\varphi_1(z)$ as (Muskhelishvili, 1963)

$$\varphi^*_1(z^*) = e^{-i(\pi-\gamma)}\varphi_1(z) = A^{1/2}\sum_{n=0}^{\infty} a^*_n\,(z^*/A)^{n/2} \tag{37}$$

where $z^* = x^* + iy^*$ and

$$a^*_n = a_n\,e^{i(\frac{n}{2}-1)(\pi-\gamma)} \tag{38}$$

The complex potential $\psi^*_1(z^*)$ can be obtained from the traction-free condition on the branch crack surfaces (Sih and Liebowitz, 1968)

$$\psi^*_1(z^*) = \chi^{*\prime}_1(z^*) = \sum_{n=1}^{\infty}\left(\frac{n}{2}+1\right) b^*_n\,(z^*/A)^{n/2} \tag{39}$$

where b^*_n is related to a^*_n as

$$\left(\frac{n}{2}+1\right) b^*_n = -\frac{n}{2}a^*_n - (-1)^n\,\bar{a}^*_n \tag{40}$$

The diplacements u^*_r, u^*_θ can be determined from

$$2\mu(u^*_r + i u^*_\theta) = e^{-i\theta^*}\left[\varkappa\varphi^*_1(z^*) - z^*\overline{\varphi^{*\prime}_1(z^*)} - \overline{\psi^*_1(z^*)}\right] \tag{41}$$

where μ – shear modulus, $\varkappa$ =3–4 ν for plane strain and (3– ν)/(1+ ν) for plane stress. By use of eq. (33) and (36)–(41), we find from eq. (35) G (r_2). Using the asymptotic expression of branch length r_2 in Appendix of Hussain, Pu and Underwood (1974) we obtain the energy-release rate for the impending branch crack

$$\begin{aligned}G'(0) &= \lim_{r_2\to 0}\frac{dG(r_2)}{dr_2} = \lim_{\varepsilon\to 0}\left(\frac{dG(r_2)}{d\varepsilon}\Big/\frac{dr_2}{d\varepsilon}\right)\\ &= \frac{\pi(\varkappa+1)}{16\mu}\sum_{n=1}^{\infty}\left(\frac{1}{4}\lambda_1^{\lambda_1}\lambda_2^{\lambda_2}\right)^{\frac{n}{2}-1}\xi_n(\overset{\circ}{f}_1 K_{In} + \overset{\circ}{f}_2 K_{IIn})\end{aligned} \tag{42}$$

where

$$K_{In} = K_{IIn} = 0 \qquad \text{when n is even}$$

$$K_{In} + iK_{IIn} = \sqrt{2}(-1)^{\frac{n}{2}-1}\,\bar{a}_{n(0)}\,e^{-i(\frac{n}{2}-1)(\pi-\gamma)} \qquad \text{when n is odd} \tag{43}$$

$$\xi_n = \frac{4}{\pi}\int_0^{\pi/2}\cos^{n+1}t\,dt = \frac{n(n-2)\cdots 4\cdot 2}{(n+1)(n-1)\cdots 5\cdot 3}\cdot\frac{4}{\pi} \qquad \text{when n is even}$$

The results obtained by Wang (1977) correspond to the first term (n=1) of the series in eq. (42). K_{I1} and K_{II1} (for n=1) are the stress-intensity factors at the tip of the vanishing branch. Crack will propagate along the direction defined by γ corresponding to maximum energy-release rate G' (0) and satisfying the condition $\overset{\circ}{\sigma}_\theta \geq 0$, and the propagation is imminent when this maximum energy-release rate reaches a material constant.

NUMERICAL RESULTS

The eqs. (31) are solved numerically for the coefficients $a_{m(0)}$ by truncating the series for m>N (n=1,2,...N). The coefficients C_{nm} is evaluated numerically from eq. (29), taking as the path of integration the half circle C' in the upper half ζ-plane (Fig. 2). Fig. 3 shows our numerical results for critical combinations of $\overset{\circ}{k}_1$ and $\overset{\circ}{k}_2$ for plane strain and Fig. 4 shows the direction angle γ of branch crack propagation. Our results for N=7 differ not appreciably from those of Wu (1978) While the critical curve of Wu (1978 c)(Fig. 3) ends at a point corresponding to α =–48.6°, the ending point in this paper corresponds to α =–54.2°. Results for N=4,5,6, differ only very slightly from those for N=7. For example, for mode II crack the critical values $\overset{\circ}{k}_{2.cr}$ for N=5,6 differ from that for N=7 only by 1.1% and 0.12% respectively.

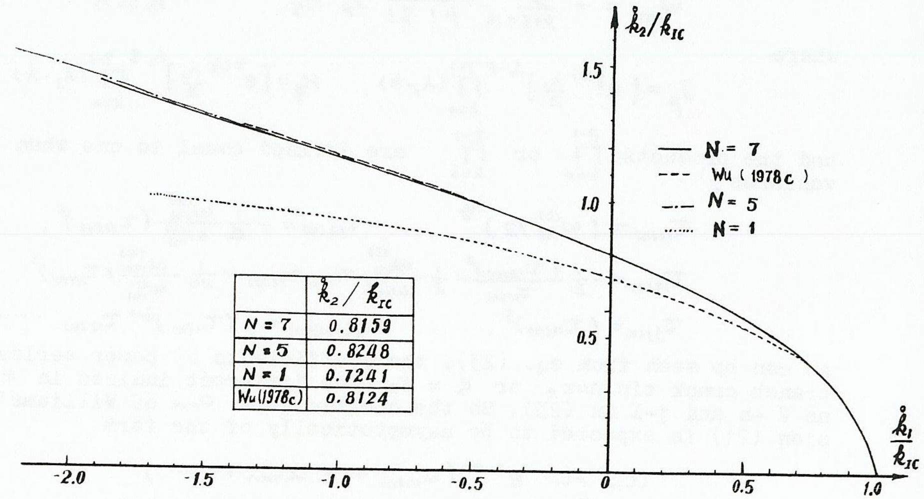

Fig. 3 Critical curve of $\overset{\circ}{k}_1$ and $\overset{\circ}{k}_2$

For $\overset{\circ}{k}_1$, $\overset{\circ}{k}_2 > o$, figures 3 and 4 agree closely with the figure 3 (for z=0) and the figure 2 (for λ =0), respectively, given by Bilby and Cardew(1975).

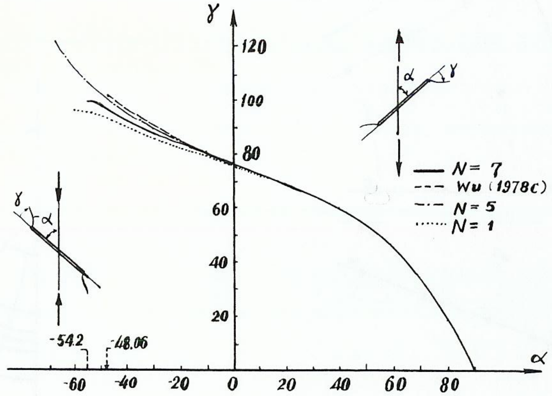

Fig. 4 Directional angle of crack propagation

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