MODELLING R RATIO EFFECTS IN FATIGUE CRACK GROWTH IN POLYMERS

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ABSTRACT

A two-stage line zone model is used to describe fatigue in polymers. An extension to a previous version is described, in which only part of the zone is completely unloaded for varying mean stress conditions (different R ratios). The possibility of the unloaded zone being sustained at the crack or zone tips is explored which leads to growth rates which are dependent on maximum K or ΔK. A residual stress distribution due to unloading is also included. Examples are given in which this model is applied to several polymers.

KEYWORDS

Fatigue; polymers; mean stress; R ratio; residual stress.

INTRODUCTION

Fatigue crack growth occurs in most polymers at stress intensity values well below those for growth under constant loading conditions. The data is rather similar in form to those for metals and is thus conveniently described in terms of the Paris law (Manson & Hertzberg, 1973), but attempts to correlate the parameters from this empirical representation have not been particularly successful. An attempt to describe fatigue in terms of parameters which can be related to other material properties has been made (Williams, 1977; Mai & Williams, 1979), in which the crack tip region was modelled as a line zone whose load carrying capacity, σ_c, was reduced to $\alpha\sigma_c$ on unloading and reloading. This leads to a two-stage line zone, as shown in Figure 1, in which reloading on each cycle leads to the growth of the zone, and if a critical displacement at the crack tip is used as a fracture criterion, stable crack growth is predicted, after a period of incubation, of the form:

$$\frac{da}{dN} = \frac{\pi}{8}\frac{1}{(1-\alpha)^2\,\sigma_c^{\,2}}\left(K^2 - \alpha\, K_c^{\,2}\right) \tag{1}$$

where K_c is the single loading fracture stress intensity factor. For $K^2 \gg \alpha K_c^{\,2}$,

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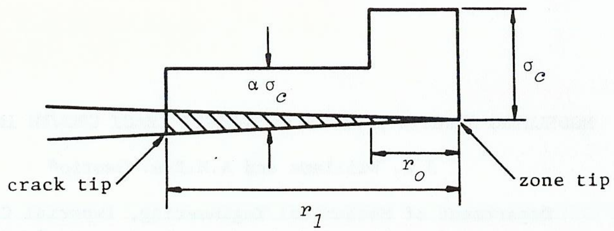


Fig. 1. Two-stage line zone model

$da/dN \propto K^2$ which gives a power of two in the Paris law, while for lower K^2 values this becomes apparently larger on log-log plots. A fatigue limit is predicted for $K^2 = \alpha K_c^2$ and graphs of da/dN versus K^2 should be linear. This has been found to be so for a wide range of polymers (Williams, 1977; Mai & Williams, 1979) but with several regions of behaviour at different K levels. The representation has also been found to be useful for including environmental effects where the environment augments the fatigue effect in α (Mai & Williams, 1979).

The original version of the analysis did, however, have shortcomings which have come to light in further applications. In particular, the effect of varying R (K_{min}/K_{max}) ratio was taken to be that the whole tip zone partially unloaded and that α was a function of R^2 such that $\alpha = 1$ when $R = 1$. Although quite a good representation of the data, this approach gave rise to a number of difficulties, the major one being that σ_c, determined from the slopes of the various da/dN versus K^2 lines, increased rapidly with R. Although modest changes could be explained (by constraint effects, for example), those observed in some materials were much too large (for example, Mai & Williams, 1979) and it has been necessary to re-examine this aspect of the model.

R EFFECTS IN THE LINE ZONE MODEL

It will now be assumed that partial unloading results in the complete unloading of part of the zone. It will further be assumed that α and σ_c are the same for this fully unloaded section and that fracture occurs at the same K_c condition as in the completely unloaded case. The limiting case of $K = K_c$ should give $da/dN = r_c = (\pi/8)(K_c^2/\sigma_c^2)$, but this is not so in equation (1). This arises because of the approximation used in the derivation of equation (1) which is valid only for $r_o/r_1 < \frac{1}{4}$ (see Figure 1) and for $r_o/r_1 > \frac{1}{4}$, which includes the limiting case of $K \simeq K_c$, the appropriate approximation is:

$$\frac{da}{dN} = \frac{\pi}{8\,(1-\alpha)^2\,\sigma_c^2}\,(K - \alpha\,K_c)^2 \qquad (2)$$

for which $da/dN = r_c$ at $K = K_c$ as required. The transition to this form of behaviour occurs only at a rather extreme condition and does not affect the subsequent discussion.

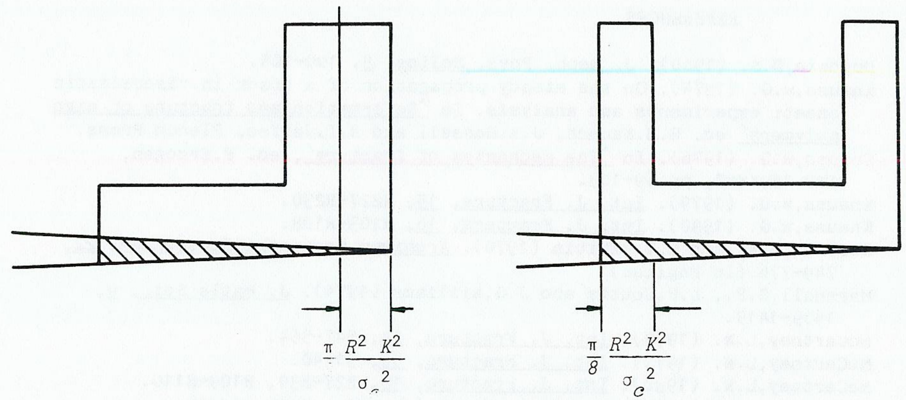
The several assumptions can only be accommodated in equation (1) by assuming that

K^2 now takes the form of an effective value dependent on R and it has been found that there is evidence of a threshold value of K, K_o, below which there is no growth. This probably arises from the residual compressive stresses at the crack tip which inevitably occur on unloading. The final form of the equation is:

$$\frac{da}{dN} = \frac{\pi}{8}\,\frac{1}{(1-\alpha)^2\,\sigma_c^2}\,[F(R)\,(K^2 - K_o^2) - \alpha\,K_c^2] \qquad (3)$$

where $F(0) = 1$.

Some possible types of behaviour which determine $F(R)$ are shown in Figure 2. In



(i) First type - zone tip sustained (ii) Second type - crack tip sustained

Fig. 2. Partially unloaded zone types

the first type (i), the non-unloaded zone is sustained at the zone tip and is thus an integral part of the fatigue zone. Unloading takes place from the zone tip backwards so that cycling would tend to result in the loaded zone moving backwards along the zone to the crack tip. On reaching the crack tip, it would, of course, fracture and be moved back to the zone tip. Such behaviour would result in a tendency to unstable crack growth. The overall average effect can be modelled by the two cases shown in Figure 2 as the first and second type which may be treated as bounds on the behaviour and result in F functions of the form:

$$F_{(i)} = 1 - (1-\alpha)^2\,R^2$$

and:

$$F_{(ii)} = (1-R)^2 + \alpha\,R^2 \qquad (4)$$

In general, type (i) will dominate when there is a single deformation mechanism which can only be sustained at the zone tip and results in F being slowly varying in R for low R values. Type (ii) can only be sustained when there is a distinct separate deformation process around the crack tip, as with shear bands or craze bunches, and gives a rapidly varying R function at low R and $da/dN \propto (1-R)^2\,K^2$, which is a dependence on ΔK. Cases in which both exist are conceivable and would require some average F to accommodate the appropriate proportions. For equal parts, for example, we would have:

$$\bar{F} = (1-R) + \frac{\alpha}{2}(3-\alpha) R^2 \quad (5)$$

For comparison with experimental data, it is convenient to express the parameters in equation (3) in terms of the slopes of the da/dN versus K^2 lines, S, and their intercepts, I. Thus:

$$S = \frac{\pi}{8} \frac{F(R)}{(1-\alpha)^2 \sigma_c^2} \quad \text{and} \quad I = \frac{\alpha K_c^2}{F(R)} + K_o^2$$

and $F(R)$ may be eliminated to give:

$$S I = \frac{\alpha}{(1-\alpha)^2} r_c + K_o^2 S \quad (6)$$

$F(R)$ may be expressed in terms of the slope at $R = 0$ since $F(0) = 1$, so that:

$$F(R) = \frac{S}{S_o} \quad (7)$$

SOME EXPERIMENTAL COMPARISONS

Figure 3 shows slope data for various polymers plotted as S/S_o versus R in accordance with equation (7). Also shown are lines corresponding to equations (4) for various values of α. Several types of behaviour are apparent. For PMMA, we have a close fit to type (i) with a low α value, while polystyrene and the modified PVC are close to type (ii) with low α values. This is sensible in terms of known behaviour since PMMA has a high shear yield stress and tends to form stable single crazes at the crack tip (Marshall, Coutts & Williams, 1974). Polystyrene, on the other hand, is much more prone to craze bunching at the crack tip (Marshall, Culver & Williams, 1973) and modified PVC does give shear yielding and multiple crazing. The unmodified PVC is not fitted by either extreme and suggests a mean F case with a low α value. Not shown on this graph are some data on several epoxy, thermoset resins in which S and I remained constant within experimental scatter up to $R = 0.6$. These are extremely brittle materials with very small plastic zones in which there is only a small reduction in K for fatigue. Thus, for $K_c^2 \simeq 0.36$ $(MN/m^{3/2})^2$ we have intercept values of approximately 0.20, giving $\alpha \simeq 0.6$. This would result in slope changes in type (i) behaviour of only 7% at $R = 0.6$, which is of the order of experimental scatter. Similarly, frequency changes in PMMA down to 1 Hz resulted in a much reduced variation in S, suggesting a substantial increase in α. It is apparent that the determination of α from variation in S is difficult for high values in type (i) and also for low values in type (ii), since the change is dominated by the $(1-R)^2$ effect. Sometimes, the derivation is possible, however, and the PMMA data at 10 Hz is shown in Figure 4, showing a good fit to type (i) behaviour with $\alpha = 0.076$ with $\sigma_c = 1.16$ GN/m^2 and $\alpha\sigma_c = 88$ MN/m^2. The reduction of this very high value to approximately the craze stress has been discussed elsewhere (Williams, 1980) and probably reflects the underlying molecular fracture mechanisms. The scatter in the PS and PVC data

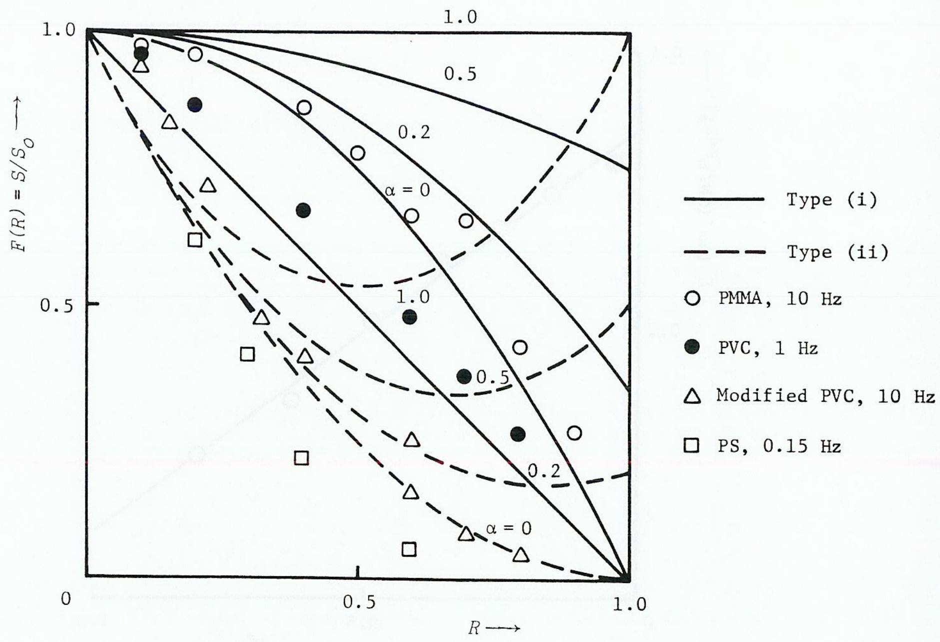


Fig. 3. Slope data for various polymers

preclude any accurate determinations of α.

The intercept (i.e. threshold) data are much less prone to scatter and this is reflected in the data plotted in accordance with equation (6), which is independent of $F(R)$. Figures 5 and 6 show lines for PMMA and PS, respectively, which give good linearity from which $\alpha r_c/(1-\alpha)^2$ and K_o^2 may be found. Table 1 list values for the materials considered here and α values of less than 0.1 are suggested for all the materials, except the epoxy. It is also worth noting that $(K_o/K_c)^2 \simeq 0.2$, which would be expected for a simple residual stress argument.

CONCLUSIONS

The additions to the basic two-stage line zone model which involve complete unloading of part of the unfatigued zone and a residual compressive stress do appear to model observed behaviour in a number of polymers. The apparent complication observed in much published data would appear to arise from quite different forms of crack tip behaviour in which the unloaded zone is either at the crack or the zone tip. There is also some evidence of widely varying values of the damage factor, α, in different materials and perhaps with frequency.

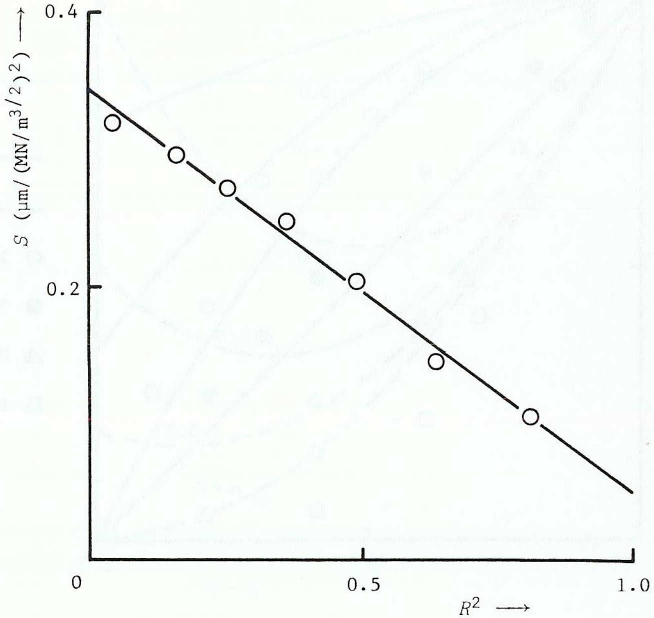


Fig. 4. Slope of da/dN versus K^2 data for PMMA at 10 Hz

TABLE 1

Material and Frequency	K_o^2 $(MN/m^{3/2})^2$	$(\alpha r_c)/(1-\alpha)^2$ (μm)	S_o $\mu m/(MN/m^{3/2})^2$	$\alpha\ K_c^2$ $(MN/m^{3/2})^2$	K_c^2 $(MN/m^{3/2})^2$	α
PMMA 10 Hz	0.18	0.020	0.34	0.059	0.70	0.08
PS 0.15 Hz	0.20	0.350	7.70	0.045	≃ 1	0.045
Modified PVC 10 Hz	0.18	4×10^{-4}	0.027	0.015	≃ 1	0.015
PVC 1 Hz	0.29	0.056	0.17	0.33	≃ 2	0.08
Epoxy 10 Hz	0	0.040	0.18	0.22	0.36	0.61

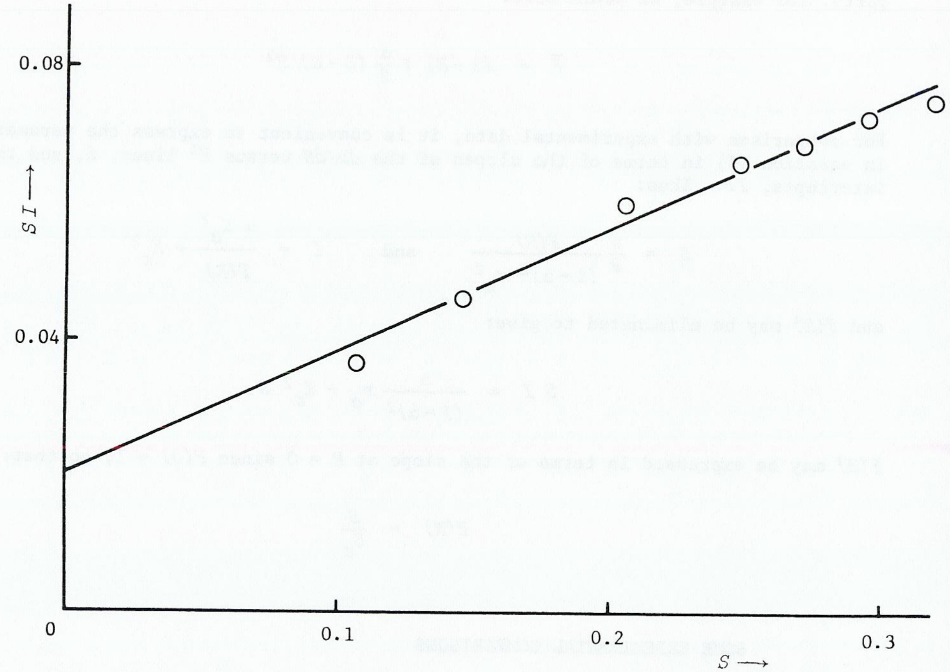


Fig. 5. Slope and intercept data for PMMA at 10 Hz

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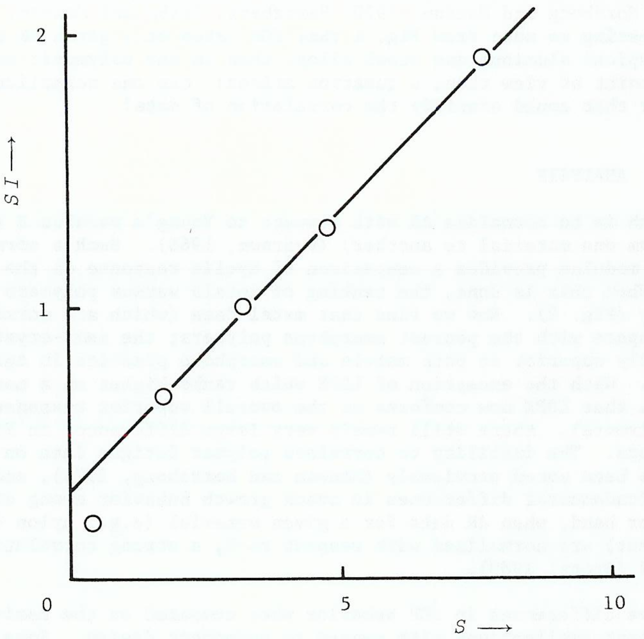

Fig. 6. Slope and intercept data for polystyrene (Mai & Williams, 1979) at 0.15 Hz