FIBRE LENGTH-STRENGTH RELATIONSHIPS AND THE FRACTURE OF COMPOSITES

M.R. Piggott

Centre for the Study of Materials, University of Toronto
Toronto, Canada M5S 1A4

ABSTRACT

The pulled-out lengths of fibres, which are observed when composites fracture, can be shown to be controlled by the fibre flaw distributions. Small flaws in the fibres give fibre strength-length relationship of the form $\sigma_{fu} = AL^{-m}$ where A and m are parameters which are different for different fibres. The pulled-out lengths can be shown to depend on A and m, together with fibre and matrix elastic constants, and the matrix shrinkage stress that arises during composite manufacture. The results explain why high modulus fibres such as boron and carbon make brittle composites, and indicate how they may be made much tougher. This can be effected by choosing a matrix with a very low shrinkage stress.

KEYWORDS

Fibre composites, fracture, toughness, length-strength relationship.

INTRODUCTION

One of the weaknesses of fibre composites is their poor toughness, especially for aligned fibre composites made with high modulus fibres such as boron or carbon. Data for reinforced polymers is reviewed by Piggott (1980), and basic processes contributing to toughness have been critically reviewed by Cooper and Piggott (1977).

Fibre pull-out during the fracturing process is the most important contributor to the work of fracture with aligned fibre composites, but conditions which favour large works of pull-out have disadvantages. Thus low fibre-matrix interfacial shear stresses (produced for example by coating the fibres with oil, Harris et al, (1971), give composites which have large works of fracture, but poor shear properties. Also, the large crack openings required to generate the work of fracture presents a serious problem (Piggott, 1978a).

Harris et al (1971) showed that fibre pull-out occurs during fracture, even when the fibres are continuous. An explanation for this has been advanced for flaw-free fibres by Piggott (1978b). However, the commonly used fibres such as glass and carbon, have flaws which will influence the processes taking place. In this paper, the influence of fibre flaws is discussed, using fibre length-strength relationships that are flaw controlled.

THEORY

Figure 1 shows a region of a propagating crack which is bridged by three fibres, one of which has just broken. Near the break, the fibre stress is quite small, and the loss of fibre stress results in it shrinking longitudinally and expanding radially. As the crack opens under increasing stress intensity factor, the fibre will slide out, and do work, U_f where

$$U_f = \pi r L^2 \tau_i \qquad (1)$$

if the interfacial shear stress is constant and equal to τ_i.

To determine the average value of L we need to know (a) how the stress in an unbroken fibre varies with distance from the crack plane, x, and (b) how the flaws are distributed in the fibres.

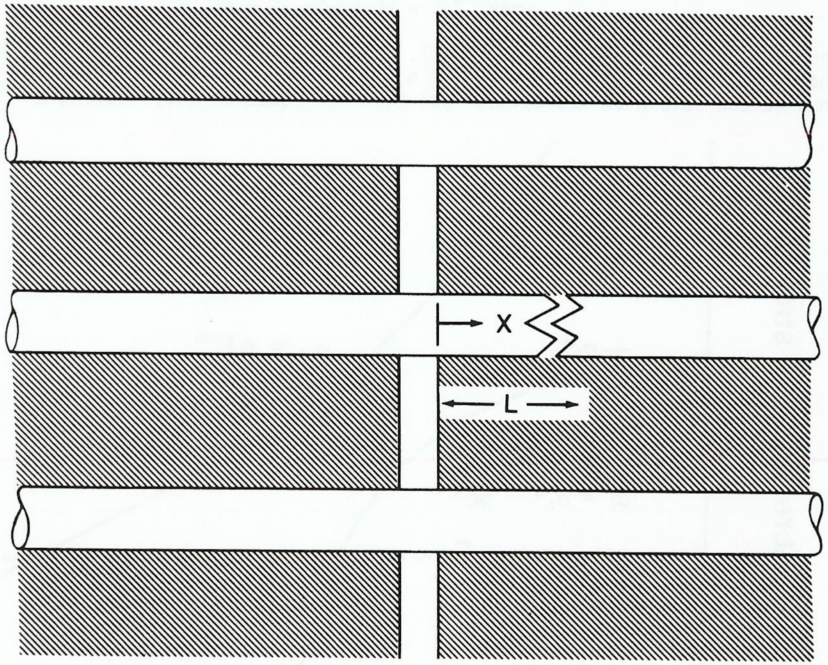

Fig. 1. Fibres bridging a crack. The central fibre has broken at distance L from the crack face.

The fibre stress-distance relationship is derived from the well known equation

$$\frac{d\sigma_f}{dx} = -\frac{2\tau}{r} \qquad (2)$$

where τ is the interfacial shear stress, which for a fibre in a polymer is not constant along the fibre length. It can be shown that at quite low applied stresses, the fibres-matrix bond fails near a discontinuity, such as a fibre end (Piggott, 1978b). Sliding takes place at the interface, governed by the coefficient of friction, μ and the normal stress, σ_{rt} where

$$\sigma_{rt} = -\nu_1 \sigma_m + \nu_2 \sigma_f E_m/E_f + \sigma_r \qquad (3)$$

ν_1 and ν_2 are Poisson's shrinkage coefficients, related to fibre and matrix Poisson's ratios, σ_m is the matrix stress in the x direction, E_m and E_f are matrix and fibre moduli, and σ_r is the residual interfacial stress that is usually present in a reinforced polymer. It is usually negative (compressive) and results from cure shrinkage of the matrix in thermosets, and differential thermal expansion between fibres and matrix in thermoplastics. Near the crack plane σ_m is zero, and since $\tau = -\mu\sigma_{rt}$, we have

$$\frac{d\sigma_f}{dx} = (\nu_2 \sigma_f E_m/E_f + \sigma_r)\, 2\mu/r \qquad (4)$$

so long as

$$\nu_2 \sigma_f E_m/E_f + \sigma_r < 0, \quad \text{i.e.}$$

$$\sigma_f < -\frac{\sigma_r E_f}{\nu_2 E_m} \qquad (5)$$

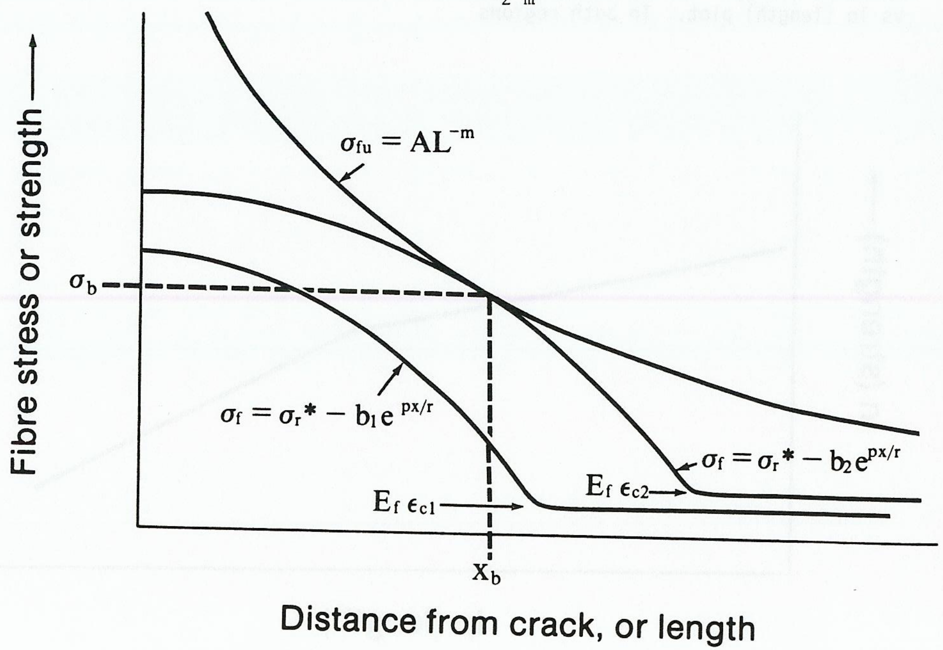

Fig. 2. Schematic drawing of fibre stress as a function of distance from the crack face, $\sigma_f = \sigma_r^* - be^{px/r}$ at two applied stress levels, together with fibre strength-length relationship $\sigma_{fu} = AL^{-m}$

Equation 4 integrates to give

$$\sigma_f = \sigma_r^* - be^{px/r} \tag{6}$$

where

$$\sigma_r^* = -\sigma_r E_f/\nu_2 E_m$$

$$p = 2\mu\nu_2 E_m/E_f$$

and b is determined by the boundary conditions.

Note that b > 0; also a > 0 so long as matrix shrinkage takes place during composite manufacture. Fig. 2 shows σ_f as a function of x. Two curves are shown, corresponding to different boundary conditions. The curves do not extend to zero stress, since in general, the fibre stress will not be zero a long distance from the crack face, but will instead be given by $E_f\varepsilon_c$ where ε_c is the composite strain remote from the crack.

The fibre flaw distribution has not, so far, been examined in detail. Instead, fibre strength vs length has been examined in detail in the case of glass by Metcalf and Schmitz (1974), and in less detail for carbon by Hitchon and Phillips (1978). The measurements generally show two regions in a ln (strength) vs ln (length) plot. In both regions

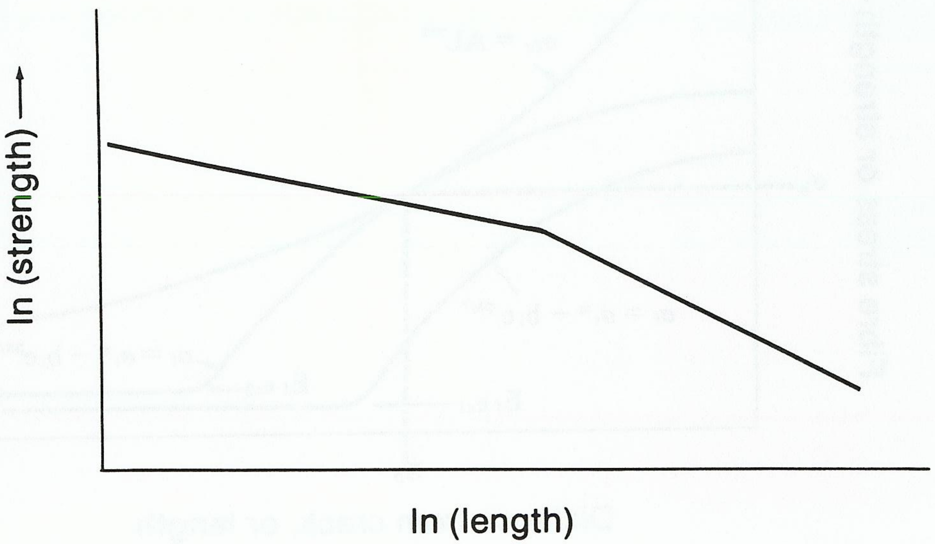

Fig. 3. Type of fibre strength-length relationship observed with glass and carbon fibres.

$$\sigma_{fu} = AL^{-m} \tag{7}$$

but A and m are different in the two regions. Thus lines with two slopes are obtained, as shown in fig. 3. A and m values are different for different fibres, or for the same fibres which have been treated in different ways.

In fig. 2, we have superimposed a curve of the form given in equation 7. It is clear that for the boundary conditions giving $\sigma_f = \sigma_r^* - b_1e^{px/r}$ the fibre stress is nowhere large enough that the breaking point is reached. However, for $\sigma_f = \sigma_r^* - b_2e^{px/r}$, the fibre stress is higher everywhere, due to an increase in load on the cracked specimen, and the fibres can break at a stress σ_b, at a distance x_b from the crack face.

For the two curves to touch as shown, $d\sigma_f/dx = d\sigma_{fu}/dL$, when $L = x_b$. Thus differentiating equation 7 and substituting the appropriate values we obtain

$$-mAx_b^{-m-1} = \frac{2\mu}{r}\left(\nu_2 Ax_b^{-m} E_m/E_f + \sigma_r\right) \tag{8}$$

For glass and carbon m is quite small. For example an E-glass gave m = 0.049 and A = 2.8 x 10^9 for L < 5.9 mm and m = 0.125 and A = 1.9 x 10^9 for L > 5.9 mm. Thus we can obtain an approximate solution for equation 8 by letting $m \simeq 0$, i.e.

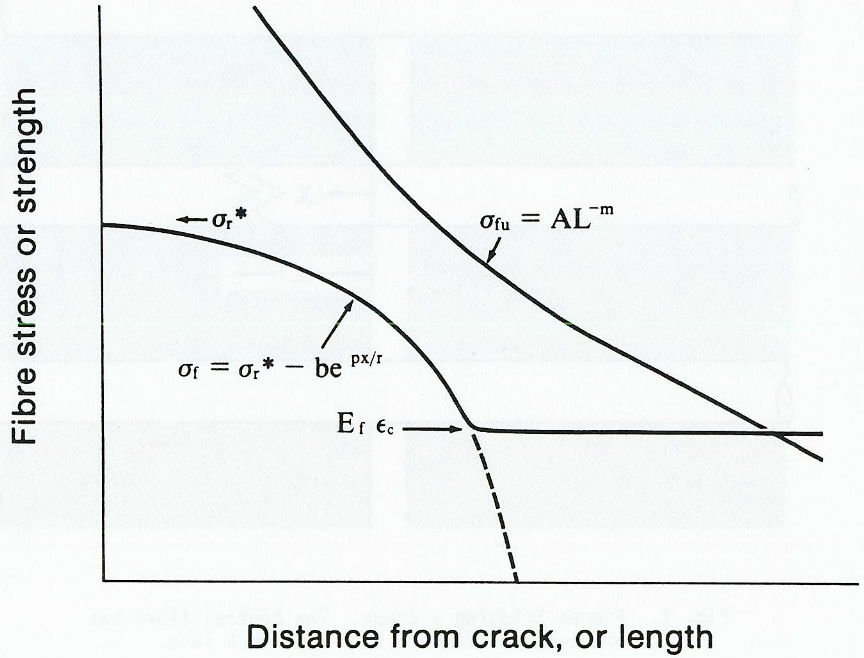

Fig. 4. Schematic drawing of fibre stress vs distance from crack face for relatively low maximum fibre stress. Also shown is fibre strength-length relationship

$$\sigma_{fu} = AL^{-m}$$

$$x_b \simeq \frac{mAr}{2\mu(-\sigma_r - \nu_2 AE_m/E_f)} \quad (9)$$

In the event that equation 9 does not have a positive solution, we have the situation shown in fig. 4. In this case we determine x_b from equation 7, using $\sigma_{fu} = E_1\varepsilon_c$ and $x_b = L$, i.e.

$$x_b = (A/E_f\varepsilon_c)^{1/m} \quad (10)$$

Since m << 1, small values of composite strain, ε_c, will involve very long fibre lengths. If the test specimen is not very large, it may not be possible to break the fibres at all. The maximum fibre stress in this case is given by $\sigma_f = \sigma_r E_f/\nu_2 E_m$ (compare this with equation 5). Clearly for σ_f to be less than σ_{fu}, we need E_f/E_m, and $-\sigma_r$, not to be very large.

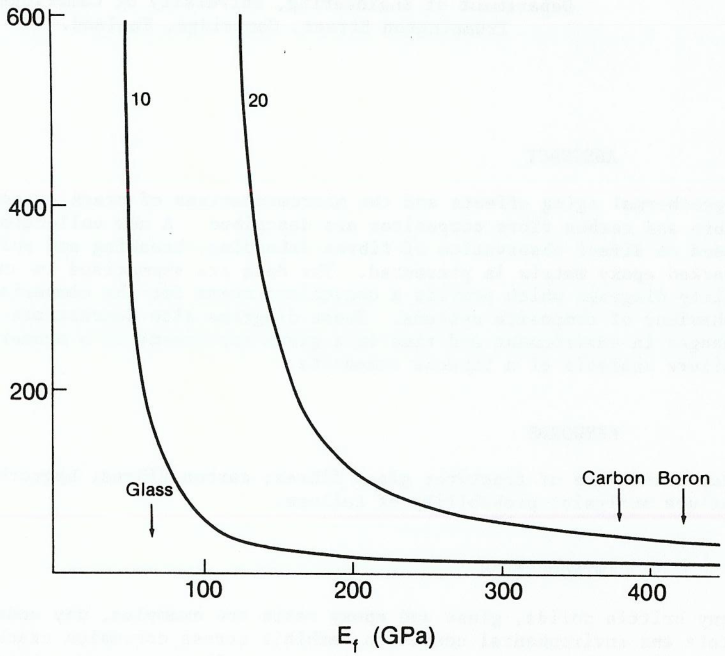

Fig. 5. Work of fracture of aligned fibre composites as a function of fibre Young's modulus for polymer matrix compressive shrinkage stresses of 10 and 20 MPa. The relative positions of boron, carbon and glass fibres are marked.

PRACTICAL IMPLICATIONS

When a fibre has broken, we assume it pulls out at constant shear stress $\tau_i = -\mu\sigma_r$. The work of fracture G, can be calculated from equation 1; for a volume fraction of fibres V_f, $G = 2V_f U_f/\pi r^2$. Replacing τ_i by $-\mu\sigma_r$, L by x_b (equation 9), and re-arranging, we obtain

$$G = \frac{V_f d m^2 A^2(-\sigma_r)}{4\mu(\sigma_r + \nu_2 AE_m/E_f)^2} \quad (11)$$

Fig. 5 shows a plot of work of fracture vs fibre modulus, where it is assumed that we can use $A = 1.9 \times 10^9$, $m = 0.125$, and $\nu_2 = \nu_f$. Results are plotted for two values of σ_r: - 0.01 and - 0.02 GPa. The effect of modulus is very marked, and similar results are obtained if we use the other values of A and m. Although we cannot directly compare carbon, boron, and glass in this graph, the results for σ_r between - 0.01 and - 0.02 GPa do fit in quite well with the observed values for G for these materials Piggott, (1980). (In carbon-epoxies it is considered likely that σ_r has a value close to - 0.02 GPa.) Thus the lack of toughness of boron and carbon reinforced polymers does appear to be due to their high moduli.

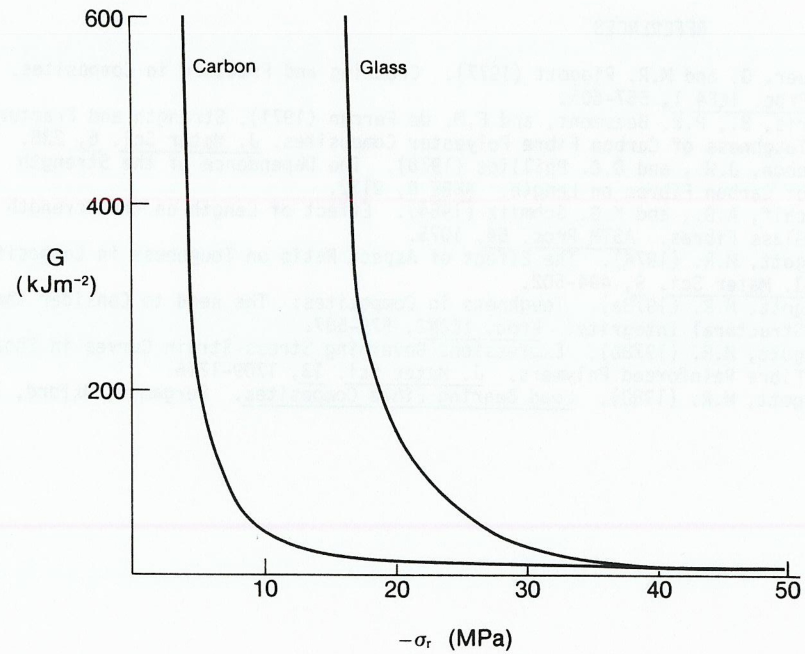

Fig. 6. Work of fracture for aligned fibre composites as a function of matrix shrinkage stress, for carbon and glass reinforced polymers.

Equation 11 suggests a way in which the toughness might be improved. Fig. 6, shows the effect of varying the residual stress. If this can be reduced to about 5 MPa, it should be possible to obtain a carbon composite which is as tough as the normal glass composite. To do this, a polymer with a very low shrinkage is required, or one that is sufficiently soft that the shrinkage stress can be removed by annealing. The toughness can be adjusted by control of the shrinkage stress, and this can be done without loss of other mechanical properties, so long as the fibres are very long Piggott, (1978b). The actual value of σ_r needed depends on A and m however, and these are usually not known for fibres that have been subject to the composite making process. The damage that this causes could well cause them to have degraded values (i.e. m higher and A lower).

CONCLUSION

The length-strength relationships that are observed with brittle fibres such as carbon and glass can be used to explain the observation that high modulus fibres give composites with low works of fracture. It also indicates a method whereby composite toughness might be improved. To do this the interfacial normal stress due to matrix shrinkage during manufacture must be reduced in a controlled fashion.

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