EXAMINATION OF SEVERAL MECHANICAL PARAMETERS TO ANALYSE THE PLASTIC FATIGUE CRACK INITIATION IN GEOMETRICAL CONCENTRATION ZONES AND MECHANICAL NOTCHES

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ABSTRACT

The geometrical concentration zones and the mechanical notches located in the structures subjected to cyclic loadings tend to increase the risk of initiation of a fatigue crack which may grow and become critical.
The engineer must have some criteria allowing him to take into consideration this damage in the structure conception or quantify it in time in the structure life.
We have chosen parameters which are easily computable by construction applied mechanics. Through a comparative study we try to point out the most suitable parameters to predict a number of initiation cycles.

KEY WORDS

Plastic fatigue, crack initiation, structure analysis.

INTRODUCTION

The study of materials cyclic plastic behaviour, the elastic and plastic calculation of strains and stresses in CT notched specimens and the assumptions relevant to the physical mecanisms of initiation process, allow to express some mechanical parameters by a mathematical relation. These parameters will be named initiation parameters Qi.
The experimental behaviour study of CT specimens allows to determine a number of initiation cycles Na related to the geometrical and physical characteristics and the loadings applied to these specimens.
The experimental results analysis allows to identify a mathematical relation between Na, number cycles of initiation and ΔQi, range of Qi variation.
At the end of the study of the best correlation, between theory and experience we can point out the characteristics values of the variables which enter into the definition of the Qi relations.

INITIATION PARAMETERS Qi

Taking into consideration the particular geometry of the CT specimen we assume the notch axis or symmetry axis is the reference axis X. So the suffix Y indicates that we consider the strain or stress component perpendicular to the axis X. This component is also named opening component.

The initiation process is bound to a small damaged volume of material located at the notch tip. The theoretical parameters Qi depend on the approximations relative to this geometrical localization.

MAXIMUM VALUE OF THE ELASTIC STRESS AT THE NOTCH ROOT (Jack and Price, 1970)

$$Q = \frac{KI}{\sqrt{\rho}} = \frac{\sqrt{\pi}}{2} \cdot \sigma_{YO} = \frac{\sqrt{\pi}}{2} \cdot K_t \cdot \sigma_{nominal} \quad (1)$$

introduction of a CRITICAL RADIUS ρ_o (Jack and Price, 1970 ; Rabbe et Amzallag, 1974)

$$Q = \frac{KI}{\sqrt{\rho}} \text{ if } \rho > \rho_o \text{ and } Q = \frac{KI}{\sqrt{\rho_o}} \text{ if } \rho \leqslant \rho_o \quad (2)$$

MAXIMUM VALUE ON θ OF THE ELASTIC STRESS $\sigma_{\theta\theta}$ AVERAGED OVER THE DISTANCE ρ^* (Neuber, 1968)

$$Q = \bar{\sigma}_Y^{\rho^*} = \frac{1}{\rho^*} \cdot \frac{KI}{\sqrt{2\pi}} \cdot \int_0^{\rho^*} \frac{dX}{\sqrt{X}.(1+\rho/2X)} = \frac{2.KI}{\sqrt{\pi(\rho+2.\rho^*)}} = \sigma_{YO}^E \cdot \frac{1}{\sqrt{1+2.\rho^*/\rho}} \quad (3)$$

MAXIMUM VALUE ON θ OF THE ELASTIC STRESS $\sigma_{\theta\theta}$ CALCULATED AT THE DISTANCE d (d'Escatha, 1978)

$$Q = \sigma_Y^E (d) = \frac{KI}{\pi.(\rho+2.d)} \cdot (1 + \frac{\rho}{\rho+2.d}) = \sigma_{YO}^E \cdot \frac{1 + d/\rho}{(1+2.d/\rho)^{3/2}} \quad (4)$$

RANGE OF THE PSEUDO FATIGUE PLASTIC STRESS $\Delta\sigma_Y^{FP}$ (Baus, Lieurade, Sanz et Truchon, 1975)

$$\Delta Q = \Delta\sigma_Y^{FP} = \sqrt{E \cdot \overline{\Delta\sigma_\infty} \cdot \overline{\Delta\varepsilon_\infty}} = K_f \cdot \Delta\sigma_{nominal} \quad (5)$$

the fig. 1 shows the method to calculate $\Delta\sigma_Y^{FP}$. In order to express Kf the authors propose the following relation :

$$K_f = 1 + q \cdot (K_t - 1) \text{ with } q = A \cdot \rho^\alpha \quad (6)$$

VALUE OF THE STABILIZED CYCLIC PLASTIC STRESS σ_Y^{STAB} (Billon, 1980)

We put forward the essential stages of the theoretical step to express σ_Y^{STAB}

. Notations $\sigma_{eqY} = \overline{\sigma_Y}$ and $\varepsilon_{eqY} = \overline{\varepsilon_Y}$ (equivalent values)

. Results of the slipping lines theory applied to the CT notched specimen and extended to a hardening material

$$\sigma_Y = \frac{2}{\sqrt{3}} \cdot \sigma_S^* \cdot [1 + \ln (1 + x/\rho)] \quad (7)$$

. Hypothesis for a rigid material (X small) $\varepsilon_{ij}^t = \varepsilon_{ij}^p$

. Behaviour model .... $\overline{\sigma_Y} = K \cdot (\overline{\varepsilon_Y})^n$

. Particular consequence .... $\overline{\sigma_{YO}} = K \cdot (\overline{\varepsilon_{YO}})^n$

. Associated hardening law .... $\sigma_S^* = K \cdot (\overline{\varepsilon_Y})^n$

. Results of the finite element analysis .... $\overline{\varepsilon_Y} = \overline{\varepsilon_{YO}} \cdot \exp - a \cdot n \cdot x/\rho$

$$\sigma_Y = \frac{2}{\sqrt{3}} \cdot \overline{\sigma_{YO}} \cdot [1 + \ln (1 + X/\rho)] \cdot \exp - a \cdot n \cdot x/\rho \quad (8)$$

. Relation of Neuber type for a given plane state

$$\overline{\sigma_{YO}} \cdot \overline{\varepsilon_{YO}} = h \cdot \overline{\sigma}_{YO}^E \cdot \overline{\varepsilon}_{YO}^E = \frac{h'}{E} \cdot (\overline{\sigma}_{YO}^E)^2 = \frac{h'}{E} \cdot K_t^2 \cdot \sigma_{nom}^2 = \frac{B}{E} \cdot \frac{KI^2}{\rho}$$

. Particular consequence .... $\overline{\sigma_{YO}} = K \cdot (\frac{1}{K} \cdot \frac{B}{E} \cdot \frac{KI^2}{\rho})^{n/n+1}$

$$\sigma_Y = \frac{2}{\sqrt{3}} \cdot K . [\frac{1}{K} \cdot \frac{B}{E} \cdot \frac{KI^2}{\rho}]^{n/n+1} . [1 + \ln (1 + X/\rho)] \cdot \exp - a.n.X/\rho \quad (9)$$

. Stabilized behaviour .... $\overline{\sigma_{Y\infty}} = K_\infty \cdot (\overline{\varepsilon_{Y\infty}})^{n_\infty}$

. Associated cyclic hardening law ...... $\sigma_{S\infty}^* = K_\infty \cdot (\overline{\varepsilon_{Y\infty}})^{n_\infty}$

$$Q = \sigma_Y^{STAB} = \frac{2}{\sqrt{3}} \cdot K_\infty . [\frac{1}{K_\infty} \cdot \frac{B}{E} \cdot \frac{KI^2}{\rho}]^{n_\infty/n_\infty+1} . [1 + \ln (1 + X/\rho)].\exp - a.n_\infty.X/\rho \quad (10)$$

VALUE OF THE CYCLIC STABILIZED PLASTIC DEFORMATION $\overline{\varepsilon}_Y^{STAB}$ (Billon, 1980)

The theoritical path to express ε_Y^{STAB} is the same as the one which expresses σ_Y^{STAB}

$$Q = \overline{\varepsilon}_Y^{STAB} = [\frac{1}{K_\infty} \cdot \frac{B}{E} \cdot \frac{KI^2}{\rho}]^{1/n_\infty+1} \cdot \exp - a \cdot X/\rho \quad (11)$$

The fatigue is bound to the cyclic variation of the loadings and the initiation parameters have to express it by their range of variation ΔQi.
To show this, KI has only to be replaced by ΔKI in the relations where Q is directly proportional to KI, that is to say in the relations (1), (2), (3) and (4). For the relations (10) and (11) where this proportionality does not exist, we assume to substitute 1/2 $\Delta\sigma_Y^{STAB}$ or 1/2 $\Delta\overline{\varepsilon}_Y^{STAB}$ to σ_Y^{STAB} or $\overline{\varepsilon}_Y^{STAB}$ and 1/2 KI to KI.

The constants E, K_∞, n_∞ are some physical characteristics which are defined by the analysis of the material behaviour. We recall that K_∞ and n_∞ are the parameters of the cyclic hardening curve. The constants B and a are identified from the results of the plastic calculation of the CT specimens by the finite element method. They also depend on the studied material nature.

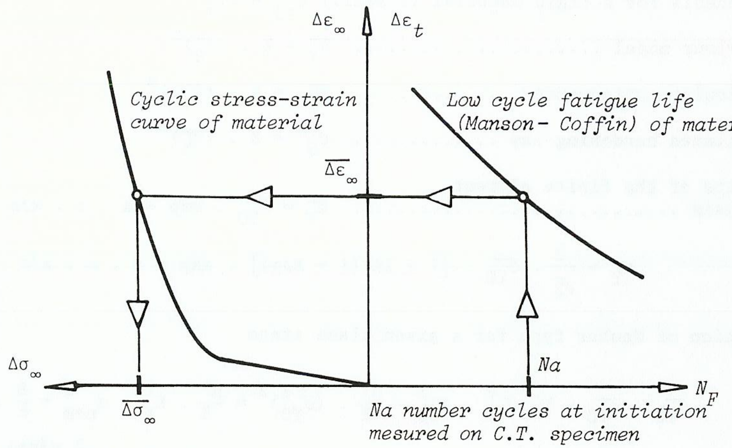

Fig. 1. Calculation method of $\Delta\sigma_Y^{FP}$

CT SPECIMENS EXPERIMENTAL BEHAVIOUR STUDY

The studied material is stainless steel type 316 L (Z3 CND 18.12) at room temperature. The experimental study has been carried out in the "Centre de Recherche d'Unieux laboratories" directed by MM. Rabbe and Amzallag. It has dealt with a large range of CT specimen with notch depths from 14 mm to 38 mm and notch radii from 0,05 mm to 4 mm. The initiation is detected as the origin of the deviation of the signal produced by a gauge displacement measuring the specimen opening fig. 2.

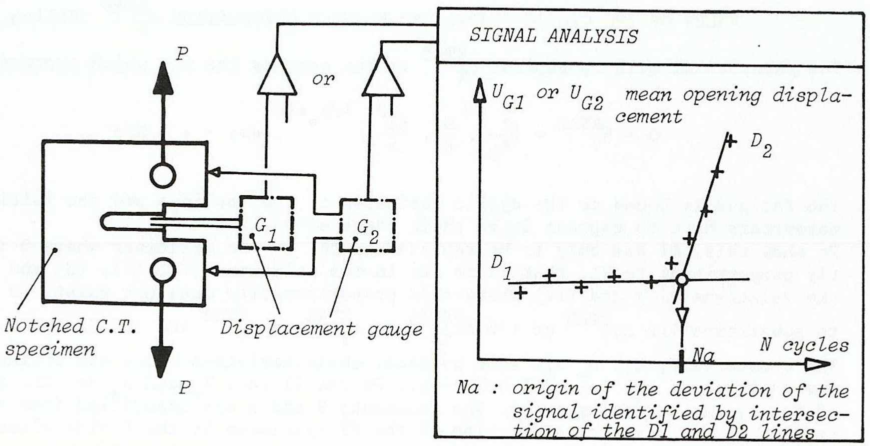

Fig. 2. Principle schema of the experiments

INTERPRETATION OF THE EXPERIMENTAL RESULTS

For each parameter expressing a stress as, $\overline{\Delta\sigma}_Y^{\rho^*}$, $\Delta\sigma_Y^E$ (d), $\Delta\sigma_Y^{FP}$, $\Delta\sigma_Y^{STAB}$, we linearize by the method of the least squares the couples (lnNa, lnΔQ) and then we obtain the equation of the mean straight line :

$$\ln Na = A_0 + A_1 \cdot \ln\Delta Q$$

For each parameter expressing a strain as $\bar{\varepsilon}_Y^{STAB}$, we linearize by the method of the least squares the couples of values (lnNa, ln1/ΔQ) and then we obtain the equation of the mean straight line :

$$\ln Na = A_0 + A_1 \cdot \ln 1/\Delta Q$$

For each analysis above we determine the mean square deviation EQ(lnNa), the maximum deviations δ^+ (lnNa) and δ^-(lnNa) from each side of the mean line and the width of the scatter band L(lnNa) = δ^+(lnNa) + δ^-(lnNa).
Calculating EQ and L for each parameter, we draw, fig. 3 and fig. 4, their variation as a fonction of variables d or ρ^*. The most significant results are related in detail in TABLE 1.

TABLE 1 Expression of Na(ΔQ). Numerical values

	$\Delta\sigma_{YO}^E$ (1)	$\Delta\sigma_Y^{FP}$	$\Delta\sigma_{YO}^{STAB}$	$\overline{\Delta\sigma}_Y^{\rho^*}$ (ρ^*=0,1)	$\Delta\sigma_Y^E$(d=0,05)	$\overline{\Delta\varepsilon}_Y^{STAB}$ (d=0,05)
A_0	28,25	39,2	59,92	36,42	36,49	-2,99
A_1	- 2,62	- 4,34	- 7,49	- 3,85	- 3,88	2,24
δ^+	1,67	0,68	1,67	1,16	0,57	0,62
δ^-	2,05	0,25	2,05	1,48	1,41	1,2
EQ	1,27	0,364	1,27	0,41	0,378	0,49
L	3,72	0,93	3,72	2,64	1,98	1,82

(1) $\sigma_{YO}^E = \bar{\sigma}_Y^{\rho^*}$ (ρ^* = 0) = σ_Y^E (d = 0)

DISCUSSION ABOUT THE RESULTS

The study of the variation of the function L and EQ confirms the theory by which the number of initiation cycles is a function of the elastoplastic strains or the elastic stresses in the case of small scale yielding at the notch tip. The existence of a minimum of the fonction L and EQ, except for $\Delta\sigma_Y^{STAB}$, is in good agreement with the concept of the characteristic distance whose value is d = 0,05 mm for the material we study.
Fig. 5 shows it is very difficult to point out the existence of a critical radius ρ_0.
The value of L and EQ of parameter $\Delta\sigma_Y^{FP}$ represents the absolute minimum ; however, a pratical use of this parameter needs a mathematical calculation of Kf or q, which, as it is showed in Fig. 6, may slightly alter the precision we had by identification with the curve of the material fatigue.

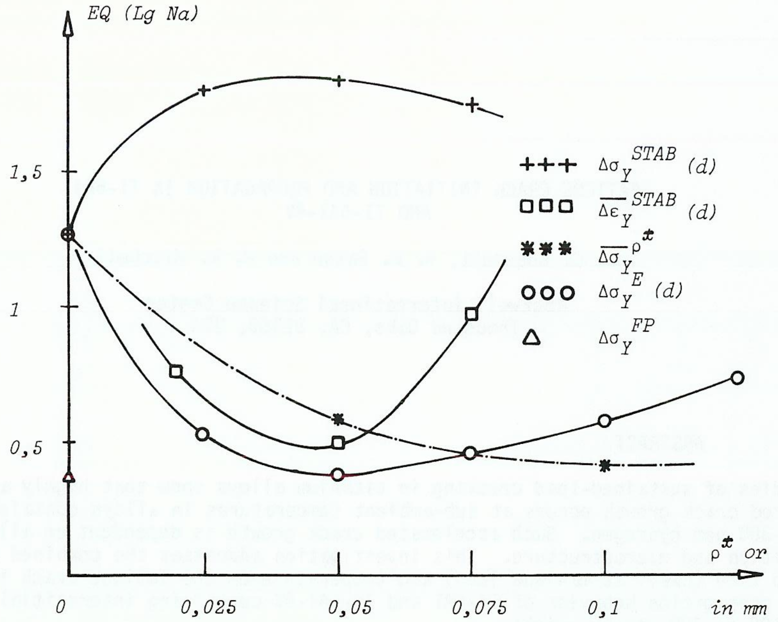

Fig. 3. Variation of EQ as a fonction of ρ* or d

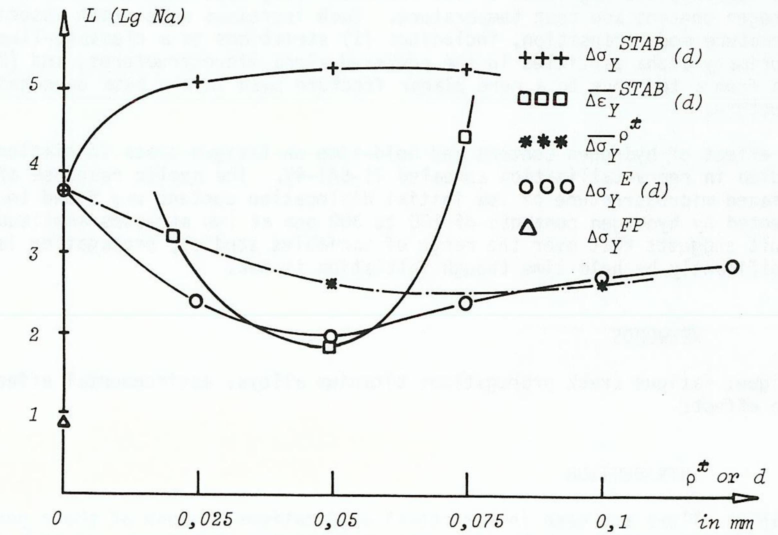

Fig. 4. Variation of L as a fonction of ρ* or d

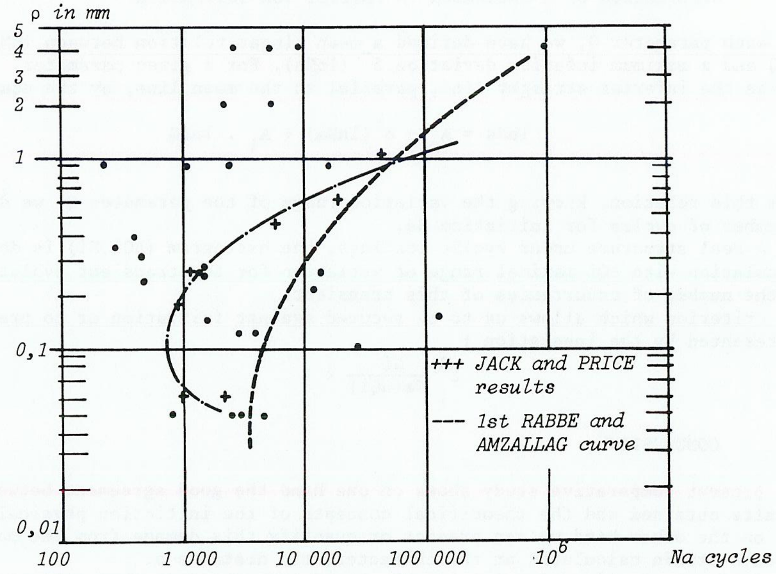

Fig. 5. Variation of ρ as a function of Na

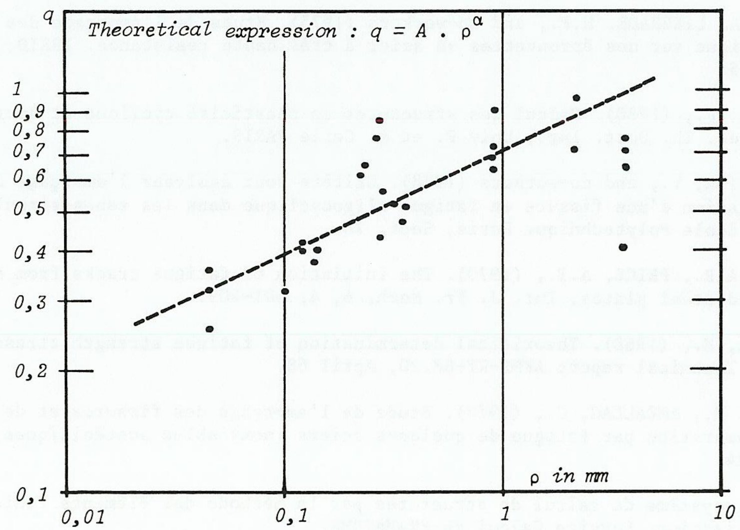

Fig. 6. Variation of q as a function of ρ

APPLICATION OF A CRITERION TO PREDICT THE INITIATION

For each parameter Q, we have defined a mean linear relation between lnNa and lnΔQ and a maximum inferior deviation δ^- (lnNa). For a given parameter, let us define the inferior straight line, parallel to the mean line, by the equation :

$$\ln Na = A_o - \delta^-(\ln Na) + A_1 \cdot \ln \Delta Q \qquad (12)$$

From this relation, knowing the variation range of the parameter Q, we determine a number of cycles for initiation Na.
For a real structure under cyclic loadings, the histogram (ΔQi,Ni) is done after calculation with ΔQi maximal range of variation for the transient evolution i and Ni the number of occurrences of this transient.
The criterion which allows us to be secured against initiation or to predict it is represented by the inequation :

$$\sum_1^R \frac{Ni}{Na(\Delta Qi)} < 1 \qquad (13)$$

CONCLUSION

The present comparative study shows on one hand the good agreement between the results obtained and the theoritical concepts of the initiation physical process and on the other hand we can predict or quantify this damage from the maximum elastic strain calculated at the characteristic distance d.
The interest of this criterion is also based upon the fact that only elastic calculation is required, this is very important to analyse some complicated structures subjected to cyclic thermomechanical sollicitations.

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